Probabilistic Cost of Light Models for Solid State Lighting in General Illumination Markets

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Introduction

The United States consumed 99.87 quadrillion BTUs of primary energy in 2006[1]. The most recent data indicate that 8% was dedicated to artificial illumination[2]. Meanwhile the past five years saw solid state lighting (SSL) technologies improve dramatically. In fact, experts have long predicted that white light emitting diodes (LEDs) have the potential to double the efficacy[3] of the most efficient artificial illumination in the market today[4]. Renewed interest in energy efficiency focuses policy makers, utilities and technology companies on efficiency improvements to general illumination, including the dynamics of the switch from vacuum tubes to semiconductors. We begin with an overview of the United States lighting market and recent lighting policy. We end with a research question and a probabilistic model of the cost of light that compares LEDs to traditional lighting technologies across various sectors of the economy.

Research Questions

Given the uncertainty about the rate of improvement of SSL, when do we expect white LEDs to be more economical than the existing competing technology? What are the key parameters that influence the cost of light in various lighting applications in various markets? If a manufacturer were to focus on one facet of SSL technology to quickly reduce the cost of light, what should it be? We attempt to answer these questions by probabilistically modeling the cost of light.

Lighting Background

United States Lighting Market

There is a dearth of public data about world energy used for lighting. Fortunately, data is available for the United States that give insight about national usage patterns. The United States has one of the world's highest per-capita energy-use, and lighting is a major contributor to that share. Of the world's approximately 460 quadrillion BTU[5] annual primary energy-use in 2005,

[1] Energy Information Administration, *Annual Energy Review 2006,* June 2007, page 3.
[2] Navigant Consulting, *U.S. Lighting Market Characterization: Volume I: National Lighting Inventory and Energy Consumption Estimate*. [Prepared for the U.S. Department of Energy], September 2002, pp 36, 50.
[3] See appendix for glossary of lighting terms.
[4] A. Romig, *Statement to U.S. Senate Committee on Energy and Natural Resources*, Albuquerque, N.M., December 2002.
[5] Energy Information Administration, *International Energy Annual*, 2007.

approximately 100 quads were consumed by the United States. Of that 100, an estimated 8 quads are contributed to lighting[6]. Moreover, lighting is estimated to account for 22% of total electricity usage.

Lighting is divided into four market sectors: residential, commercial, industrial and outdoor stationary. Each sector is dominated by one of the "big-three" lighting technologies in terms of annual lighting service. According to a 2002 Department of Energy sponsored study[6], fluorescent lamps[7] account for a majority of the lumen-share[7] in the United States [Figure 1].

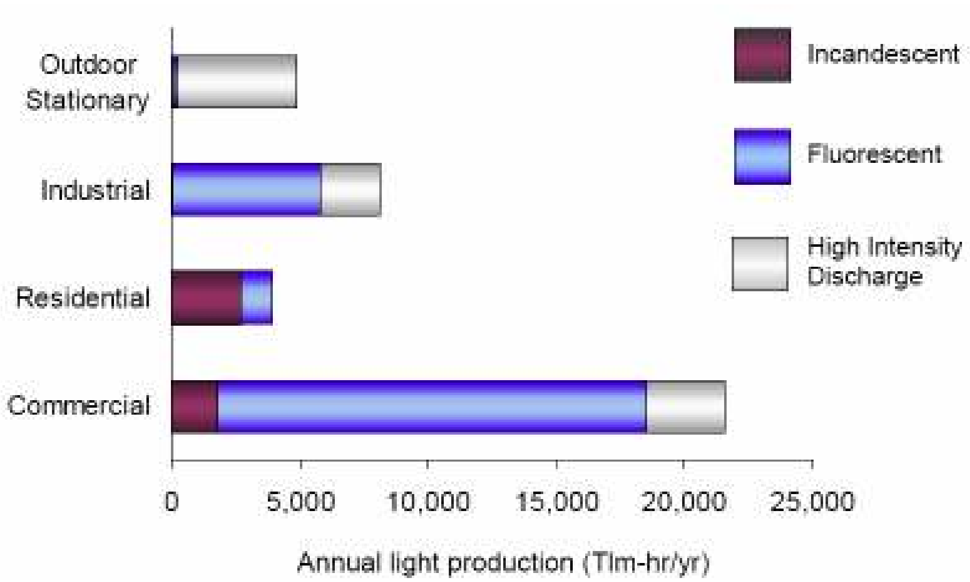

Figure 1:Annual US Lighting Service by Sector and Source [Navigant 2002]

Taking Figure 1 and dividing each technology's light production by its corresponding average efficacy yields the total energy consumed in each sector attributed to each technology [Figure 2]. These two charts show that while fluorescent lamps are the clear leader in lumen share in the United States, a large amount of the energy used for artificial illumination is dedicated to the less-efficacious incandescent technology.

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[6] Navigant Consulting, *U.S. Lighting Market Characterization: Volume I: National Lighting Inventory and Energy Consumption Estimate*. [Prepared for the U.S. Department of Energy], September 2002.
[7] See appendix for glossary of lighting terms

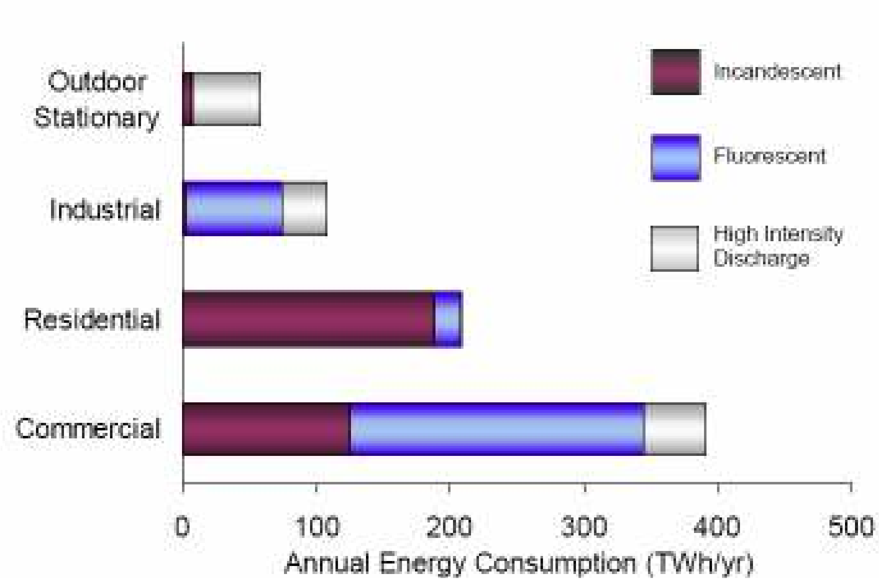

Figure 2: Annual US Energy Consumption by Sector and Source [Navigant 2002]

Externalities

Two market externalities are often attributed to artificial illumination. Since electricity powers lamps, all external costs associated with power production are also associated with artificial illumination. For example, using a 100 Watt bulb for an hour causes a marginal increase in air pollutants released from the natural gas fired power plant that supplied the 0.1 kWh of electricity to power it. Light pollution is the other commonly noted externality—no longer solely the complaint of stargazers and astronomers, recent studies show the effect of light pollution on human health[8]. Light pollution legislation is mostly limited to state and local government, but lighting energy-use legislation is common at the federal level.

Recent United States Lighting Policy

The Energy Policy Act of 2005

This legislation was signed into law in August of 2005 by President George W. Bush. It specifically addressed energy efficient lighting in three ways: it created a next generation lighting initiative, it modified tax code, and it outlined an energy efficiency study of using passive solar radiation to supplement artificial illumination.

The next generation lighting initiative's objective is *to develop advanced solid-state organic and inorganic lighting technologies based on white light emitting diodes that, compared to incandescent and fluorescent lighting technologies, are longer lasting, are more energy-efficient and cost-competitive, and have less environmental impact.*[9] The initiative mandates that the Secretary of Energy award research grants across national laboratories, universities, and industry

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[8] Many research reports on light and health are available from the Lighting Resource Center at Rensselaer Polytechnic Institute http://www.lrc.rpi.edu/programs/lightHealth/index.asp

[9] United States Congress, *Energy Policy Act of 2005*, sec. 931.

for the advancement of SSL technologies, and to lay provisions for the Next Generation Lighting Industry Alliance (NGLIA). The NGLIA is an SSL research consortium tasked to promote *the understanding, implementation, and adoption of semiconductor light sources in specialty and general lighting systems*.[10] It is very active today and has members that include multinational corporations that conduct SSL RD&D, infrastructure or manufacturing activities within the United States.

The Secretary of Energy is also to initiate a program of RD&D for *lighting systems that integrate sunlight and electrical lighting in complement to each other in common lighting fixtures for the purpose of improving energy efficiency*, ostensibly targeted to advance daylighting technologies.[11] Tax code is modified to award deductions for: energy efficient commercial buildings that, among other requirements, meet new minimum lighting efficiency standards, and also for hybrid solar lighting implemented by businesses. Finally this bill holds the Secretary of Energy responsible for conducting a study to determine the 25 year energy efficiency potential of "passive solar technology"[12] such as daylighting.

The Energy Independence and Security Act of 2007

The *Energy Independence and Security Act of 2007* was signed into law by President George W. Bush on December 19th, 2007. It also promotes energy efficient lighting through three basic means: minimum efficacy standards for specific lighting technologies; the prescription of lighting technologies and efficiency standards for use by public institutions; the establishment of research and development of passive solar lighting technologies such as daylighting.

One common misconception about this legislation is that it outlaws incandescent bulbs. In fact it sets specification performance thresholds for *general service incandescent lamps,*[13] a distinction that excludes many incandescent lighting products such as traffic signal bulbs, reflector bulbs, 3-way bulbs, and numerous others. It does set the following efficiency standards for lamps that meet the "general service incandescent" definition, which applies only to lamps that range from 310 Watts to 2600Watts [Table 1].

Table 1: Efficiency standards for general service incandescent bulbs

Rated Lumen Ranges	Maximum Rated Wattage	Minimum Rated Lifetime	Effective Date
1490-2600	72	1,000 hrs	1/1/2012
1050-1489	53	1,000 hrs	1/1/2013
750-1049	43	1,000 hrs	1/1/2014
310-749	29	1,000 hrs	1/1/2014

One can compare the effect of this standard on the typical household incandescent bulb. General Electric sells its *GE Basic A19* 60 Watt incandescent bulb that is rated at 865 lumens[14]. Under the new standard, such a bulb will need a power rating of no more than 43 Watts, a 28%

[10] http://www.nglia.org/index.html last accessed April 2nd, 2008.
[11] United States Congress, *Energy Policy Act of 2005*, sec. 931.
[12] ibid, sec. 1826.
[13] United States Congress, *Energy Independence and Security Act of 2007*, sec. 321.
[14] http://www.gelighting.com/na/home_lighting/products/ Product Number 41026, accessed April 3rd, 2008.

reduction, by January of 2014. It is reasonable to believe these goals will be met by incandescent bulbs that use new energy conserving infrared-reflective coatings. However, the efficiency standards do not stop with this table. The legislation later stipulates that general service lamps meet a minimum efficacy standard of 45 lumens per Watt by January 1, 2020. This aggressive target would require that same 865 lumen bulb operate at roughly 19 Watts, a 68% reduction.

Certain government institutions must make use of energy efficient lighting where feasible. Public buildings that are new construction, are updated, or are acquired *shall be equipped, to the maximum extent feasible as determined by the Administrator, with lighting fixtures and bulbs that are energy efficient*[15]. Energy efficient replacement bulbs and luminaires must be considered during the course of routine maintenance. Lighting that is *energy efficient* is either Energy Star certified, or meets the requirements for Energy Star certification. Lastly, with few exceptions, a *general service incandescent lamp shall not be purchased or installed in a Coast Guard facility by or on behalf of the Coast Guard*[16], after January 1st, 2009.

Finally, this legislation encourages research in certain advanced lighting technologies. Section 605 tasks the Secretary of Energy to establish an RD&D program for daylighting and solar light pipe technology. And in accordance with the Energy Policy Act of 2005, the Secretary is responsible to establish Bright Tomorrow Lighting Prizes to encourage more rapid development of SSL[17]. These prizes consist of: $10 million for an SSL package that can replace a 60W incandescent Edison bulb; $5 million for an SSL package that can replace a PAR type 38 halogen bulb; $5 million for an SSL package that among other specifications can produce at minimum1200 lumens at no less than 150 lumens per Watt with a CRI[18] of at least 90.

Predicted Technological Advances

The big-three traditional light sources are mature technologies that underwent many refinements and improvements over their many decades of development. Incremental improvements are expected, but it is unlikely[19] that a game-changing improvement in efficacy and/or cost is imminent. Since current LED technology is operating so far below its theoretical limits, it is projected to make rapid efficacy improvements [Figure 3]. Moreover, since sales volumes are low and the technology is young, manufacturing costs are also projected to make rapid improvement. To demonstrate this in a fashion akin to Moore's Law[20] Dr. Roland Haitz, a retired scientist from Agilent Technologies and Hewlett Packard, gathered data and noted the

[15] United States Congress, *Energy Independence and Security Act of 2007*, sec. 323
[16] ibid, sec. 522
[17] ibid, sec. 655
[18] See appendix for a glossary of lighting terms.
[19] There is a possibility that next-generation incandescent bulbs with new infrared-reflective coatings could significantly increase in efficacy. There is motivation for this advancement from the Energy Independence and Security Act. Therefore, to be conservative, we assume in our analysis that general service incandescent lamps achieve the aggressive performance 2020 threshold of 45 lumens per Watt.
[20] Moore's Law is the exponential rate at which the number of transistors can be placed on an integrated circuit. It was named after Gordon Moore, a co-founder of Intel, who predicted it in 1965. It has held up well with IC progress over 50 years.

historic trend of LED cost and performance improvement [Figure 4]. The trends show that while lighting output was increasing by a twenty-fold each decade, the unit cost of these devices was decreasing by a factor of ten. Since few data points exist for white LEDs and their theoretical limit looms, our analysis will rely on an industry expert's specific beliefs about future efficacy and cost improvements rather than projecting "Haitz's Law" forward.

