A Spatial Channel Model for Multiple-Antenna Systems

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Abstract – Conventional multi-input multi-output (MIMO) models are not efficient to assess the benefits of multiple-antenna systems with area limitation on antenna arrays in realistic propagation environments. Physical models, though capturing these factors, are too complex for tractable analysis. This paper presents a spatial channel model that takes into account these factors without sacrificing analytical tractability. In the model, the Green function for radiation is adapted to account for the array area limitation, and the clustering of propagation paths observed in recent spatial channel measurements is exploited to obtain a concise description of channel scattering.

I INTRODUCTION

Multiple-antenna systems improve the performance by exploiting the scattering nature of physical environments. They separate scattered paths into parallel spatial channels to increase the data rate (spatial multiplexing gain or degrees of freedom), resolve scattered paths where data symbols can be coded across these paths to improve the link quality (diversity gain), and focus the energy transfer on directions of strong scattered paths to extend the range (power gain). All these benefits can be obtained simultaneously. A fundamental question therefore arises, given an *area* limitation on antenna arrays and a *scattering condition*, what is the available performance gain from using multiple-antenna systems? Conventional spatial channel models, however, are not adequate to answer this question. Earlier results focus on the statistical MIMO model where the number of transmit and receive antennas is given (not the array area). Both scattering condition and antenna spacing are captured by the correlation across all pairs of transmit and receive antennas. This approach has two basic inadequacies:

- The number of parallel spatial channels available is limited by the minimum of the number of transmit and receive antennas. This limit can be achieved by either large antenna spacing or a very scattered environment. However, when the area of wireless terminals is limited, increasing the antenna spacing decreases the number of antennas and the number of parallel spatial channels available is not addressed by the MIMO theory.
- The scattering condition is usually inferred by the correlation across antennas, for example, the i.i.d. fading model and the correlated fading model. However, systems with larger antenna spacing in a less scattered environment yield the same correlation as systems with smaller antenna spacing but in a more scattered environment. As a result, using the antenna correlation to quantify the scattering condition is ambiguous. Furthermore, those assumptions about the correlation are often adjusted to make the problem analytically tractable and thus lack of direct connection to the underlying scattering mechanisms.

This paper addresses these inadequacies in existing analytical channel models. The proposed spatial channel model has two key components:

- It is built on the Green function for radiation in electromagnetic theory in order to capture the array area limitation. This Green function is commonly derived for a given shape of the radiating system (antenna array) in a particular coordinate system. To derive a generic spatial channel model, we adopt a coordinate-free approach and the corresponding Green function is derived in [1].

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- To characterize channel scattering, the model focuses on how scattering sources affect the direction of propagation of radiated field and how much of the radiated field reaches the receiver. Similar to any physical model, this approach can place the analytical tractability in jeopardy. Observations from spatial channel models are therefore incorporated to reduce the complexity without losing critical information about the channel.

As indicated, the proposed model emphasizes a separation of antenna array and channel scattering characteristics which is not addressed in the MIMO model. This separation of controllable quantities such as the number of antennas and their spacing from the uncontrollable quantity, the channel itself, allows system designer to better understand and optimize the system performance.

The rest of the paper is organized as follows. Section II presents the proposed channel model. Its analytical tractability is addressed in Section III. Finally, we conclude this paper in Section IV.

II CONTINUOUS MULTIPLE-ANTENNA CHANNEL MODEL

We consider continuous arrays which are composed of an infinite number of antenna elements separated by infinitesimal distances so as to decouple the scattering condition from the number of antennas and their relative positions on the arrays. Each antenna element is composed of three dipoles oriented orthogonally to each other. This antenna topology is often referred as a tripole where arbitrarily polarized electric fields can be generated. In a frequency non-selective fading channel, the transmit and receive signals, $\mathbf{x}(\mathbf{p})$ and $\mathbf{y}(\mathbf{q})$ which are 3×1 complex vectors due to the use of tripole antenna elements, at a particular time are related by

$$\mathbf{y}(\mathbf{q}) = \int \mathbf{C}(\mathbf{q},\mathbf{p})\,\mathbf{x}(\mathbf{p})\,d\mathbf{p} + \mathbf{z}(\mathbf{q})$$

