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Education

- *Ph.D.*: Stanford University, Department of Philosophy, June 2009 (expected).
- *B.A.*: University of Minnesota at Morris, 2001. Majors: Mathematics, Computer Science, Philosophy.

Academic Interests

- **Areas of Specialization:** Logic (mathematical, philosophical), Philosophy of Mathematics.
- **Areas of Competence:** Epistemology, Philosophy of Science.

Dissertation

Title: *Formal Proofs and Refutations*.

Reading Committee:

- Grigori Mints (Stanford University)
- Solomon Feferman (Stanford University)
- Johan van Benthem (Stanford University)
- Jeremy Avigad (Carnegie Mellon University)

Teaching Experience

Stanford University: Graduate Teaching Assistant

- PHIL 152/252: *Computability and Logic* (2008, 2006)
- PHIL 151/251: *First-order Logic* (2008, 2007, 2006)
- PHIL 150/250: *Basic Concepts in Mathematical Logic* (2006, 2005)

Stanford University Educational Program for Gifted Youth: Instructor

- Non-Euclidean Geometry (2008, 2007)

Professional Activity

- *Assistant Editor* for the journal *Formalized Mathematics* (2008–present).
- *Graduate Student Assistant* for:

  - Stanford Mathematical Logic Seminar (2007–present), and
  - Stanford Workshop on Logical Methods in the Humanities (2008–present).
- *Assistant Editor* for the Stanford Encyclopedia of Philosophy (2007–present).

Awards

- Stanford Center for Teaching and Learning *Centennial Teaching Assistant Award*, 2008.
- *Fulbright Scholar*, Rényi Institute of Mathematics of the Hungarian Academy of Sciences, 2002–3.
- *Scholar of the College*, University of Minnesota at Morris, 2000.

Publications

- "A formal proof of Euler's polyhedron formula", *Studies in Logic, Grammar and Rhetoric* **15**(28), 2008 (in press).
- "The vector space of subsets of a set based on disjoint union", *Formalized Mathematics* **16**(1), 2008 (in press).
- "Euler's polyhedron formula", *Formalized Mathematics* **16**(1), 2008 (in press).
- "The rank+nullity theorem", *Formalized Mathematics* **15**(3), 2007, pp. 137–142.

Presentations

- "A refined proof of Euler's polyhedron formula", *Stanford Mathematical Logic Seminar* (June 2008).
- "A formal proof of Euler's polyhedron formula",
  - *Association for Symbolic Logic/American Philosophical Association Joint Meeting* (March 2008);
  - *Stanford Mathematical Logic Seminar* (November 2007); and
  - *Radboud University Nijmegen Brouwer Seminar* (May 2007).
- "Finite sets and Gödel's incompleteness theorem", *Stanford Mathematical Logic Seminar* (January 2008).
- "Formalizing Euler's polyhedron formula", *Seminar on Formal Mathematics*, University of Bonn (April 2007).
- "Problems and prospects for computer-checked formal proofs", *Stanford Workshop on Logical Methods in the Humanities* (February 2007).
- "What can we do with formal mathematical texts?", *Stanford Humanities Center Workshop on Logical Methods in the Humanities* (November 2005).
- "Kuhn's challenge to the thesis that relativity theory is a generalization of Newtonian mechanics", *Logic in Hungary 2005* (August 2005).
- "An analysis of first-order completeness from a computer-checked proof of Gödel's completeness theorem", *First World Congress on Universal Logic* (unilog 2005), Montreux (April 2005).
- "Tarski's elementary geometry", *Stanford Mathematical Logic Seminar* (May 2004)
- "Decision methods for arithmetical universal-existential sentences" (with Patrick Girard), *Stanford Mathematical Logic Seminar* (October–November 2003).