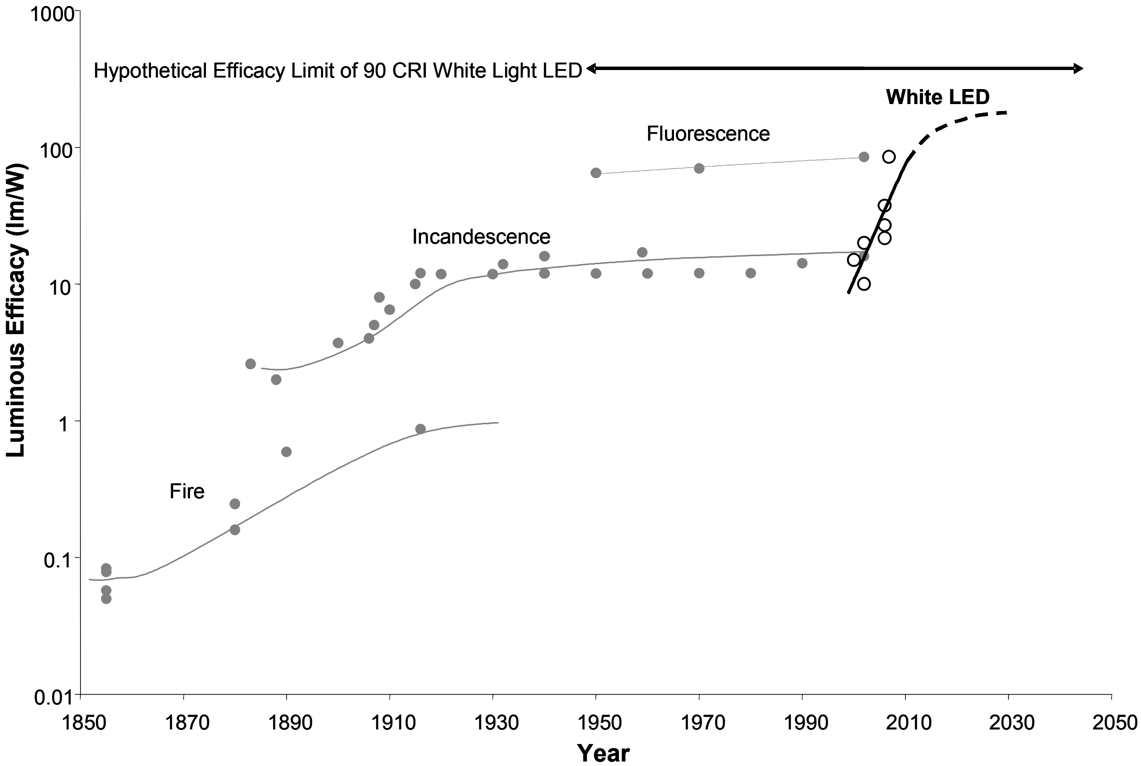

Figure 3: Historical Lighting Efficacy [data courtesy of Jeff Tsao, Sandia National Laboratory]

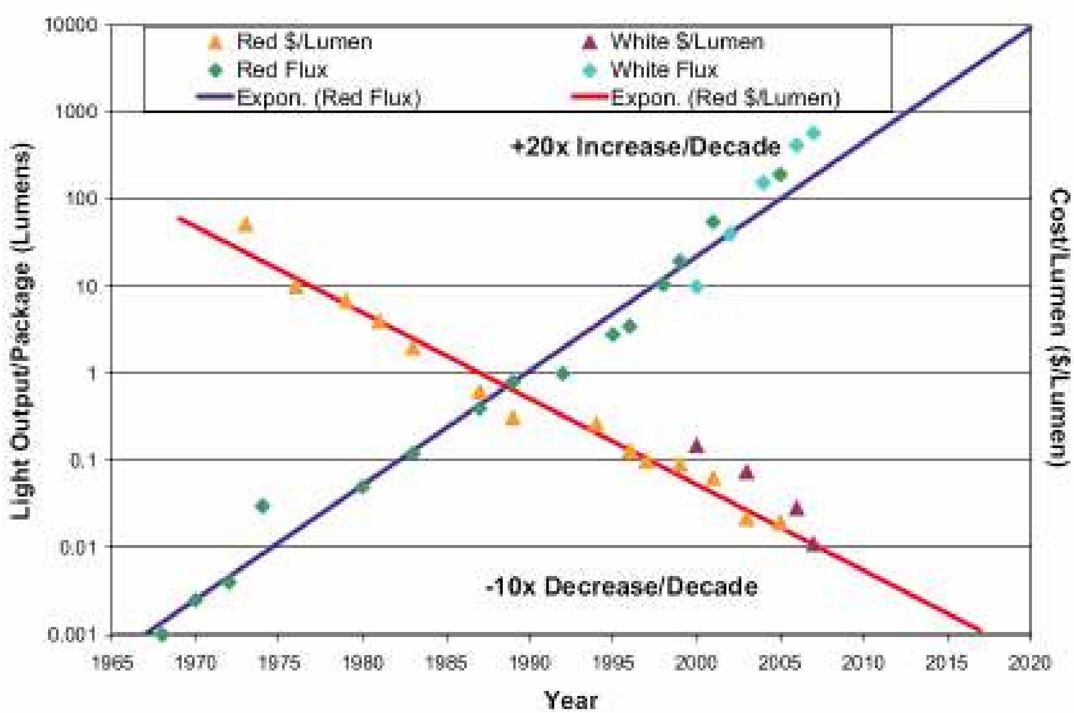

Figure 4: Haitz's Law: Model of LED Output Increase and Cost Decrease[21]

Probabilistic Cost of Light Model

There is much uncertainty in the timing of when SSL might begin to significantly penetrate general illumination markets. This research attempts to elucidate that uncertainty by modeling the total cost of lighting service (including capital, maintenance, and electricity costs) of different technologies at various points in the future. Once we model these costs we can determine whether a decision-maker would consider such an investment economical if she applied a specific financial decision process. Three such financial decision processes are in common use in industry: *first cost*, *payback period*, and *discounted cash flows*. In addition we will demonstrate a fourth less-frequently used approach called decision analysis.

We acknowledge that there are many nonmonetary attributes that consumers consider when making lighting purchase decisions like dimmability, perceived light quality, and luminaire style. Consumers' slow acceptance of compact fluorescent bulbs (CFLs) in the United States strengthens this assertion. However, most of the non-financial barriers to acceptance of CFLs, such as the legacy of the buzzing, flickering, poor color rendering fluorescent tubes of a quarter-century ago[22], as well as the bulkiness and delayed starts of current CFL offerings are not relevant to today's white LED technology. That is not to say that some barriers aren't relevant to LED market penetration—customer awareness, marketing, and retail availability name just a

[21] Navigant Consulting, *Multi-Year Program Plan FY'09-FY'14 Solid State Research and Development,* prepared for the DOE, March 2008, figure 2-4.
[22] Pacific Northwest National Laboratory, *Compact Fluorescent Lighting in America: Lessons Learned on the Way to Market*, June 2006.

few. To be clear, this analysis only attempts to give insights by modeling the *cost* of light and ignores other factors that might influence a purchase decision.

There are four sectors of interest in this study: residential, commercial, industrial and outdoor stationary. For each sector we compare at least one lighting technology with white LEDs:

- *850 lumen 60 Watt incandescent bulbs and 13 Watt compact fluorescent lamps (CFL) in the residential sector:* Incandescent bulbs are the dominant illumination technology in United States residences, contributing well over half of the lighting service. CFLs contribute only a few percent of the total residential lighting service for the United States. However, they are aggressively being marketed to customers by utilities and corporations alike. Therefore, we believe it is also a useful market-technology comparison for SSL.
- *2300 lumen 4 foot T-8 fluorescent tubes in the commercial sector:* According to the 2002 Lighting Market Characterization, this market-technology combination accounted for an estimated 18% of the total United States commercial share of lumen-hrs (12% of electricity) and 10% of the total lumen-hours produced in the United States[23]. Moreover, if white LED technology can unseat 4 foot T-8 fluorescent tubes, then it will likely be able to unseat T-12 tubes as well. These older technology tubes constitute *over 50 %* of the lumen share in the commercial sector and some argue are they are currently in process of being obsoleted by T-8 tubes. There are also many lumen-hours being delivered by T-8 and T-12 lamps in the industrial sector, so if LEDs become a viable alternative to T-8 tubes, they also might gain a strong foothold in the industrial sector.
- *33,100 lumen 400 Watt metal halide in the industrial sector:* Metal halide lamps are estimated[24] to account for 20% of the lumen-share and 7% of the electricity in the United States' industrial sector. While the largest lumen-share in this sector belongs to the linear fluorescent tube, we focus on the second-place shareholder, metal halide HID. We do this because the commercial sector linear fluorescent analysis serves as a reasonable surrogate for the industrial sector.
- *22,000 lumen 200Watt high pressure sodium in the outdoor stationary sector:* This technology dominates the lumen share of the outdoor stationary sector at an estimated[24] 70%. Its share of electricity consumption is 45%.

We choose these five technologies because they stand out as the most likely competition for LEDs in the years to come, and they are very common in adjacent sectors. For example, if white LEDs can compete well in the outdoor stationary sector with metal halide, then they are also likely to compete well against metal halide technologies in use in the commercial and industrial sectors.

Model Background

General Assumptions

Our model begins with a decision-maker in a specific sub-market weighing two or more lighting purchase alternatives. Each lighting alternative can be analyzed via its characteristic costs with

[23] Navigant Consulting, *U.S. Lighting Market Characterization: Volume I: National Lighting Inventory and Energy Consumption Estimate*. [Prepared for the U.S. Department of Energy], table September 2002.
[24] Navigant Consulting, *U.S. Lighting Market Characterization: Volume I: National Lighting Inventory and Energy Consumption Estimate*. [Prepared for the U.S. Department of Energy], September 2002.

each of the three aforementioned approaches. We assume the decision is made either for new construction, for remodel, or to replace an existing lamp and ballast[25] assembly that has no remaining life. This begins the analysis with a clean slate since the decision-maker does not carry forward any residual benefits from past lighting decisions.

We ignore the cost of luminaire features such as diffusers, lenses, and mounting hardware. Any differences in luminaires that house LEDs and those that house competing technologies are application-specific and therefore difficult to model. Moreover, we believe that most such luminaire cost differences will be insignificant. One consequence of excluding the luminaire is that the coefficient of utilization, an industry measure of the efficiency of the lamp/luminaire system's ability to deliver lumens, is removed from the analysis.

As white LED luminaires commercialize, they may suffer design issues such as difficulty managing waste heat[26]. We assume that any such design issues are resolved without cost increases to the LED or its driver[25] electronics, and that any modifications to luminaires cause insignificant increases in cost. On the other hand, we also ignore design optimization in package size, so our model does not account for any improvements to device cost or efficacy due to such optimization. Another potential advantage we omit from our model is the inherent directionality of LED light. This feature might enable more efficient LED fixture designs that accomplish lighting design constraints while requiring fewer lumens than competing technologies.

Modeling Technological Improvements

There are various means for lighting technologies to improve with future design advances. We model four such parameters: *lamp efficacy* and *cost*, and *driver electronics efficiency* and *ballast/driver cost*. We consider the lifespan of all modeled technologies as fixed at the 2008 level—for example, 2008 vintage CFLs are modeled with a life of 8000 hours as are 2009 vintage CFLs and every subsequent future year's offering.

Lamp efficacy is universally taken to be *initial* lumens per Watt. For bulbs and tubes, cost is measured in dollars per lamp, but in the case of LEDs it is measured in dollars per 1 Watt device. Driver efficiency is dimensionless, with a maximum value of one, and ballast/driver cost is measured in dollars per Watt. We project the future values of all four measures by means of a simple exponential function as shown in equation (1), where P_0 is the value of that parameter today, P_f is the *best* we expect that it will ever become, and r is the rate that P_0 approaches P_f [Figure 5]. In the case of efficacy and efficiency P_0 is less than P_f, so the function is increasing in time. For the two cost parameters, P_0 is greater than P_f, so the function is decreasing in time.

$$P(t) = \left(P_0 - P_f\right)\cdot \exp\left(-rt\right) + P_f \qquad (1)$$

[25] See appendix for glossary of lighting terms.
[26] discussion with Terry Clark, CEO Finelite, Inc., 10/26/2007

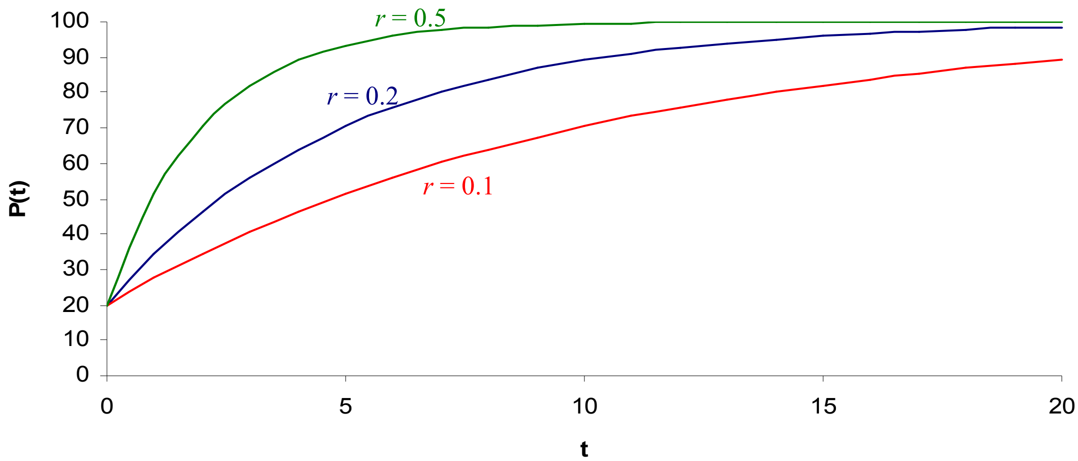

Figure 5: example of three increasing exponential functions, varying *r*, where P_0 = 20 and P_f = 100.

Modeling Uncertainty in LED improvements

We model the rate of improvement, r in equation (1), of LED efficacy and LED cost as random variables. This is what makes our model probabilistic. In order to apply appropriate probability distributions to $r_{efficacy}$ and r_{cost} for LEDs, we assessed the knowledge of a recognized LED industry expert. We first worked with him on the following definitions for each of the uncertainties:

$r_{efficacy}$: *The best fit exponential curve (as defined in equation (1)) of the average initial efficacy [lumens per Watt] of high-power[27], 90+ CRI, warm-white[28] LED devices sold in the United States each calendar year by top tier manufacturers such as Cree, Lumileds, Nichia, and Osram.*

r_{cost}: *The best fit exponential curve of the average price [$2008 per one Watt device] of high-power, 90+ CRI, warm-white LED devices sold in the United States each calendar year by top tier manufacturers such as Cree, Lumileds, Nichia, and Osram.*

He chose 45 lumens per Watt to be the initial average efficacy for warm-white LEDs sold in the last calendar year by top-tier manufacturers. He predicted 160 lumens per Watt as the highest efficacy that such LEDs would achieve in the future. This set the P_0 and P_f terms, respectively. We then used a spreadsheet chart with a slider that controlled the $r_{efficacy}$ term so that he could adjust it to match his beliefs in response to questions about the rate of future efficacy improvements. Likewise, he chose $3 per 1 Watt device for the initial price and $.50 per 1 Watt device for the final price. Again we constructed a spreadsheet, this time with a slider to control r_{cost}.