where $\mathbf{z}(\mathbf{q})$ is the additive noise. The channel response $\mathbf{C}(\mathbf{q}, \mathbf{p})$, a 3×3 complex matrix, gives the signal arrived at point $\mathbf{q}$ of the receive array due to a unit point source applied at point $\mathbf{p}$ of the transmit array. It can be further expanded as

$$\mathbf{C}(\mathbf{q},\mathbf{p}) = \iint \mathbf{A}^{H}(\hat{\boldsymbol{\kappa}},\mathbf{q})\,\mathbf{H}(\hat{\boldsymbol{\kappa}},\hat{\mathbf{k}})\,\mathbf{A}(\hat{\mathbf{k}},\mathbf{p})\,d\hat{\mathbf{k}}\,d\hat{\boldsymbol{\kappa}}, \quad (\mathbf{q},\mathbf{p}) \in \mathcal{A}_r \times \mathcal{A}_t \qquad (1)$$

The transmit/receive array response $\mathbf{A}(\hat{\mathbf{k}},\mathbf{p})$ maps the signal on the array to the radiated field in the direction $\hat{\mathbf{k}}$, and $\mathcal{A}_t$ and $\mathcal{A}_r$ denote the surface of the transmit and receive arrays respectively (array area limitations). The $\mathbf{H}(\hat{\boldsymbol{\kappa}},\hat{\mathbf{k}})$ is the scattering response which gives the signal received from direction $\hat{\boldsymbol{\kappa}}$ due to an impulse radiated to direction $\hat{\mathbf{k}}$. In this paper, we distinguish the scattering response $\mathbf{H}(\hat{\boldsymbol{\kappa}},\hat{\mathbf{k}})$ from the channel response $\mathbf{C}(\mathbf{q},\mathbf{p})$. The former describes the scattering in the physical environment while the latter includes the array responses as well.

The electric field at $\mathbf{k}$ due to a point source at $\mathbf{p}$ is given by [1]

$$\mathcal{G}(\mathbf{k},\mathbf{p}) = \frac{j\eta e^{j2\pi r/\lambda}}{2\lambda r}\left[\left(\mathbf{I}-\hat{\mathbf{r}}\hat{\mathbf{r}}^{H}\right)+\frac{j}{2\pi r/\lambda}\left(\mathbf{I}-3\hat{\mathbf{r}}\hat{\mathbf{r}}^{H}\right)-\frac{1}{(2\pi r/\lambda)^{2}}\left(\mathbf{I}-3\hat{\mathbf{r}}\hat{\mathbf{r}}^{H}\right)\right]$$

where $\mathbf{r} = \mathbf{k} - \mathbf{p}$ and r denotes the magnitude of $\mathbf{r}$. The Green function consists of three terms: far-field, intermediate-field and near-field. Only the far-field term corresponds to the radiated field as it falls off inversely as the distance apart r, and hence its power follows the inverse square law. The power of the remaining two terms falls off much faster than r^{-2} so they do not contribute to electromagnetic radiation. The array response is therefore

$$\mathbf{A}(\hat{\mathbf{k}},\mathbf{p}) = \frac{j\eta e^{j2\pi r/\lambda}}{2\lambda r}\left(\mathbf{I}-\hat{\mathbf{r}}\hat{\mathbf{r}}^{H}\right) \qquad \text{and} \qquad \mathbf{r} = d_0\hat{\mathbf{k}} - \mathbf{p}$$

The reference distance d_0 is chosen such that $d_0 \gg p$ for all position vector on the array, the far-field region. This implies that the array response can be simplified to

$$\mathbf{A}(\hat{\mathbf{k}},\mathbf{p}) = \frac{j\eta e^{j2\pi d_0/\lambda}}{2\lambda d_0}\left(\mathbf{I}-\hat{\mathbf{k}}\hat{\mathbf{k}}^{H}\right)\exp\left(-j2\pi\hat{\mathbf{k}}^{T}\mathbf{p}\right)$$

The $\mathbf{I}-\hat{\mathbf{k}}\hat{\mathbf{k}}^H$ is a 3×3 rank 2 matrix and constrains the oscillation direction of radiated field to be perpendicular to its propagation direction. The term $\exp(-j2\pi\hat{\mathbf{k}}^T\mathbf{p})$ relates the propagation direction $\hat{\mathbf{k}}$ of the radiated field to the excitation position $\mathbf{p}$ on the array.

