Stanford University Mathematical Organization (SUMO) Speaker Series

An Infinity of Worlds: Exploring Manifolds and Their Classification

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Abstract

Though Earth is a sphere (regardless of what some corners of the internet may try to tell you!), in daily life we often act as though our immediate vicinity is a plane: we lay out streets on coordinate grids, speak of the cardinal directions as a pair of axes, and rely on maps to provide accurate information about our surroundings.

This approximation is justified as the sphere is an example of what mathematicians call a "Manifold," a space which near every point locally looks like $\mathbb{R}^n$, but globally may be more complicated.

Manifolds are ubiquitous in modern mathematics, from the familiar low dimensional examples of curves and surfaces in calculus, to higher dimensional abstract examples in geometry, physics, data science and beyond. They come in a bewildering variety - and even the basic question of "what kinds of manifolds are possible in each dimension" remains an active area of research.

Over the past almost two centuries, incredible progress has been made on this classification. This has resulted in a rather clear picture in low dimensions, and a clear demarcation of the obstacles we must confront as the dimension increases. And while the arguments involved can get quite technical, the big-picture story is a beautiful interplay of shape, symmetry and geometry which deserves to be more widely known. Together we will visit some of the best understood parts of this ongoing story.