Next we went through a process of conditioning questions followed by series of trade-off questions designed to elicit his beliefs about the rate of efficacy improvement while avoiding cognitive biases. Finally we reviewed the uncertainty assessment to verify that it properly

[27] *high power* LEDs are generally understood by the SSL industry to be 1-3 Watt LED packages
[28] *warm-white* is generally understood by the lighting industry to refer to light with a correlated color temperature around 3000 degrees Kelvin. See appendix for definition of correlated color temperature.

reflected his beliefs. This process was pioneered in the 1960's and 1970's at the Stanford Department of Engineering Economic Systems, and SRI[29]. We approximate his probability assessments with a least-squares fit of a lognormal distribution, shown as curves in Figure 6 and Figure 7.

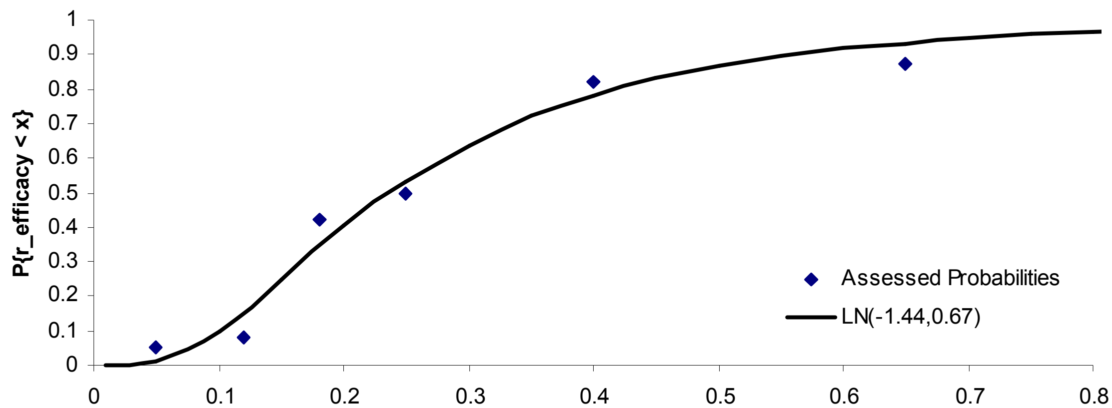

Figure 6: Expert-assessed probabilities for the rate of improvement of white LED efficacy

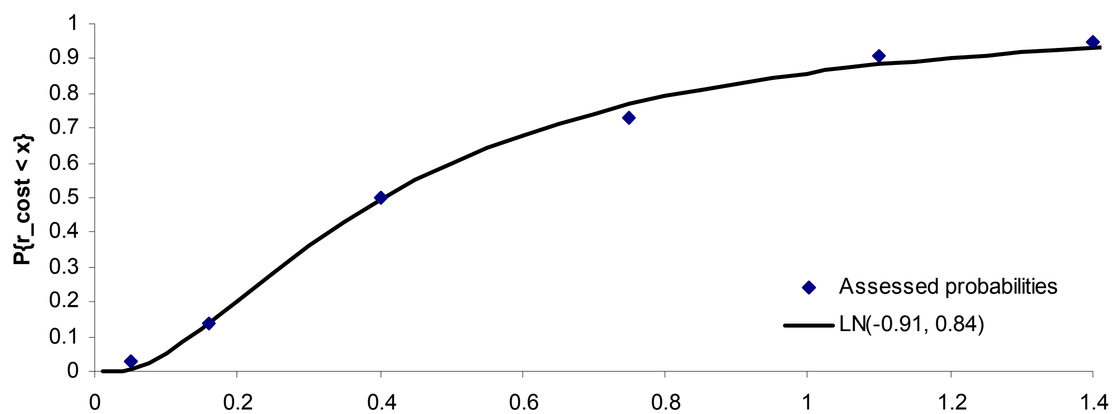

Figure 7: Expert-assessed probabilities for the rate of improvement of white LED cost.

Financial Decision Making Methodology

First Cost

This simplest of methods is exactly what it seems. The purchase decision is based a single consideration, namely the initial cost of the investment. In our model, this first cost includes the lamp, ballast/power supply, and initial installation cost only. We generally do not recommend the first cost approach for making important financial decisions because it does not consider future cash flows such as future electricity and maintenance costs that are a consequence of the

[29] Morgan, M.G., Henrion, M., *Uncertainty, A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*, Cambridge University Press, 1995, pp 141-146.

lighting investment decision. We include it as a descriptive model because it is a very common decision-making method in the residential and commercial sectors[30].

Payback Period

This slightly more sophisticated method is measured in time rather than money. *Payback period* is the length time that an investment takes to recoup its initial cost. Therefore, if $100 is initially invested, and savings of $50 are expected to be achieved each year as a result of the initial investment, then the payback period is $100/$50, or *two years*. Since this paper's analysis considers a decision with only negative cash flows, the *payback period* of an investment can be calculated only as a comparison between one technology and a default course of action. In the case of two lighting technologies, the first (default) with a lower initial cost and the second with a lower operating cost, the payback for the second will be the length of time it takes for the difference between the annual operating costs of the investments to equal the difference between their initial costs. Take the investment illustrated in Table 2 as an example. The difference in initial costs is $40 and the difference in annual operating cost is $10. Therefore, the payback period for choosing lighting technology Y over lighting technology X is $40/$10, or *four years.* We include payback period as a descriptive method of how financial decisions for lighting investments are sometimes made, but we generally do not recommend this method for important financial decisions because it does not account for the decision-maker's time preference (the discount rate is implicitly 0%) nor does it account for his risk preference.

Table 2: Example: payback period for lighting technology Y with technology X as the default

payback period = 4 years	Initial cost	Annual operating cost
lighting technology X	$60	$25
lighting technology Y	$100	$15

Discounted Cash Flow

Discounted cash flow uses the concept of a discount rate to account for the decision-maker's time preference on money, for example, $10 spent five years from now with a 7% annual discount rate is valued at $7.13, or $\$10 / (1 + 0.07)^5$. Discounted cash flow requires a few additional parameters than the aforementioned two methods, namely, a discount rate and a time horizon for the investment. If we take the example of lighting technology Y detailed in Table 2 over a three year time horizon and a 7% discount rate, the discounted cash flow[31] is $139.36:

$$\$100 + \frac{\$15}{(1+0.07)} + \frac{\$15}{(1+0.07)^2} + \frac{\$15}{(1+0.07)^3} = \$139.36$$

By the same method, lighting technology X has a lower discounted present cost of $125.60. Therefore given the discount rate and time horizon stated, the decision-maker should choose lighting technology X. Discounted cash flow serves as a useful descriptive tool about how decision-makers might invest in a specific lighting technology. Note that in our model, when a decision-maker chooses a lighting technology, she purchases a lamp and ballast/driver system and then commits to a policy of persevering with that technology should a lamp reach the end of its lifespan before the end of the investment time horizon. We assume the ballast/driver survives

[30] discussion with Terry Clark, CEO Finelite, Inc., 10/26/2007

[31] This analysis assumes the annual operating cost is incurred at the end of each calendar year and the initial investment was made at the beginning of the current year.

through the decision-maker's time horizon and is never replaced. While discounted cash flow is the best of the three methods in accounting for a decision maker's preferences, it has one weakness in that it accounts for neither uncertainty nor the user's risk preference. We therefore recommend decision analysis as an approach for making important lighting decisions.

Decision Analysis

Decision analysis refines the discounted cash flow approach by including uncertainty as well as risk preference. Again, imagine technology Y from Table 3 over a three year time horizon and a 7% discount rate, only this time the annual cash flow is believed to have a 40% chance of being \$15, and a 60% chance of being \$5. If the decision maker is risk-neutral[32] over these monetary prospects (most business owners should be), this expenditure is valued at \$123.62:

$$\$100+\frac{0.4\cdot\$15+0.6\cdot\$5}{(1+0.07)}+\frac{0.4\cdot\$15+0.6\cdot\$5}{(1+0.07)^2}+\frac{0.4\cdot\$15+0.6\cdot\$5}{(1+0.07)^3}=\$123.62$$

Note that we do not use decision analysis as a descriptive decision making method in our model.

Model Flowchart

The model was originally designed to compute the discounted cash flow of a lighting investment, which is the most complex of the three models. However, by changing some input parameters, it can be customized to any of the three approaches[33]. Figure 31 details the logic that the model uses to compute a run of 10,000 simulations of the cost of light from white LEDs. Since improvements in the other three technologies are modeled deterministically, they can be computed with one pass to yield a single cost of light. For LEDs, each simulation begins by sampling from the two lognormal distributions on efficacy and cost, tallies all of the capital, maintenance, and electricity costs over the time horizon, and records a single cost of light which is recorded and later displayed with the other 9,999 simulations as a histogram.

[32] For decision analysis with a risk-averse decision maker, see *Foundations of Decision Analysis* by R. A. Howard, and A. Abbas, which is scheduled for release Summer 2008.

[33] To compute *first cost*, we use the same model, but set the bulb usage to zero hours per day. To compute whether a lighting technology met a *payback period* of 2 years, we set the time horizon to 2 years and set the discount rate to 0%. Then we subtracted the total cost of LEDs to the total cost of reference technology. A technology meets the payback criteria if this difference is negative, otherwise it doesn't.

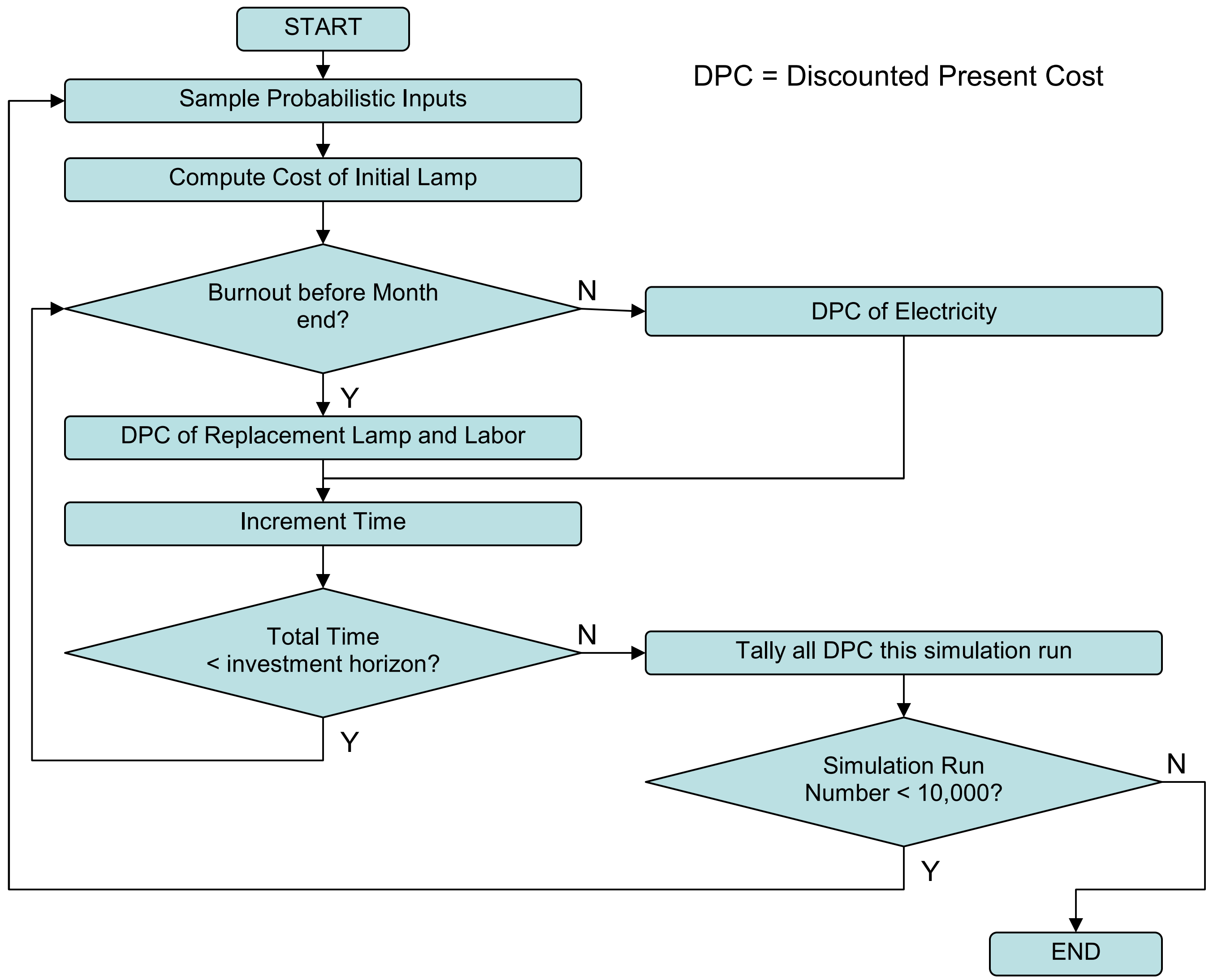

Figure 8: Flowchart of Discounted Cash Flow Model

Model Inputs

General

Certain model inputs are used in every scenario that we ran for this paper. In all cases, the baseline year is 2008 and we look at the cost of light for an investment made in any given year from 2008 to 2022. We take the annual rate of inflation to be 2.5% compounded annually—we use this term in order to work in 2008 dollars. We model the marginal cost of electricity as $0.10 per kWh in 2008, increasing faster than inflation at a rate of 4% annually. Hardware costs are all displayed in $2008, and labor costs are assumed to increase exactly at the rate of inflation. Electricity bills are paid at the end of each month, but the burn-out and replacement of lamps happens in continuous time.

Technology Specific

We set the lifespan of a white LED lamp at 50,000 hours. We assumed LEDs to come in arrays of 1 Watt devices, so to reach the target luminous flux, we computed [equation 2] the minimal number of 1 Watt LED devices n, with efficacy e_{LED}, to achieve the threshold number of lumens, F. Due to this discretization, we model the output of the LED lamps as weakly greater than a sub-market's specified nominal output. Note that the brackets indicate the *ceiling* function.

$$n = \left\lceil \frac{F}{e_{LED}} \right\rceil \tag{2}$$

The remaining parameters are shown in Figure 9. We determined the LED driver efficiency parameter via a least-squares fit to the projections made in Navigant, 2008[34]. Since the LED device parameters are probabilistic, the charts show curves that represent three different percentiles to give an indication as to the range of the uncertainty. Note that the LED device performance projections from Table 4-2 of the Navigant 2008 study are more optimistic than our expert's.

