Math Problems

Prove that you can choose 2^k numbers from the set $\{1, 2, 3, \ldots, 3^k-1\}$ in such a way that the chosen set contains no averages of any two of its elements.

The integers from 1 to 999999 are partitioned into two groups: those integers for which the nearest perfect square is odd and those integers for which the nearest perfect square is even. For which group is the sum of all its numbers greater?

Last week's math mingle contained the following interesting problem:

If one wishes to color the integers so that every two integers that differ by a factorial get different colors, what is the fewest number of colors necessary?

It turns out the answer is 4. There is a paper solving the problem and a generalization of it where you replace the factorials by any infinite increasing sequence of integers:

http://poisson.dm.unipi.it/~daurizio/4Colors.pdf

Kinkonkan

Place exactly one diagonal mirror in each bolded region such that if a ray of light were shot out from each letter on the border of the grid, the ray of light would reflect off of the mirrors and hit the other corresponding letter on the border of the grid. The number next to each letter specifies exactly how many mirrors the ray of light must reflect off of before hitting the other corresponding letter.