From spatial channel measurements [2], we observe that scattered paths are clustered around a number of angular intervals in the propagation space. For example, indoor channel measurements reported that there are on average of 3 clusters, each with angle varying from 20° to 40°. Instead of encapsulating all scattered paths individually such as in the ray-tracing model, we use the union of these angular intervals to parameterize the scattering condition which reduces the complexity in the analysis without losing critical information. Thus, the scattering response must satisfy

$$\mathbf{H}(\hat{\kappa},\hat{\mathbf{k}})\neq 0, \qquad \text{only if } (\hat{\kappa},\hat{\mathbf{k}})\in\Omega_r\times\Omega_t$$

where Ω_t and Ω_r are the union of the angular intervals subtended by the scattering sources as observed from the transmitter and the receiver respectively (see Figure 1). To quantify the scattering condition, we use the measures, $|\Omega_t|$ and $|\Omega_r|$ which are the respective solid angles of Ω_t and Ω_r. These measures have the desired property of being insensitive to the array configuration. The underlying scattering mechanism (reflection/refraction and diffuse scattering/diffraction) then characterizes the well-conditionedness of $\mathbf{H}(\hat{\kappa},\hat{\mathbf{k}})$ within $\Omega_r\times\Omega_t$.

III DISCRETE REPRESENTATION FOR THE CONTINUOUS MODEL & PERFORMANCE ANALYSES

The proposed channel model, as it is continuous, may seem abstract and cumbersome at first. A mathematical tool that makes it more convenient for analysis is now addressed. As the scattering response $\mathbf{H}(\hat{\kappa},\hat{\mathbf{k}})$ is sandwiched between

$$\mathbf{A}(\hat{\mathbf{k}},\mathbf{p}),\quad (\hat{\mathbf{k}},\mathbf{p})\in\Omega_t\times\mathcal{A}_t \qquad \text{and} \qquad \mathbf{A}(\hat{\mathbf{k}},\mathbf{p}),\quad (\hat{\mathbf{k}},\mathbf{p})\in\Omega_r\times\mathcal{A}_r$$

which are non-zero and square integrable, concepts from functional analysis are applied to obtain the following spectral decomposition:

$$\begin{aligned}&\mathbf{A}(\hat{\mathbf{k}},\mathbf{p}),\quad (\hat{\mathbf{k}},\mathbf{p})\in\Omega\times\mathcal{A}\\ &=\sum_{n=1}^{\infty}\sigma_n\eta_n(\hat{\mathbf{k}})\xi_n^*(\mathbf{p})\left[\hat{\mathbf{u}}_1(\hat{\mathbf{k}})\hat{\mathbf{u}}_1^T(\hat{\mathbf{k}})+\hat{\mathbf{u}}_2(\hat{\mathbf{k}})\hat{\mathbf{u}}_2^T(\hat{\mathbf{k}})\right]\end{aligned}$$

In the expression, $\mathcal{A}$ can be the transmit or receive array surface, and Ω is the corresponding scattering interval. The sets $\{\eta_n(\hat{\mathbf{k}})\}$ and $\{\xi_n(\mathbf{p})\}$ are orthonormal, $\{\sigma_n\}_{n=1}^{\infty}$ is a sequence of non-increasing positive numbers, and $\hat{\mathbf{u}}_i(\hat{\mathbf{k}})$'s are 3×1 orthonormal vectors. This expansion is equivalent to the singular value decomposition on finite dimensional matrices and σ_n's are the singular values.

Expanding the scattering response $\mathbf{H}(\hat{\kappa},\hat{\mathbf{k}})$, input signal $\mathbf{x}(\mathbf{p})$ and output signal $\mathbf{y}(\mathbf{q})$ in terms of these orthonormal functions yields a discrete representation. Defining