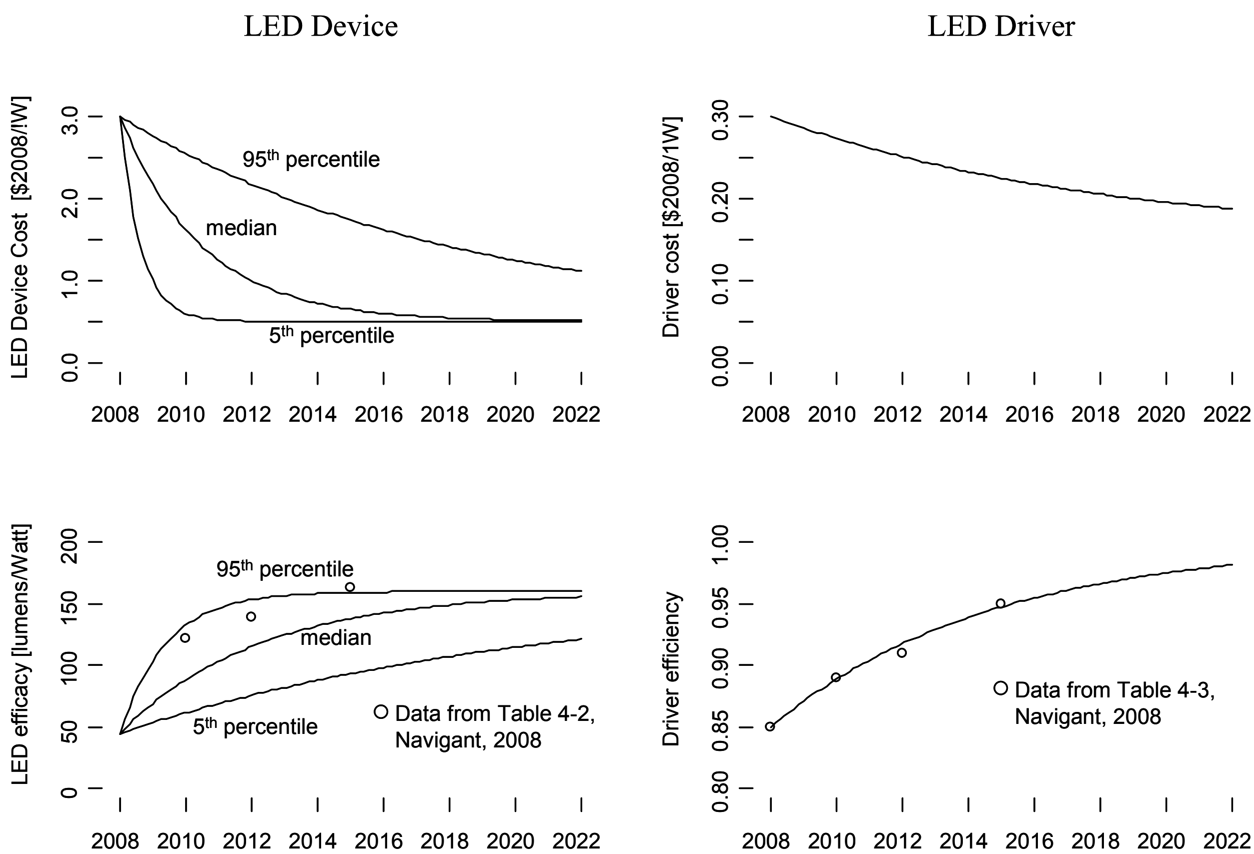

Figure 9: LED-specific model inputs

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[34] Navigant Consulting, *Multi-Year Program Plan FY'09-FY'14 Solid State Research and Development,* prepared for the DOE, March 2008, figure 2-4.

We use a standard 60W bulb[35] to determine the input parameters for an incandescent that outputs 865 lumens, and has a lifespan of 1000 hours. We optimistically project incandescent efficacy to improve at a rate enabling them to achieve the 2020 federal efficacy threshold of 45 lumens per Watt. Since incandescent lamps operate without a power supply or ballast, we only considered lamp cost and efficacy [Figure 10]. We again made a conservative assumption that the rapid technological improvement in incandescent technology would have no effect on the cost of the bulb.

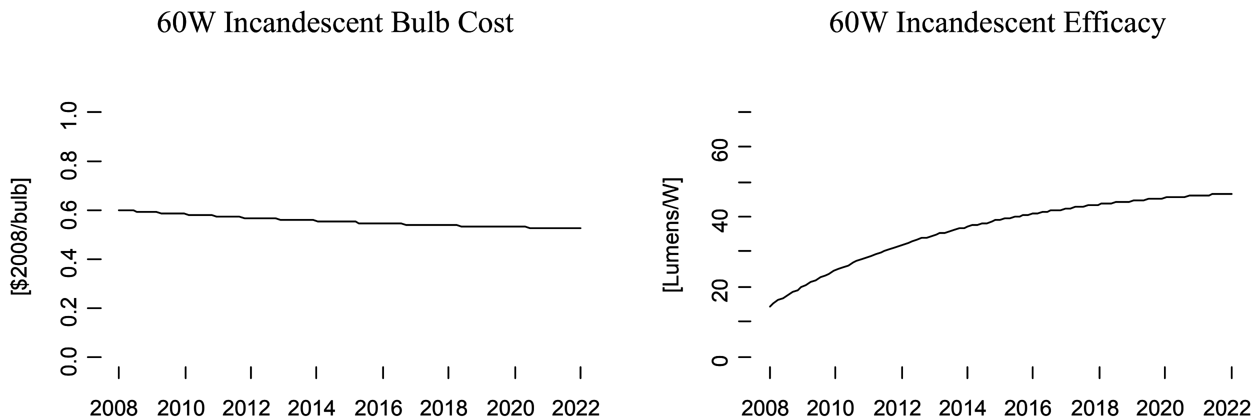

Figure 10: Incandescent-specific model inputs

We determine our CFL input parameters in most part from the performance characteristics of a typical retail bulb[36] that outputs 825 lumens at 13 Watts, and has a lifespan of 8000 hours. Since CFLs are commonly sold as lamp-ballast systems, we model the cost and efficacy together [Figure 11].

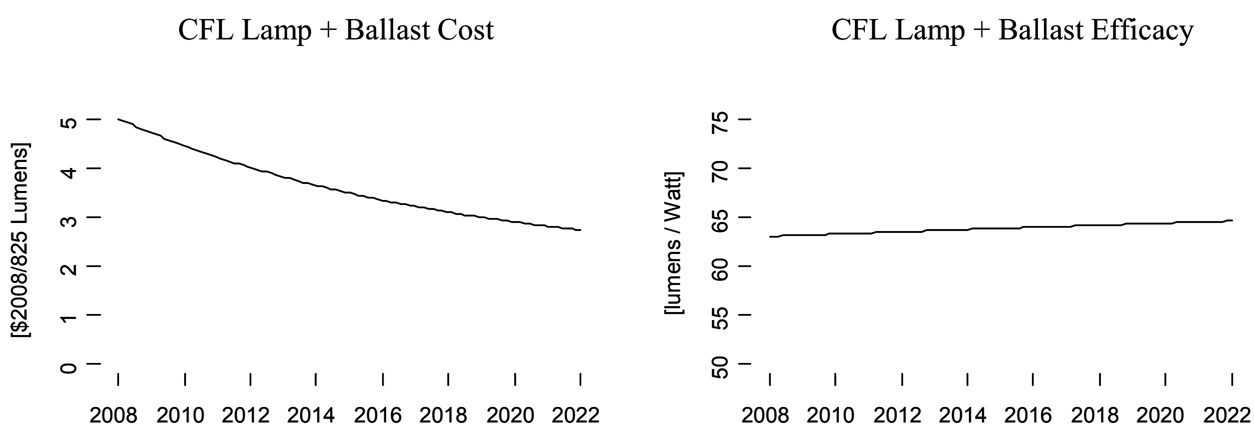

Figure 11: CFL-specific model inputs

[35] http://www.amazon.com, GE p/n 41026, $28.56 for 4 x 12 pack, roughly $.60 per bulb, accessed May 4th, 2008.
[36] http://www.amazon.com, GE p/n 16460, $17 for a 3-pack, accessed April, 2008. Other brands were a bit cheaper at 12 per 3-pack, so we took $5 to represent the cost of a CFL lamp + ballast in 2008.

We set the parameters of a four foot T-8 fluorescent lamp according to a commonly available tube and ballast combination[37]. The lifespan is 20,000 hours and the output is 2300 lumens (2650 lumens * 0.87 ballast factor) at 29 Watts. We model the tube and ballast cost evolutions separately and the efficacy evolution as a system [Figure 12].

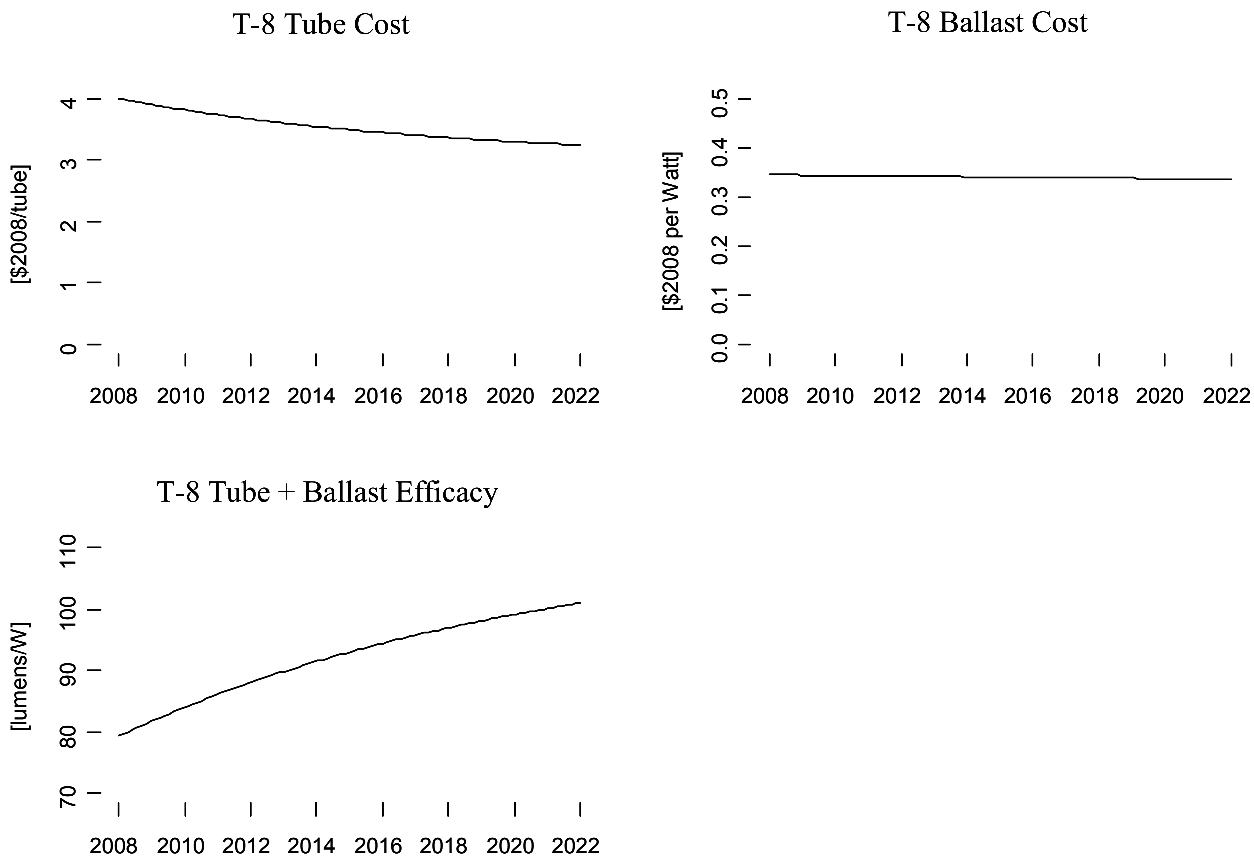

Figure 12: T-8 fluorescent-specific model inputs

We determine our metal halide HID input parameters in from the performance characteristics of a nominal 400W bulb/ballast combination[38] with at 90% power factor that outputs 33,100 lumens with a CRI of 65. The bulb has a specified lifespan of 15,000 hours. We model the efficacy of the bulb/ballast system [Figure 13].

[37] http://www.amazon.com, GE T-8 tube p/n 16263 $8/2pack, accessed April 2008; http://www.1000bulbs.com GE ballast p/n 49772, $20 for a 2-lamp ballast, normalized to $20/(29W * 2lamps) = $.34 per Watt per lamp.

[38] http://www.1000bulbs.com, GE bulb p/n 43828, $16.85 each, Sola ballast p/n E-871-W-411 listed at $44.24 or $.10/Watt. Accessed May 10th, 2008.

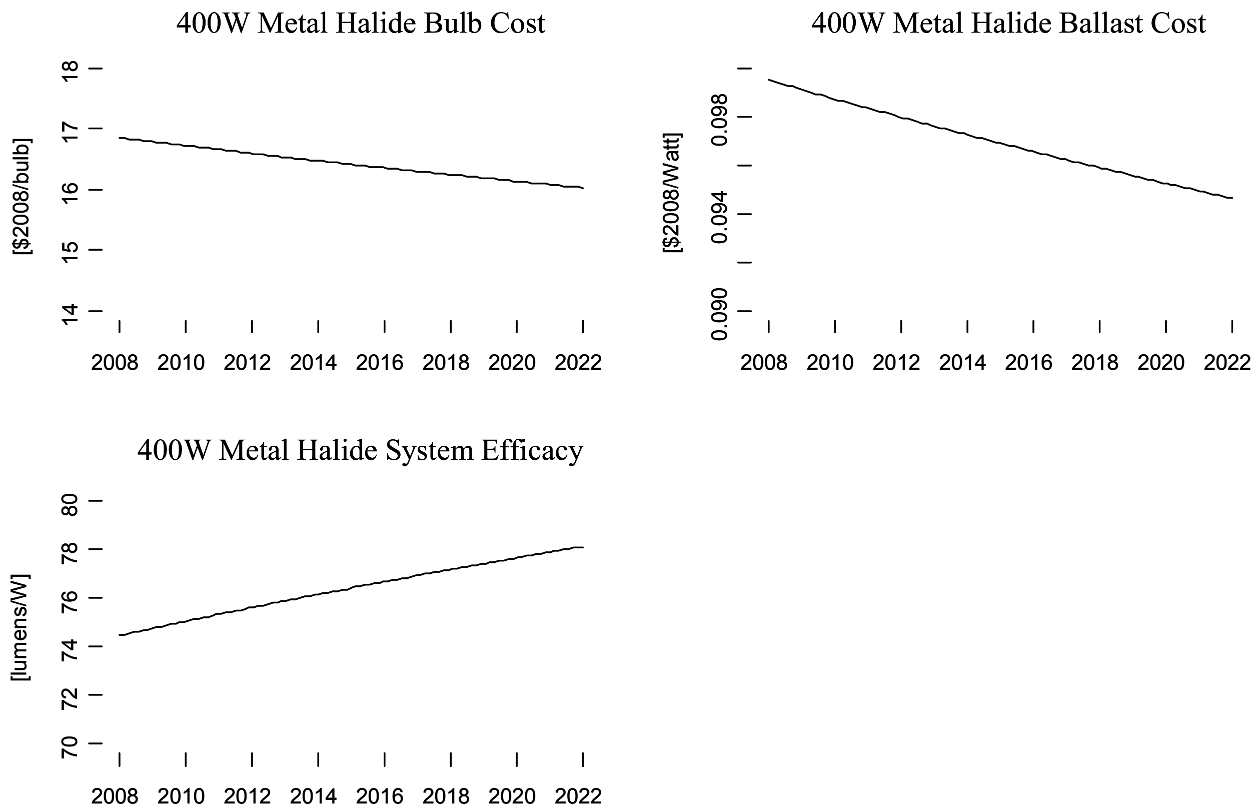

Figure 13: 400W Metal Halide-specific model inputs

The high pressure sodium lamp/ballast system[39] we choose outputs 22,000 lumens with a nominal 200W and a power factor of 90%. The bulb has a specified life span of 20,000 hours and a CRI of only 22. Again, we model efficacy of the bulb/ballast system [Figure 14].