Coursework

Philosophy

- Metaphysics and Epistemology (*John Perry*)
- Philosophy of Language (*John Perry*)
- Philosophy of Science (*Michael Friedman, Thomas Ryckman*)
- Kant's First Critique (*Lanier Anderson*)
- Hegel's *Phenomenology* (*Allen Wood*)
- Quine (*Dagfinn Føllesdal*)
- Plato's Philosophy of Mathematics (*Julius Moravcsik*)

Logic

- Proof Theory A (*Solomon Feferman*) and B (*Grigori Mints*)
- Recursion Theory A (*Solomon Feferman*) and B (*Ruy de Queiroz*)
- Set Theory A and B (*Sergei Tupailo*)
- Model Theory A and B (*Grigori Mints*)
- Automated Reasoning (*Zohar Manna*)
- History of Set Theory (*Solomon Feferman*)
- Advanced Modal Logic (*Johan van Benthem*)
- Epsilon Calculus (*Grigori Mints*)
- Constructive Mathematics (*Grigori Mints*)
- Finite Model Theory (*Solomon Feferman*)
- Higher-order Logic (*Marc Pauly*)

Dissertation Summary: *Formal Proofs and Refutations*

Two questions drive the dissertation:

- *What can one discover in a formal mathematical theory?*
- *What more do we know of a mathematical theorem when it has been formally proved than that it is provable?*

These questions spring from the provocative philosophy of mathematics of Imre Lakatos. These questions are tackled in two ways: by evaluating the philosophical foundations of Lakatos's work, and by studying contemporary work in formal mathematics.

The dissertation has a technical part and a philosophical part. The first part considers some philosophical problems raised (or brought into focus) by formal mathematical proofs. The second technical part attempts to answer mathematical questions raised in the first part. The bridge between the two is a formal proof of a famous mathematical result known as *Euler's polyhedron formula*, whose history Lakatos has reconstructed and which serves as the central example for his philosophy of mathematics. The aim of the dissertation is to explore some of the philosophical problems suggested by such formalization efforts.

The dissertation is timely because, owing to developments in logic and computing in the last half-century, the concept of a formal proof, which used to be at best a model of mathematical argumentation, has become more concrete and practical. It has now become possible to actually formalize significant mathematical proofs. These contemporary developments are a source of problems for a philosophy of mathematics that is sensitive to mathematical practice. This movement within the philosophy of mathematics is to no small degree inspired by Lakatos's work. The time is ripe for returning to some of the basic philosophical problems that Lakatos and other philosophers pointed to long ago, and to examine new problems that come from the development of what might be called *proof technology*, tools for helping mathematicians construct and evaluate proofs.

In chapter 1, I lay out some of the main questions and problems about formal proofs and explain how they are related to central issues within mainstream philosophy, particularly epistemology and philosophy of science.

The development of proof is based on classical 19th and 20th century results in mathematical logic but depends crucially on computers. Chapter 2 surveys the variety of uses of computers in mathematical practice and discusses the variety of philosophical problems they pose.

The next step in our discussion of formal proofs will be a critical evaluation of the philosophy of Imre Lakatos. His *Proofs and Refutations* (1963) attacks formalist philosophies of mathematics. Since much proof technology is to some extent based on or requires a certain formalist view of mathematics, the question naturally arises how Lakatos's philosophy bears on these developments. Chapter 3 addresses these concerns.

Chapter 4 continues the discussion started in chapter 3 by focusing more specifically on some epistemological problems suggested by formal proofs, such as the question of defining *rigor* and the problem of whether and how one improves one's justification for a mathematical claim through formalization of a proof of it.

The cornerstone of Lakatos's *Proofs and Refutations* is a history of a particular mathematical theorem known as *Euler's polyhedron formula*, which is a certain geometrical-combinatorial claim with a rather colorful history. I have formalized a proof of this mathematical result; chapter 5 contains a discussion of the proof and its formalization.

Thanks to the work carried out in chapter 5, we have that Euler's polyhedron formula (understood in a certain abstract or combinatorial way that is explained in chapter 5) is a first-order consequence a certain first-order theory of sets. Because of the peculiarities of the particular proof technology with which the formal proof was carried out, the theory of sets that is used is much stronger than what is intuitively required for Euler's theorem. A natural proof-theoretic question thereby arises: how can we do better? Are the strong assumptions really necessary? In chapter 6, I carry out the project of trying to identify a weaker theory in which to carry out a formal proof Euler's formula.

In chapter 7, I again carry out some formal work that is naturally suggested by the formal proof. We design a certain first-order theory of sets, called simply *polyhedron theory*, and discuss some metamathematical questions that can be asked of it.

Finally, in chapter 8, I return to some of the issues that Lakatos raised in connection with formal proofs in light of the formal work that is carried out in chapter 5. We then have the resources for taking on the two questions that were initially asked. I show that Lakatos's philosophy, its strong reservations against 'formalism' notwithstanding, applies quite naturally to formal mathematics.