MS&E 235, Internet Commerce

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Lecture 3: Introduction to Game Theory

Primer on Game Theory

Imagine N players (in this case bidders) $1, 2......., N$. Each player has a set of actions (Bids) that he can take. Also, each player has some private information that is not known to the other players such as his/her valuation v_i (how much each click is worth to them).

The strategy for each player is a probability distribution over the actions. The strategy may either be a pure strategy (uses a single action) or a mixed strategy (uses multiple actions).

In general, payoff for player i is $f_i(b_1, b_2, ...b_N)$.

In the case of a first price auction, payoff is:

$$= v_i - b_i \text{ if } b_i \text{ is largest}$$

$$= 0 \text{ otherwise}$$

In the case of a second price auction, payoff is:

$$= v_i - 2^{nd} \text{ largest bid if } b_i \text{ is largest}$$

$$= 0 \text{ otherwise}$$

Equilibria

It becomes important to understand the concept of Equilibria in the context of Internet Commerce as games are played by players in real-time. In conventional commerce, there is usually some lag before the players/market reacts to an action whereas in internet commerce, reactions can even be instantaneous.

Dominant Strategy Equilibrium

Strategies $s_1, s_2, ..., s_N$ such that for every player i, s_i is an optimum strategy regardless of others strategies.

Second Price Auction

Making a bid $b_i = v_i$ is the unique dominant strategy equilibrium. (Note: the second price auction is truthful only if done once as if it is going to be repeated, players have an incentive to not bid truthfully the first time to get a sense of what others are bidding).

English Auction

An example of this auction is the one used by eBay. In this auction, buyers name a maximum price they are willing to pay and eBay keeps incrementally bidding on the buyers behalf and bids the lowest possible amount to get the good. eBay also allows the seller to mention a reserve price and if the winning bid is less than the reserve price, the seller keeps the good. Such a mechanism also deters schilling (discussed below). In such auctions, you cannot easily bid higher than your valuation without risking the possibility of having to buy the good.

Note on Schilling

Schilling is fraudulent bidding by the seller (using an alternate registration) or an associate of the seller in order to inflate the price of an item which in the case of the english auction would be to bid a higher price for the item you are selling from a false account that you create.

Next Price Auction

Let $CTR_{i,j}$ = click through rate for advertisement i in position j. Also assume that $CTR_{i,j} = p_j * q_i$ where p_j is the position factor (depends on the slot at which the ad is displayed) and q_i is the quality factor (depends on the quality of the ad). Assume $p_1 = 1$ which implies $q_i = CTR_i$

1. Assume k slots, arrange advertisers in decreasing order of $q_i * b_i$.
2. If A is at position j and B is at position $j + 1$, then cost per click for A equals $(q_B * b_B)/q_A$. This means that cost per click for A is the minimum amount that A has to bid to remain in its current position. Such an auction has elements of a truthful auction as the price that a bidder has to pay is not dependent on his/her bid.

Example

Consider three bidders X,Y and Z. Their quality factors q_i, their bids b_i and resulting cost-per-click are tabulated below.

Bidder	q	b	PPC
X	.5	.5	.2
Y	.1	.5	No Slot
Z	.1	1	.5

Payoffs

π_i is the payoff per impression for advertiser i

$$= CTR_{i,j}(v_i - \text{ price per click for } i)$$

Example: Let $v_A = 1$, $v_B = .95$ and $v_C = .1$. Let $q_A = q_B = q_C = .1$ and $p_1 = 1$ and $p_2 = .8$ (2 slots). Consider situations where $b_B = .95$ and $b_C = .1$. There are three possibilities for bid by A namely $> .95$, between .1 and .95 or $< .1$ (It wont win in this case and payoff = zero).

If $b_A = 1$ say, then it gets the position 1 so price per click is .95, CTR is .1 and therefore, payoff is $.1 * .05 = .005$.

If $b_A = .5$, then it gets the second position so price per click is .1, $CTR = .08$ and therefore payoff is $.08 * (1 - .1) = .072$

Therefore, in this auction there is no dominant strategy equilibrium as $A's$ strategy depends on what $B\&C$ have bid. The right thing to bid depends on what the market is doing.

Nash Equilibrium

A set of strategies $s_1, s_2, ..., s_N$ is in a nash equilibrium if no player has an incentive to unilaterally change strategies (weaker notion but always exists).

Example: Let $v_A = 1$, $v_B = .95$ and $v_C = .1$. Let $q_A = q_B = q_C = .1$ and $p_1 = 1$ and $p_2 = .8$ (2 slots). Consider situations where $b_B = .5$, $b_C = .1$ and $b_A = .11$.

Now, $\pi_B = .1 * (.95 - .11) = .084$ and $\pi_A = .08 * (1 - .1) = .072$. Suppose, A wants to get $B's$ place. Two possibilities are:

1. A over cuts B: Suppose $b_A = 1$, $\pi_A = .1 * .5 = .05$. Since this is lesser than the previous case, A has no incentive to over cut B.

2. B undercuts A: Suppose $b_B = .105$, $\pi_B = .08 * (.95 - .1) = .08 * .85 = .068$ which is lesser than the previous value. Therefore B also has no incentive to undercut A.

This is an example of a nash equilibrium. However, there might be more than one nash equilibria. The goal of any player is to push the system toward an equilibrium in which he has a higher payoff.