[39] http://www.1000bulbs.com, PlusRight bulb p/n L02009111202, $11.85 each, Sola ballast p/n E-SCA00W200 listed at $46, or $.21/Watt. Accessed May 10th, 2008.

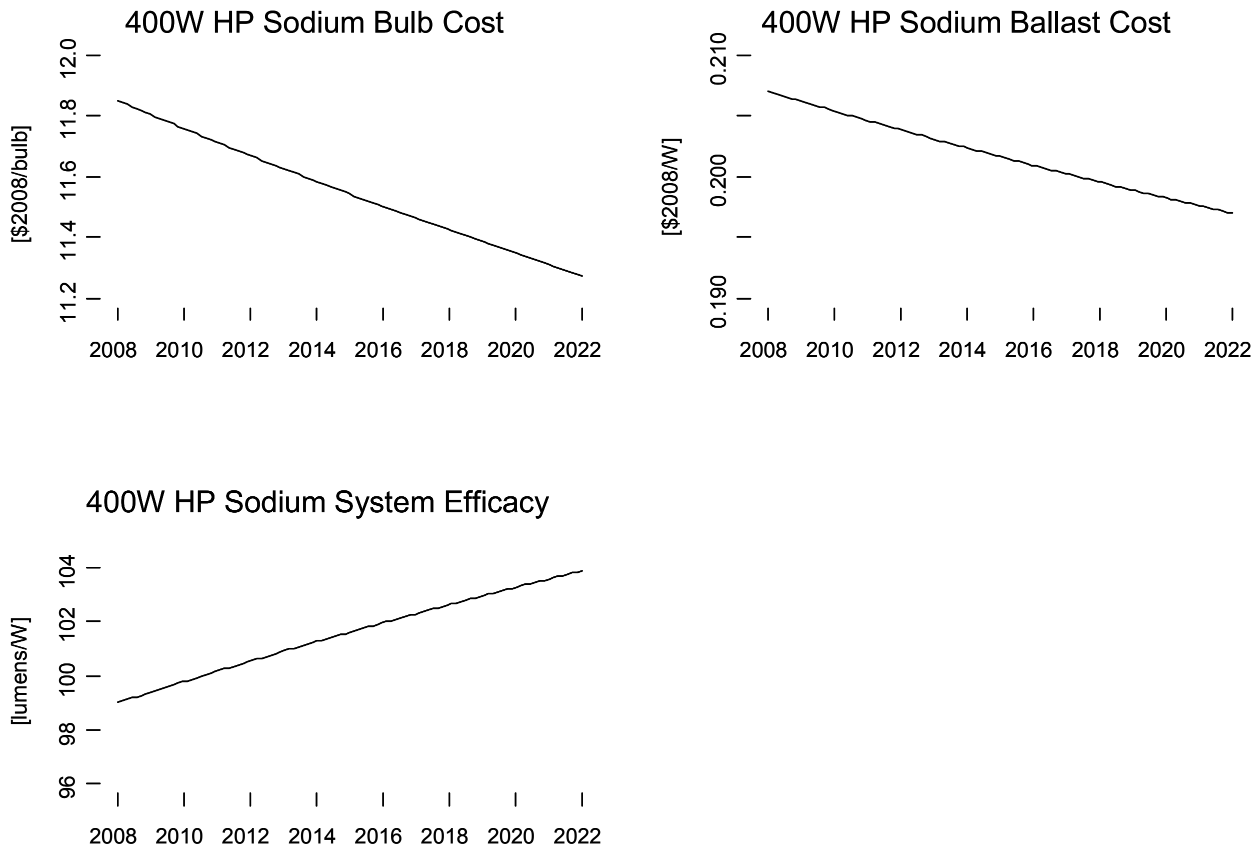

Figure 14: High Pressure Sodium-specific model inputs

US Sub-Market Specific

In the residential, 850 lumen scenario, we compare white LEDs to a 60 Watt incandescent bulb and a 13W CFL. The residential sub-market uses general service incandescent lamps and CFLs on average approximately two hours per day. For the discounted cash flow approach, we set the time horizon for the investment at six years, an approximation of the mean time between moves for an adult United States citizen[40]. We take the labor cost at $30 per hour and the time to replace the lamp at 0.1 hours. Therefore, it costs $3 in maintenance to replace the lamp (this should include the incremental time to purchase the lamps, and to remove and dispose of the burnt-out lamp).

There are a few key differences in the commercial sub-market. Namely, the daily lamp usage is much higher at approximately 10 hours. We set the time horizon at seven years as an approximation of the average tenancy of a commercial business[41]. Since commercial luminaires are often set in high ceilings, we set the replacement time at 0.5 hours, making for a maintenance cost of $15 each time a lamp expires.

[40] http://www.census.gov/population/www/pop-profile/geomob.html, accessed 5/1/2008. If at age 19, one can expect 9.2 remaining moves, and the average life span is 75 years, then the average time between moves is roughly 6 years.

[41] discussion with Terry Clark, CEO Finelite, Inc., 10/26/2007

We chose a bit higher labor cost and a longer time horizon for the industrial market while keeping the replacement time and discount rate the same as the commercial sector's. Data suggest that the average time of usage for a metal halide bulb in this sector is 14 hours.

The outdoor stationary sector uses high pressure sodium lamps approximately 11 hours per day. The labor cost and replacement time suggest that it costs $100 for replacing a bulb in a streetlamp, an estimation that includes the cost of transportation, labor, and equipment[42].

Table 3: US Sub-Market model input summary

Sub-market	Output [Lumens]	Daily Usage[43] [hours]	Time Horizon[44] [years]	Labor Cost [$2008/hour]	Time to Replace [hours]	Discount Rate
Residential	850	2	6	30	0.1	0.10
Commercial	2,300	10	7	30	0.5	0.07
Industrial	33,100	14	10	50	0.5	0.07
Outdoor Stationary	22,000	11	10	100	1	0.07

Output

Since the cost of light computation for LED applications are probabilistic, the discounted present cost of an LED investment is a histogram rather than a point estimate. Since all other technologies are deterministic, we show a single point estimate indicated by a colored leader for their discounted present cost for a given year of investment. Note that the term *investment year* indicates the point in time when the investment decision is made, for example *investment year 2012* means that the decision maker made the investment decision in 2012.

Output for Residential, 850 Lumen Sub-Market

We initially use the *first cost* financial decision process to compare an 850 lumen white LED lighting system to a 60 Watt incandescent bulb and a 13 Watt CFL. Since incandescent bulbs are inexpensive (they require no supporting ballast/driver), this technology will likely *always* be the best alternative for our first-cost decision maker in this sub-market.

[42] discussion with Jim Helmer, director, San Jose Department of Transportation, March 27th, 2008.
[43] Navigant Consulting, *U.S. Lighting Market Characterization: Volume I: National Lighting Inventory and Energy Consumption Estimate*. [Prepared for the U.S. Department of Energy], September 2002, pg 37.
[44] The time horizon only comes into play for the discounted cash flow calculation.

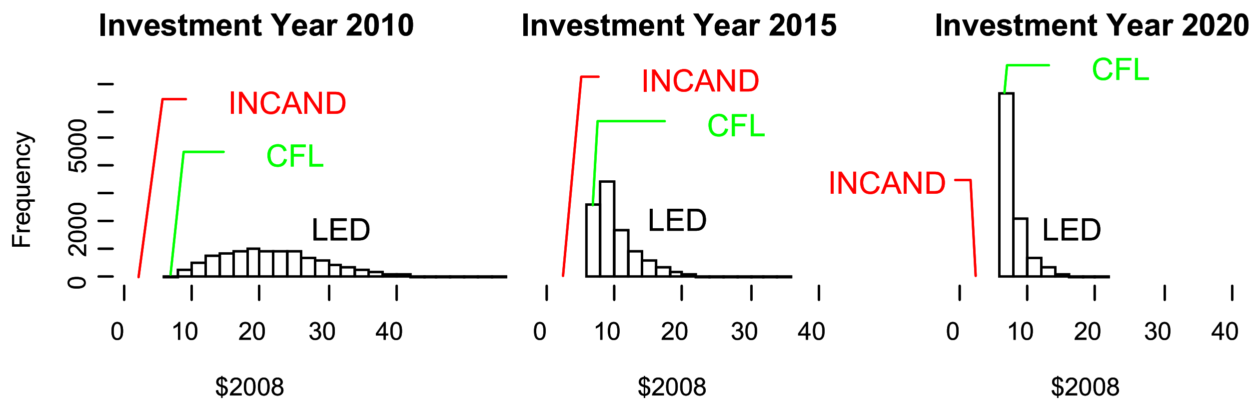

Figure 15: First Cost, Residential, 850 Lumen Sub-Market

A *payback period* financial decision process with only negative cash flows (like a lighting decision) requires a reference technology with which to compare a white LED lighting system. We choose the CFL as a comparison point for this sub-market. If our decision maker considers a two-year payback period as her decision criterion, then in the near term, she will not pick LED-based technology. However, our model suggests that LED-based technology may appear favorable in the distant future.

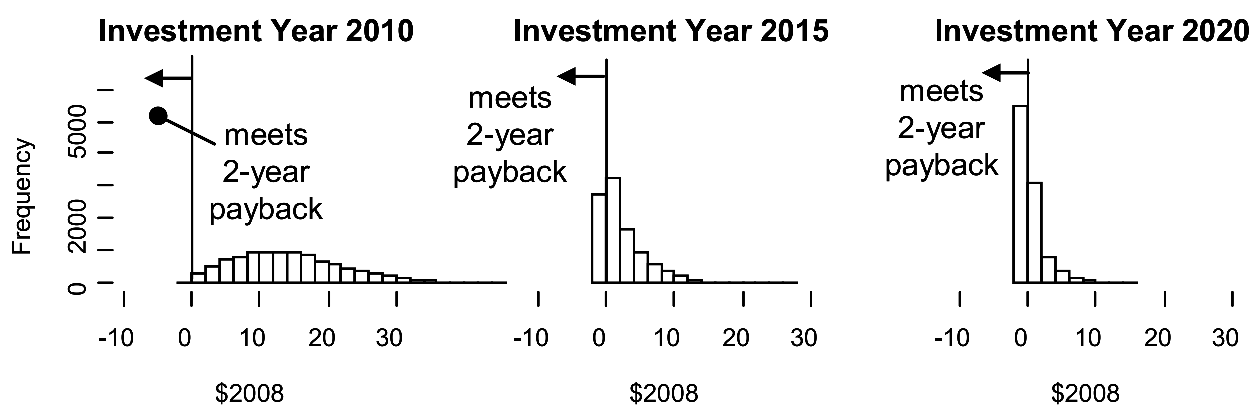

Figure 16: Two-year Payback Period, with CFL as the baseline investment

Using a discounted cash flow approach, CFLs are the lighting technology of choice in the near term. However, as time passes and white LED technology improves, many of the mid and long-term simulations show it to be the best alternative. It might be of surprise that incandescent bulbs are the last choice in all modeled years, but are not too far afield. Again, this is because we modeled the aggressive incandescent improvement scenario mandated by federal legislation.

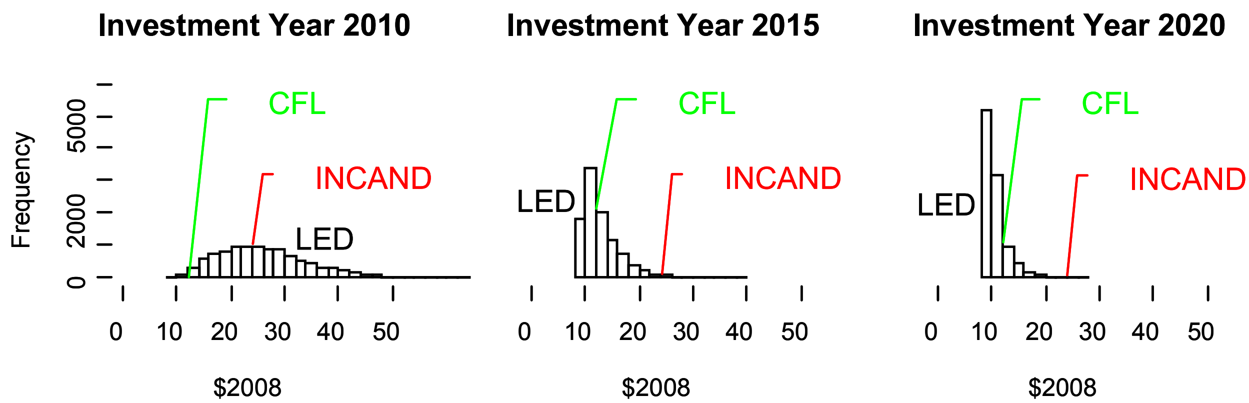

Figure 17: Discounted Present Cost, Residential, 850 Lumen Sub-Market

Output for Commercial, 2300 Lumen Sub-Market

We start with the *first cost* financial decision process to compare a 2300 lumen white LED lighting system to a four-foot 29W T-8 fluorescent tube/ballast system. The analysis suggests that a first-cost decision maker would likely choose T-8 fluorescent technology now and in the distant future.