$$\mathbf{H}_{nm}=\iint\eta_{r,n}^*(\hat{\kappa})\left[\hat{\mathbf{u}}_1(\hat{\kappa})\hat{\mathbf{u}}_1^T(\hat{\kappa})+\hat{\mathbf{u}}_2(\hat{\kappa})\hat{\mathbf{u}}_2^T(\hat{\kappa})\right]\mathbf{H}(\hat{\kappa},\hat{\mathbf{k}})\left[\hat{\mathbf{u}}_1(\hat{\kappa})\hat{\mathbf{u}}_1^T(\hat{\kappa})+\hat{\mathbf{u}}_2(\hat{\kappa})\hat{\mathbf{u}}_2^T(\hat{\kappa})\right]\eta_{t,m}(\hat{\mathbf{k}})\,d\hat{\mathbf{k}}d\hat{\kappa}$$

$$\mathbf{x}_m=\int\mathbf{x}(\mathbf{p})\,\xi_{t,m}^*(\mathbf{p})\,d\mathbf{p},\quad \mathbf{y}_n=\int\mathbf{y}(\mathbf{q})\,\xi_{r,m}^*(\mathbf{q})\,d\mathbf{q},\quad \text{and}\quad \mathbf{z}_n=\int\mathbf{z}(\mathbf{q})\,\xi_{r,m}^*(\mathbf{q})\,d\mathbf{q}$$

the input-output model in (1) can be expressed as

$$\mathbf{y}_n=\sum_{m=1}^{\infty}\sigma_{r,n}\sigma_{t,m}\mathbf{H}_{nm}\mathbf{x}_m+\mathbf{z}_n,\quad n\in\{1,2,\cdots\}$$

Note that $\mathbf{H}_{nm}$'s are 3×3 matrices but of *rank 2* only. Now, the performance of a multiple-antenna system is critically determined by

- the distributions of $\sigma_{r,n}$ and $\sigma_{t,m}$,
- the rank of $\mathbf{H}_{nm}$, and
- the well-conditionedness of the infinite dimensional matrix $[\mathbf{H}_{nm}]$.

The distributions of $\sigma_{r,n}$ and $\sigma_{t,m}$ are entirely determined by the physical constraints: array area $\mathcal{A}_t$ and $\mathcal{A}_r$, and scattering condition Ω_r and Ω_t. Figure 2 plots the distribution of σ_n^2 for a linear array of various lengths in a physical environment with 3 scattering clusters, each of angles 30°. The number of non-negligible σ_n^2 limits the number of parallel spatial channels available. In [1] we show that the spatial degrees of freedom is

$$\frac{1}{\lambda^2}\min\{2|\mathcal{A}_t||\Omega_t|, 2|\mathcal{A}_r||\Omega_r|\}$$

The rank of $\mathbf{H}_{nm}$ gives the polarization gain which is always 2 and obviously is independent of the array geometry and channel scattering. The well-conditionedness of $[\mathbf{H}_{nm}]$ depends on the underlying scattering mechanisms and is addressed in [3].

IV CONCLUSIONS

In this paper, we present a more realistic but analytically tractable spatial channel model which has been used in [1] and [3] to determine the spatial degrees of freedom and the information theoretical capacity. Currently, we are setting up a time-domain spatial channel measurement campaign to characterize how Ω_t/Ω_r change with the propagation range, carrier frequency and physical environments, the variation of Ω_t/Ω_r over the frequency bandwidth of the system, and the correlation on Ω_t/Ω_r between users in a network environment. This characterization together with the proposed model helps the design of practical multiple-antenna systems.

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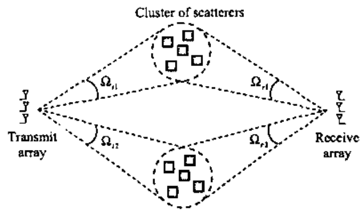
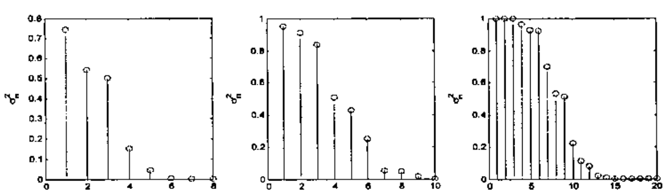


Figure 1 – Illustrates the clustering in transmit and receive signals.



(a) $\mathcal{A} = 2\lambda^2$ (b) $\mathcal{A} = 4\lambda^2$ (c) $\mathcal{A} = 8\lambda^2$

Figure 2 – Plots the distribution of σ_n^2 for different $\mathcal{A}$ in a channel with 3 clusters, each of angle 30°.