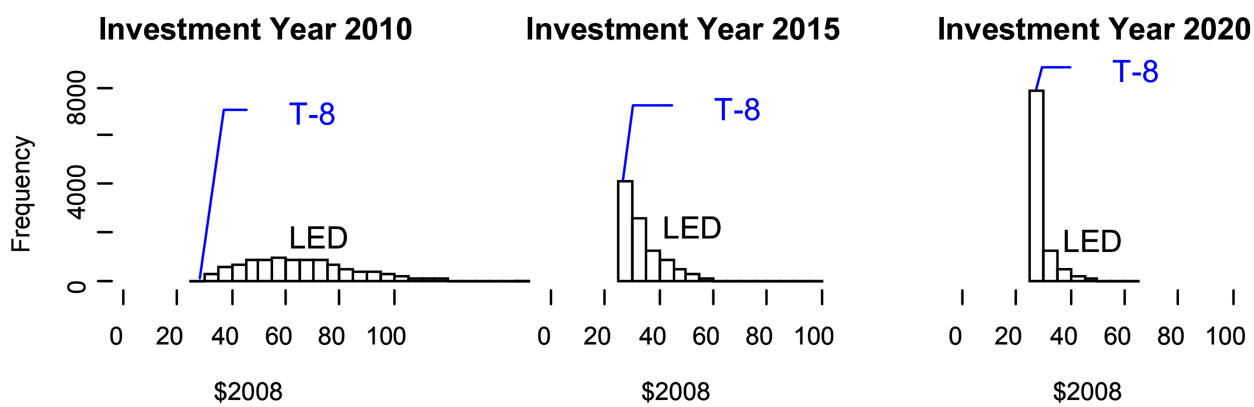

Figure 18: First Cost, Commercial, 2300 Lumen Sub-Market

Our two-year *payback period* decision-maker who uses a T-8 fluorescent as a reference, will not choose a white LED-based lighting system in the near term but would likely choose it given strong LED technological advancement scenarios in the mid and far-term.

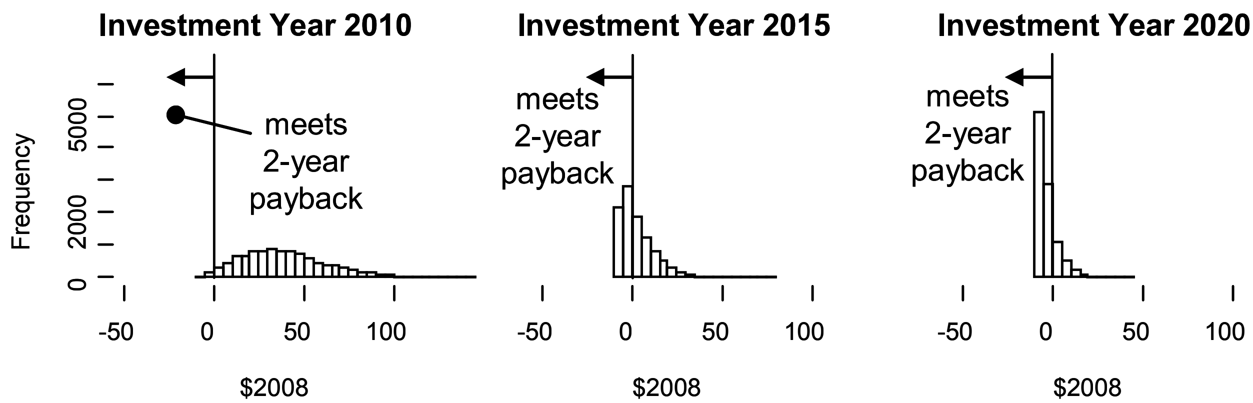

Figure 19: Two Year Payback, Commercial, 2300 Lumen Sub-Market

Our decision maker using a *discounted cash flow* approach would not likely choose a white LED-based lighting system in the near term, but would likely choose it in the mid and long-term. Thus the model highlights that while LEDs will eventually benefit from best-in-class efficacy, the up-front cost of an LED system is not likely to best that of a T-8 lighting system.

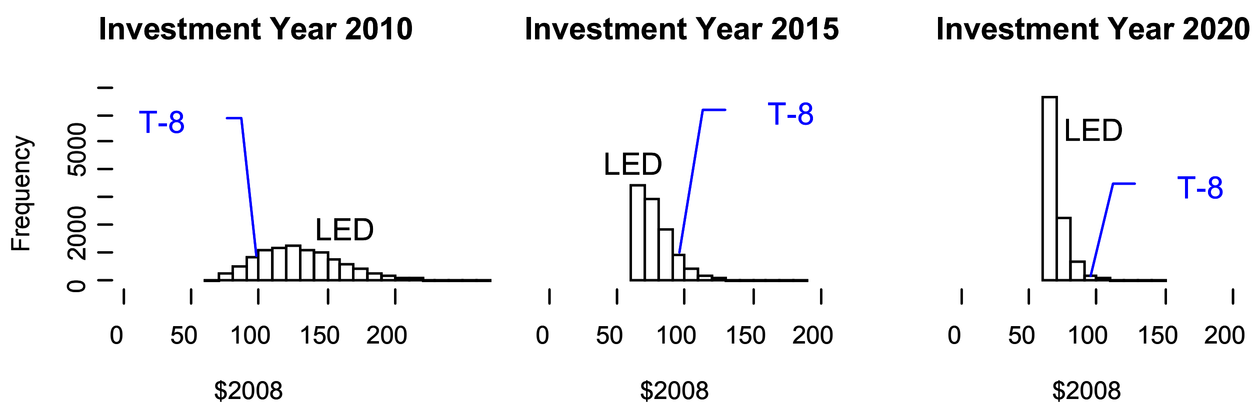

Figure 20: Discounted Present Cost, Commercial, 2300 Lumen Sub-Market

Output for Industrial, 33100 Lumen Sub-Market

We compare a white LED lighting system to a 400 Watt metal halide lamp and ballast for this sub-market. Our model predicts that a *first cost* financial decision maker in this industrial sub-market will opt for metal halide illumination now and in the future. That technology uses very inexpensive lamp and ballast hardware that is likely to be less costly than a white LED lighting system even into the distant future.

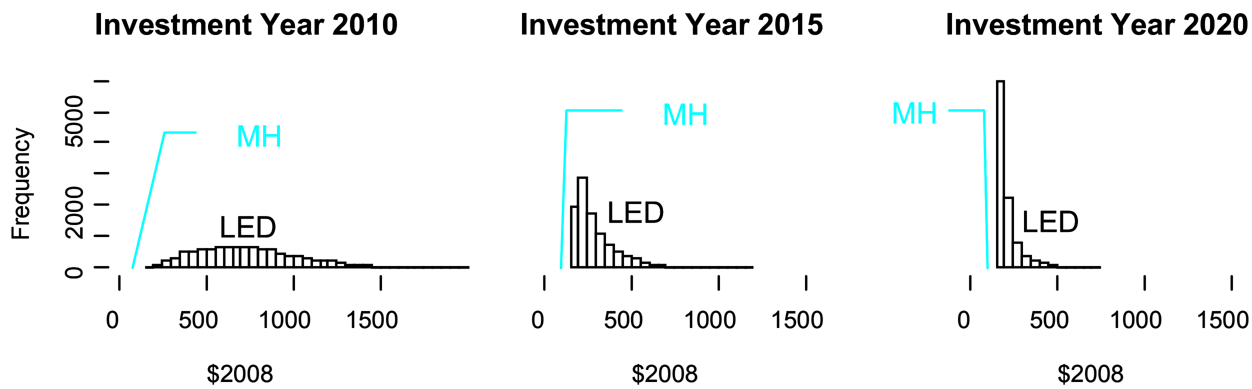

Figure 21: First Cost, Industrial, 33100 Lumen Sub-Market; MH = metal halide

However, when using a two year payback criterion, our decision maker is likely to choose a white LED-based system in the distant future. The near term favors the metal halide system and the mid term is quite uncertain.

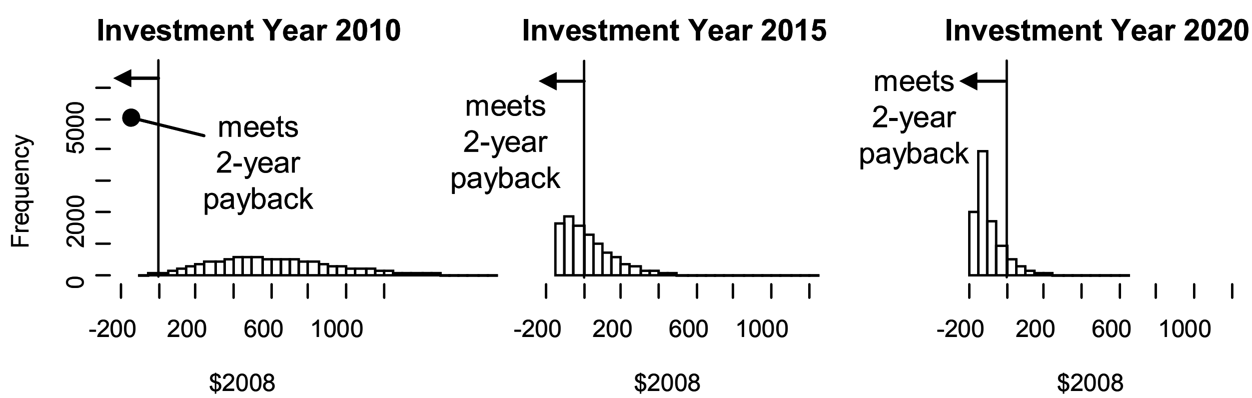

Figure 22: Two Year Payback, Industrial, 33100 Lumen Sub-Market

Once again, best-in-class efficacy enables the white LED system to fare best in a discounted cash flow comparison. The model indicates that our decision maker is likely to favor it over metal halide in the mid-term future.

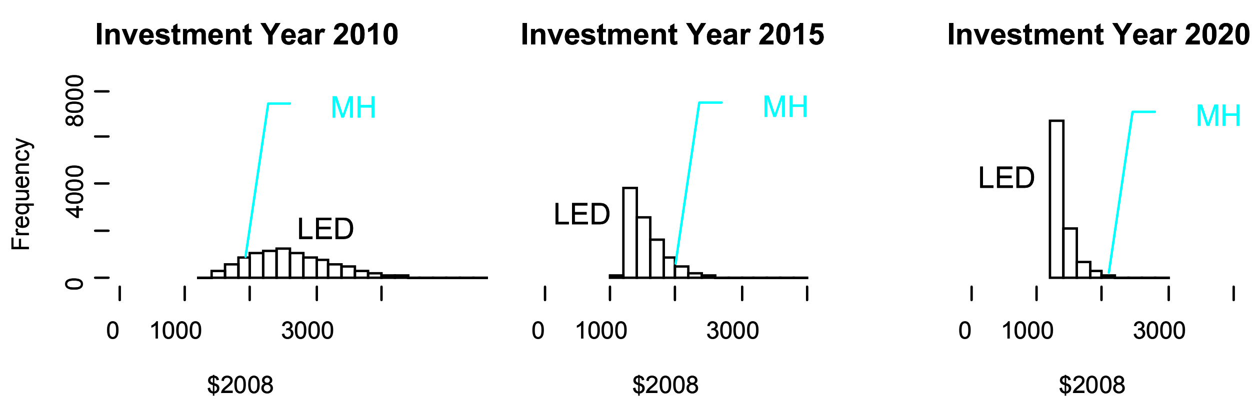

Figure 23: Discounted Present Cost, Industrial, 33100 Lumen Sub-Market; MH = metal halide

Output for Outdoor Stationary, 22000 Lumen Sub-Market

For the outdoor stationary sub-market we choose a high-pressure sodium lamp and ballast system as a comparator. High pressure sodium, like its HID brother has very inexpensive up-front costs. Our model indicates that it will be the preferred alternative for a *first cost* decision maker well into the future.

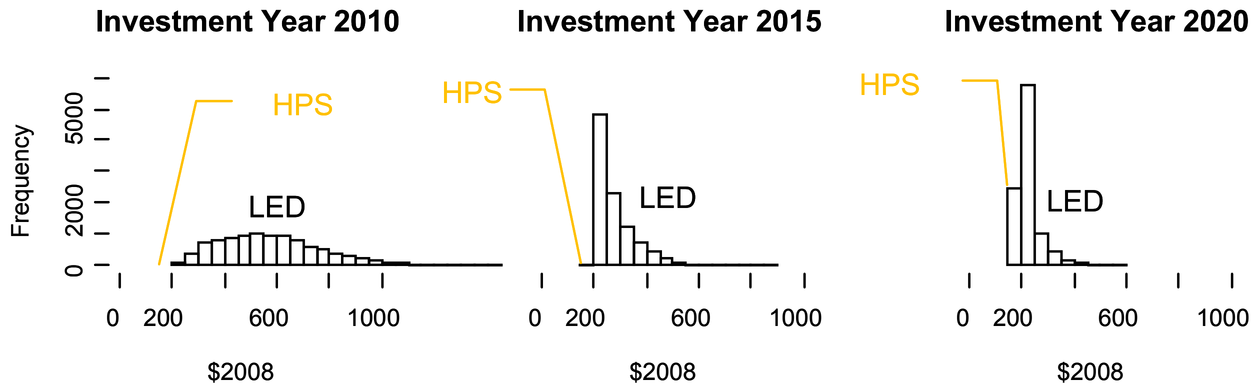

Figure 24: First Cost, Outdoor Stationary, 22000 Lumen Sub-Market; HPS = high pressure sodium

Since today's high pressure sodium lamps have high efficacy and low up-front costs, they are modeled to be the technology of choice for this sub-market for a *two year payback* decision maker. Like all comparisons in the output section our decision maker does not consider factors outside of costs when deciding. This is important because the high pressure sodium system we modeled has a CRI of only 22, compared to our white LED CRI of 90+. Therefore, it is possible that other concerns such as safety might swing our two year payback decision maker in favor of a white LED system.

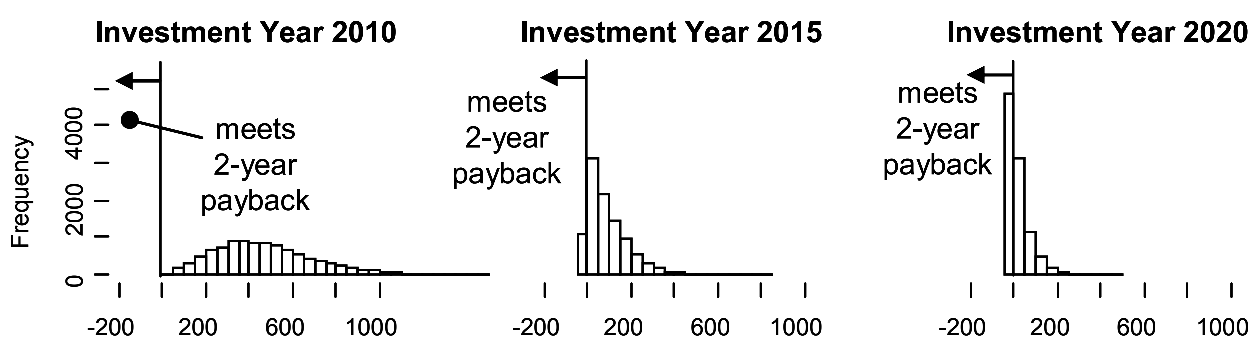

Figure 25: Two Year Payback, Outdoor Stationary, 22000 Lumen Sub-Market

The model predicts that our *discounted present cost* decision maker will likely shift to a white LED lighting system in the mid term future. This is contingent upon a scenario of nominal to strong LED efficacy improvement.

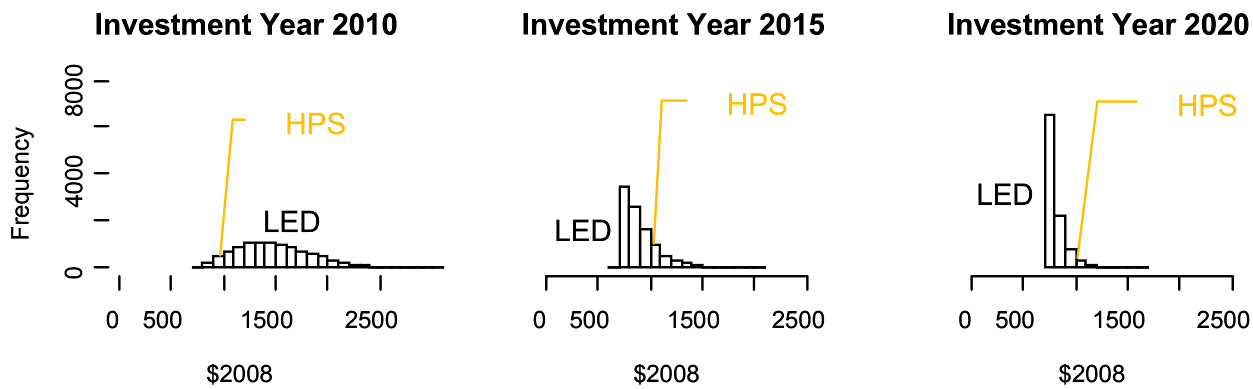

Figure 26: Discounted Present Cost, Outdoor Stationary, 22000 Lumen Sub-Market; HPS = high pressure sodium

Sensitivity Analysis

We tested each of the two random variables, *rate of cost reduction per one Watt white LED device* and *rate of LED efficacy improvement* to get a sense for which parameter has the larger effect on the cost of light in investment year 2013 (five years hence). For each sector, we separately set the two parameters to three levels each: the 5th, 50th (median), and 95th percentiles. Note that since we do not vary the parameters simultaneously, the model still outputs histograms due to the uncertainty of the unadjusted parameter.

We use the first cost method and the two-year payback method to model a decision-maker's lighting decision for the residential and commercial sectors respectively. We apply the discounted cash flow approach for both the industrial and outdoor stationary sectors. We choose these four decision methodology/sub-market combinations because we believe they most accurately reflect current decision makers' behavior.

Sensitivity for Residential 850 Lumen Sub-Market

The residential sector is clearly more sensitive to the cost reduction rate than it is to efficacy [Figure 27]. This is intuitive since electricity costs as well as downstream maintenance and replacement costs are ignored by our first-cost decision maker. It is interesting to note that there is still some sensitivity to efficacy. This is because higher efficacy translates into fewer 1 Watt devices, therefore a lower upfront cost, in order to meet the 850 Lumen design requirement. Recalling the earlier results of the competing technologies, only a 95% cost reduction begins to compete with the CFL, and no white LED scenario can compete with an incandescent bulb on first cost.

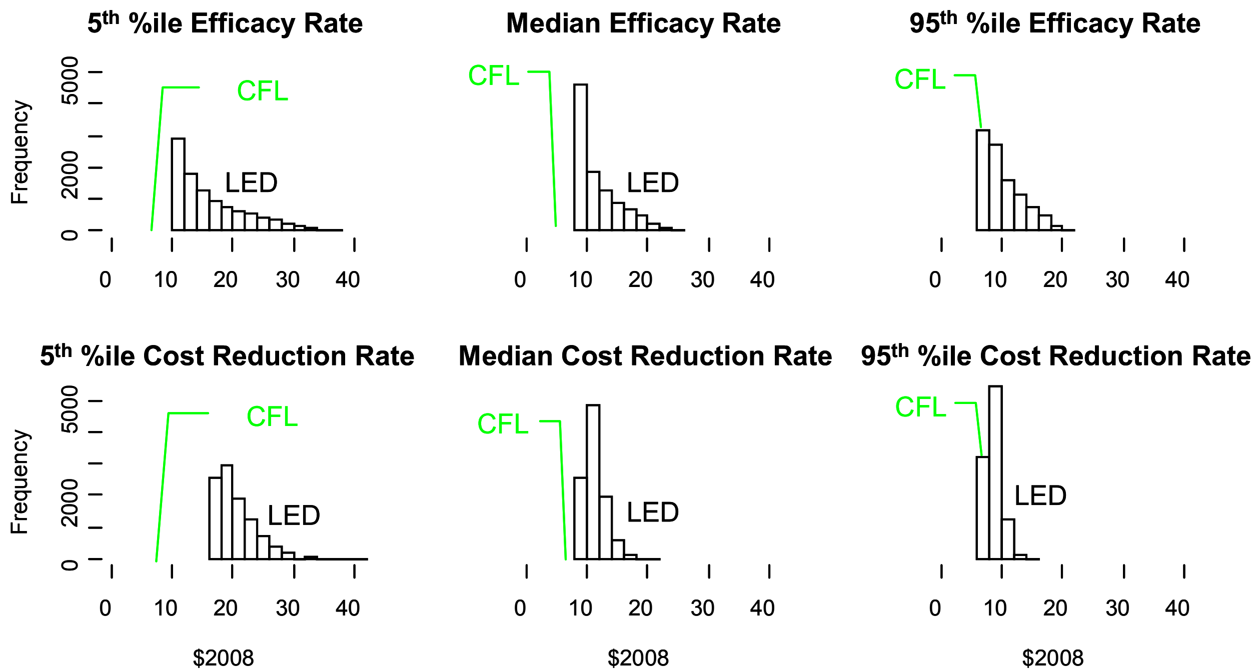

Figure 27: Sensitivity Analysis; First Cost Method, Residential Sector, Investment Year 2013

Sensitivity for Commercial 2300 Lumen Sub-Market

Efficacy plays a larger role with this sub-market than with residential decision makers since electricity costs begin to factor into the decision [Figure 28]. However, this two-year payback decision looks to be just about equally sensitive to *efficacy* and *cost per 1 Watt device*.

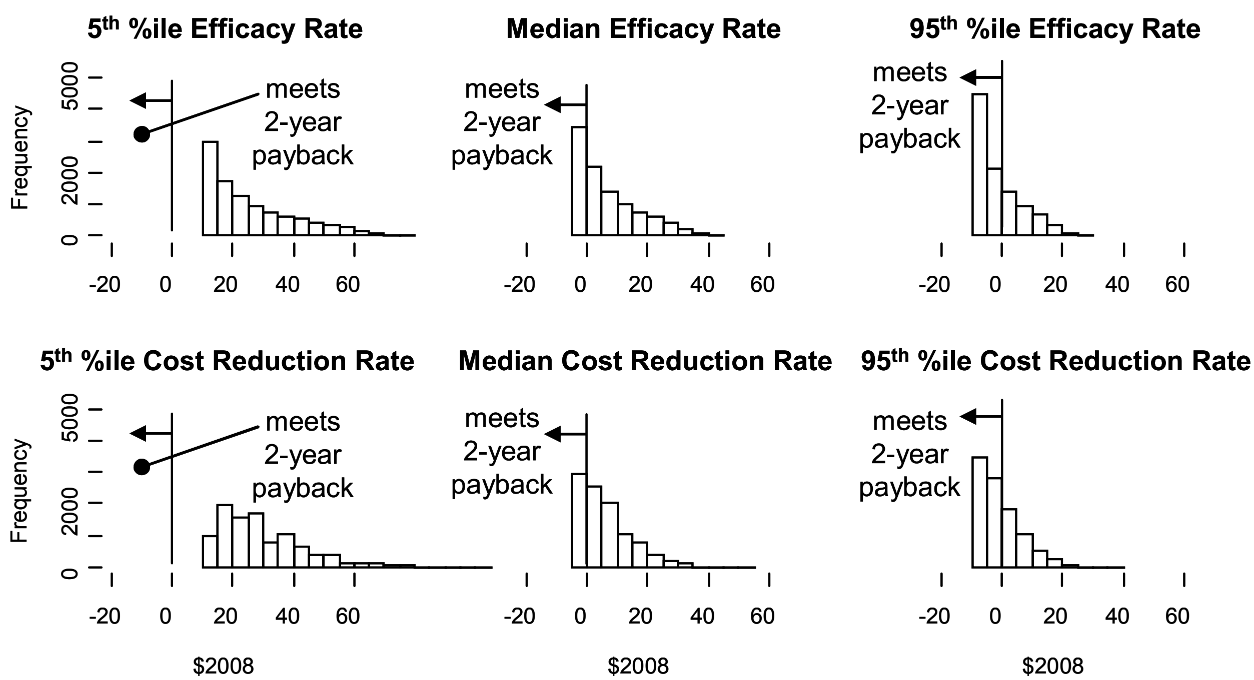

Figure 28: Sensitivity Analysis; Two-Year Payback Method, Commercial Sector, Investment Year 2013

Sensitivity for Industrial 33100 Lumen Sub-Market

Now that electricity, maintenance, and replacement costs are included over a longer duration, the rate of efficacy improvement becomes the dominant factor. The model predicts that if the 95th percentile rate of efficacy improvement were achieved, a discounted cash flow decision maker would almost certainly choose LED technology over metal halide HID technology by 2013.

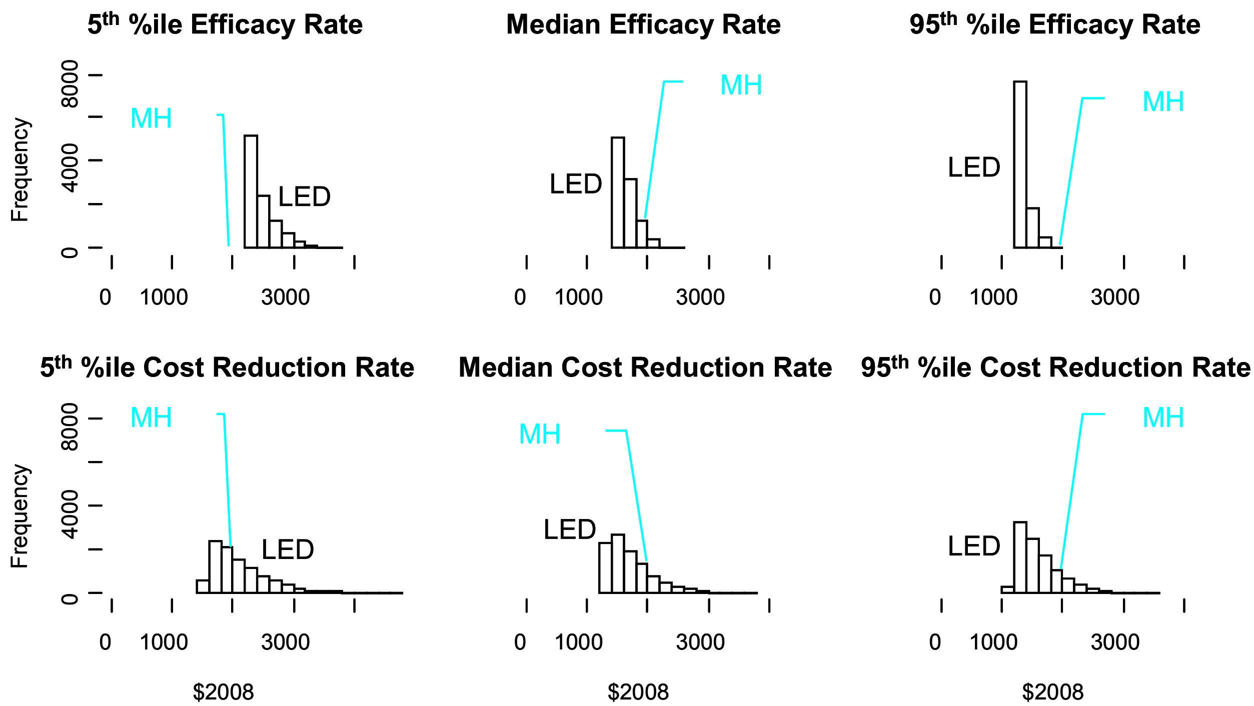

Figure 29: Sensitivity Analysis; Discounted Present Cost; Industrial Sector, Investment Year 2013

Sensitivity for Outdoor Stationary 22000 Lumen Sub-Market

The outdoor stationary analysis yields similar results [Figure 30]. Rate of efficacy improvement is once again more influential than the rate of cost reduction of the 1 Watt white LED device. Like the industrial sector analysis, the model predicts that if the 95th percentile rate of efficacy improvement were achieved, our discounted cash flow decision maker would almost certainly choose LED technology (this time over high pressure sodium technology) by 2013.

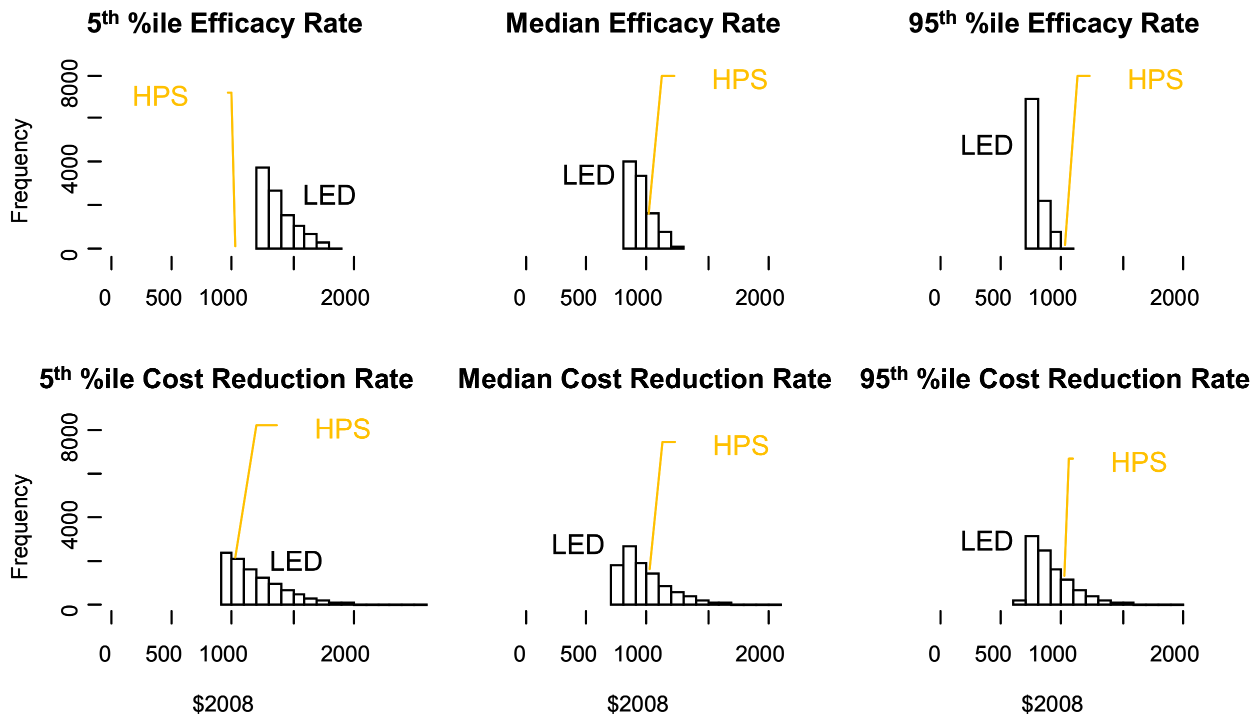

Figure 30: Sensitivity Analysis; Discounted Present Cost, Outdoor Stationary Sector, Investment Year 2013

Conclusions

Taking what we know now about improvements in lighting technologies, our model predicts that white LED lighting systems will first become cost competitive with high output metal halide lighting systems. The model predicts this tipping point to be at least five years into the future assuming that the decision makers in the industrial sector use a more sophisticated decision process than *first cost,* which we believe is likely. The next most vulnerable lighting technology is the 22000 lumen high pressure sodium system we modeled for the outdoor stationary sector. Although this technology may fall earlier to white LED systems as a consequence of the low CRI characteristic of sodium vapor lamps, our model predicts that cost will not likely be the deciding factor until the mid to distant future. Furthermore, it indicates that CFLs in the residential sector and T-8 fluorescent tubes will retain their cost advantage over white LED systems well into the future regardless of the financial decision making method. Although incandescent bulbs have such poor efficacy, they remain the technology of choice for a *first cost* decision maker in the residential sector given their low up-front cost.

Finally, our sensitivity analysis indicates that a white LED manufacturer looking to penetrate general illumination markets should focus its technological development differently depending on the specific sub-market it wishes to pursue. White LEDs will most quickly become cost competitive in the industrial and outdoor stationary sectors through rapid efficacy improvement. However, in the substantial markets of residential and commercial lighting where less-sophisticated financial decision making methods than discounted present cost are common, we recommend rapid cost improvement as a means of quickly becoming cost-competitive.

Appendix

Glossary of Lighting Terms

Lumen: The SI unit of luminous flux defined as the light emitted by a one candela point source passing through a unit solid angle. More simply, a lumen is a measure of the perceived power of light. Most commercially available lamps specify output in lumens. For example, a 60 Watt incandescent light bulb delivers approximately 850 lumens.

Lamp: A term used by the lighting industry to mean a device, such as a light bulb, tube or semiconductor package(s) that emits light. Not to be confused with the *luminaire* that sits on one's nightstand.

Luminaire: Used by the lighting industry to mean an assembly that might include a lens, diffuser, reflector, structural elements, etcetera that, when combined with a lamp and ballast/driver, comprise a completed lighting appliance.

Ballast: A device designed to control the flow of electric current in a circuit. Ballasts are used in lighting applications to deliver lamp-starting currents and to control a steady-state current. They are required in applications such as fluorescent lighting where the lamp does not regulate its own current.

Driver: Control circuitry that converts one form of electrical power to another. Used primarily to convert AC line power to low-voltage DC, which is required for solid state lighting devices such as LEDs. Sometimes it is called a *power supply*.

Efficacy: A measure of the lighting efficiency of a device or system measured in units of *lumens per Watt*. It is distinguished from the classical use of the term efficiency in that it includes only the output of visible radiation (lumens) and not the total power of the radiation. While theoretical maximum efficiency equals one, theoretical maximum *efficacy* equals 683 lumens per Watt in well-lit, or *photopic,* applications.

Color rendering index (CRI): A measure on a scale of 0 to 100, of the ability of a given light source to reproduce the colors of 14 pre-defined targets[45] in comparison with a reference light source. CRI is often used as a means to quantify the quality of a light source, although some experts criticize its use.

Correlated color temperature: According to the International Commission on Illumination it is *the temperature of the Planckian radiator whose perceived colour most closely resembles that of a given stimulus at the same brightness and under specified viewing conditions.* More simply, when a black-body radiator is raised to a high enough temperature it glows a certain color. This allows different colors of white light from any source to be described with the temperature (in degrees Kelvin) that a black-body radiator would have to reach to emit that same color. For example, a warm-white lamp might have a correlated color temperature of 3000K, whereas a cool-white lamp might be 5000K.

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[45] International Commission on Illumination, *Method of Measuring and Specifying Colour Rendering Properties of Light Sources*, Publication 13.3, 1995.

Natural Illumination

> *I say it is impossible that so sensible a people, under such circumstances, should have lived so long by the smoky, unwholesome, and enormously expensive light of candles, if they had really known, that they might have had as much pure light of the sun for nothing.* – Benjamin Franklin, Letter to the Editor of the Journal of Paris, 1784

Illumination sources can be divided into two categories, artificial and natural, where the distinction "natural" indicates visible radiation from the sun. This paper focuses on artificial illumination, but some new trends in natural illumination are worthy of note. The first is the practice of *daylighting*, an architectural technique using windows and reflective surfaces in order to provide illumination to the interior of a building. While daylighting is indeed an ancient practice, modern attention on energy efficient buildings has reawakened interest. The United States Green Building Council's Leadership in Energy and Environmental Design, or *LEED* standard, includes a new-construction credit if 75% of spaces within a building are daylit[46]. In addition, improved computer aided design tools for daylight simulation help today's architect make more informed design tradeoffs regarding the use of daylight.

A more recent daylighting technique developed at Oak Ridge National Laboratory is called *hybrid solar lighting*. Hybrid solar lighting uses a heliostatic dish [Figure 31] in combination with fiber optics to pipe natural light from building rooftops to internal luminaires. Some advantages to such a system over windows or skylights are: it delivers daylight without compromising the building's thermal envelope; it prevents unwanted solar heat gain; it can deliver daylight to difficult-to-reach locations of a building's interior. Demonstration systems have been active since 2006 and complete systems are now available for purchase.

Figure 31: HSL3000 from Sunlight Direct LLC

[46] United States Green Building Council, *LEED for New Construction*, ver. 2.2, credit 8.1.

Artificial Illumination

Three competing technologies make up the bulk of general illumination in the United States: incandescent, fluorescent, and high intensity discharge (HID). A fourth, the white LED, enjoys little market share but holds much promise for future market penetration. Each technology has its idiosyncrasies: some are long-lived, others have excellent color-rendering ability, still others have wide-ranging efficacies.

Table 4 offers a side-by-side comparison of the four technologies of interest in this paper across four important lighting metrics. Note that the two entries that are projected to increase the most dramatically in the coming decade are the efficacy and cost of the white LED. These are also the parameters whose exact rate of improvement is the most difficult to predict—an uncertainty that we later include in our model.

Table 4: Illumination Device Metrics

Technology:	efficacy [lumens/W]	cost [$/k-lumen]	CRI	Life [k-hour]	Commercial Advent
Incandescent	10-20	.20-1	100	0.8-4	1900s
Fluorescent	35-100	.50-5	60-85	8-30	1940s
HID	40-150	1-2	10-85	7.5-20	1930s
White LED	25-85	25-50	70-90+	35-50	2000s

Incandescence

Incandescent lamps emit light when a resistive element is Joule-heated until the point that its black-body radiation becomes visible. Such lamps are characterized by low efficacy because such a large percentage of the energy is radiated as heat, not light. They also have a relatively short lifespan, a high color rendering index, and a very low cost of manufacture. Incandescent bulbs were commercialized in the latter half of the 19^{th} century, and the modern Tungsten filament bulb emerged in the first decade of the 20^{th}. Incandescent lamps are unique in that it does not require a ballast or an electrical power supply to operate on line power. This feature helps keep the cost of this technology low.

Fluorescence

Fluorescent lamps radiate light via a more complex process. Electricity excites mercury vapor in order to create a plasma that in turn emits ultraviolet radiation. This invisible radiation is down-converted to visible light when it strikes phosphors painted on a glass tube causing them to *fluoresce*. Fluorescent lamps are characterized by medium to high efficacy, medium color rendering, moderate manufacturing cost, and long life. Commercialization of fluorescent lamps commenced in the late 1930's, and it is now the most popular source of light in the United States, as measured in lumen-hours.

High Intensity Discharge

HID lamps produce light by way of an electric arc struck between two electrodes. This technology is characterized by high efficacy, long life, generally poor color rendering, and medium cost. HID lamps tend to be used in applications where efficiency is valued, yet the lamp need not be cycled on and off frequently. Hence the outdoor stationary market is dominated by HID lamps.

Solid State Lighting

SSL includes any lighting technology that uses LEDs or organic light emitting diodes (OLED) as a source of illumination. LEDs produce light when the p-n junction of a diode is forward-biased causing electron-hole pairs to recombine and release photons. The wavelength of these photons is a characteristic of the junction material, which is designed to emit a specific color. Since LEDs are by nature monochromatic yet white light is by definition multi-chromatic, achieving solid state white light requires special treatment. One approach is to use an array of red, green, and blue LEDs, allowing for the possibility of dynamic tuning the color temperature of the white light. The more common approach however is to use a blue-light LED in tandem with a yellow phosphor-coated lens. LEDs are characterized by a very long lifespan, very small size, good color rendering, medium efficacy, mechanical durability and very high lamp (device) cost. Because of their small size, long life and relatively high efficacy, LEDs are preferred in monochromatic expensive-to-maintain lighting systems like traffic signals and exit signs. Their small size makes them common in portable white-lighting applications like flashlights, camping headlamps, and bicycle lights.

Model Abstraction

Technologies:

Incandescent	Compact Fluorescent	White Light Emitting Diode	High Intensity Discharge
I	CFL	LED	HID

Constants:

Symbol	Description	Units
TH	Time Horizon of interest	[years]
U_L	Lamp usage	[hours/day]
MC_E	2008 marginal cost of electricity	[$2008/kWh]
R_E	Annual rate of electricity price increase	[$/year]
MC_L	Marginal cost of labor	[$2008/hour]
T_R	Time needed to replace a lamp	[hours]
R_D	Discount Rate	dimensionless
R_E	Annual rate of increase in electricity prices	dimensionless
$Infl$	Annual rate of inflation	dimensionless

Functions of Technology, T:

Symbol	Description	Units
$F(T)$	luminous flux of bulb	[lumens]
$l_B(T)$	bulb rated life span	[hrs/bulb]
$r_L(T)$[47]	**annual rate of change of lamp cost**	dimensionless
$r_e(T)$[**Error! Bookmark not defined.**]	**annual rate of change of lamp efficacy**	dimensionless
$r_\eta(T)$	annual rate of change of driver efficiency	dimensionless
$r_{Dr}(T)$	annual rate of change of driver cost	Dimensionless

[47] For LEDs, the annual rate of change of the lamp cost and efficacy is modeled as a random variable

Functions of Technology T, and Time t, (years from 2008):

Symbol	Description	Units
$c_L(T,t) = (c_L(T,0) - c_L(T,\infty)) \cdot \exp(-r_L(T) \cdot t) + c_L(T,\infty)$	lamp cost	[$2008/bulb]
$e_L(T,t) = (e_L(T,0) - e_L(T,\infty)) \cdot \exp(-r_L(T) \cdot t) + e_L(T,\infty)$	luminous efficacy	[lumens/W]
$c_{Dr}(T,t) = (c_{Dr}(T,0) - c_{Dr}(T,\infty)) \cdot \exp(-r_{Dr}(T) \cdot t) + c_{Dr}(T,\infty)$	Driver cost	[$2008/driver]
$\eta_{Dr}(T,t) = (\eta_{Dr}(T,0) - \eta_{Dr}(T,\infty)) \cdot \exp(-r_{Dr}(T) \cdot t) + \eta_{Dr}(T,\infty)$	Driver efficiency	dimensionless

Formulae:

Symbol	Description	Units
$\tau_m(T) = \dfrac{l_B(T)}{365 \cdot U_B}$	Expected time until bulb reaches end of life	[yr/bulb]
$n_B(T) = \left\lfloor \dfrac{TH}{\tau_m(T)} \right\rfloor$	Expected number of bulbs replaced within TH	[total bulbs]
$E_t = \dfrac{365}{1000} u_B \cdot F(T) \cdot \dfrac{t_n - t_{n-1}}{e_{\mathbf{B}}(T, t_0 + k \cdot \tau_m(T)) \cdot \eta_{PS}(T, t_0 + k \cdot \tau_m(T))}$	Energy used in time period, t_n by the k[th] purchased lamp given no burnout in $[t_{n-1}, t_n)$	[kWh]
$E_t = \dfrac{365}{1000} u_B \cdot F(T) \cdot \left(\dfrac{t_0 + k \cdot \tau_m(T) - t_{n-1}}{e_{\mathbf{B}}(T, t_0 + k \cdot \tau_m(T)) \cdot \eta_{PS}(T, t_0 + k \cdot \tau_m(T))} + \dfrac{t_0 + t_n - k \cdot \tau_m(T)}{e_{\mathbf{B}}(T, t_0 + (k+1) \cdot \tau_m(T)) \cdot \eta_{PS}(T, t_0 + (k+1) \cdot \tau_m(T))} \right)$	Energy used in time period, t_n given the k[th] lamp burns out in $[t_{n-1}, t_n)$	[kWh]
$C_E(T,t) = 0.365 \cdot U_{\mathbf{B}} \cdot E_t \cdot MC_E \cdot \sum_{t=0}^{TH} \left(\dfrac{1 + R_E}{(1 + R_D)(1 + Infl)} \right)^t$	Total cost of electricity over time horizon	[$2008]
$C_M(T) = T_R \cdot MC_L \cdot \sum_{k=0}^{n_B} \dfrac{1}{(1 + R_D)^{k \cdot \tau_m(T)}}$	Discounted maintenance costs	[$2008]
$C_C(T,t) = \sum_{k=0}^{n_B} \dfrac{c_L(T,t) + c_{Dr}(T,t)}{(1 + R_D)^{k \cdot \tau_m(T)}}$	Discounted capital costs	[$2008]
$DPC = C_E(T,t) + C_M(T) + C_C(T,t)$	Discounted present cost	[$2008]