

**The Fourth International Conference on  
Complementarity Problems**

**Dedicated to the Memory of  
Carlton E. Lemke and George B. Dantzig**

Stanford University

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**ABSTRACTS OF TALKS**

## **WeA2** SUFFICIENT MATRICES BELONG TO L

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In this paper, we establish a significant matrix class inclusion that seems to have been overlooked in the literature of the linear complementarity problem. We show that  $P_*$ , the class of sufficient matrices, is a subclass of L. In the course of demonstrating this inclusion, we introduce other new matrix classes that forge interesting new connections between known matrix classes.

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## **ThA2** COMPLEMENTARITY-BASED MULTI-RIGID-BODY DYNAMICS SIMULATION

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We discuss recent results in hard constraint time stepping schemes for the simulation of multi-rigid-body dynamics. We present such schemes that achieve constraint stabilization and we discuss convergence of schemes that solve convex subproblems.

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## **WeM1** DECOMPOSITION AND SYNCHRONIZATION SCHEMES FOR SOLVING ASYMMETRIC USER EQUILIBRIUM

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The asymmetric user equilibrium can be formulated as a multi-commodity network flow problem with non-separable travel cost, and further a nonlinear complementarity problem (NCP) defined on disaggregated link flows. The NCP formulation has a special structure such that the defining set is a Cartesian product of a number of lower-dimension sets. Therefore, certain decomposition scheme, e.g. the Gauss-Seidel decomposition, can be applied. To further improve the convergence performance, a synchronization nonlinear programming problem (NLP) is developed by deriving a particular merit function for the decomposition scheme. Through solving the synchronization NLP, an optimal step size, instead of the conventional full step, is computed for obtaining the next iterate in the Gauss-Seidel algorithm. The structure of the

NCP formulation also motivates one to adopt the origin-based algorithm (OBA) which has been shown to be very efficient for solving symmetric user equilibrium. Some of the implementation issues for incorporating OBA in the proposed “decomposition + synchronization” scheme will be discussed in this presentation. Numerical examples are finally provided on large scale user equilibrium problems to test the presented model and solution approach.

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**WeA1** INTERIOR-POINT ALGORITHMS, PENALTY METHODS, AND EQUILIBRIUM PROBLEMS

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In this talk, we consider the question of solving mathematical programs with equilibrium constraints as nonlinear programs, using an interior-point approach. These problems pose theoretical difficulties for nonlinear solvers, including interior-point methods. We examine the use of penalty methods to get around these difficulties, and present substantial numerical results. We go on to show that penalty methods can resolve some problems that interior-point algorithms encounter in general.

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**TuM2** ON THE DIRECTIONAL DIFFERENTIABILITY OF THE TRAJECTORIES OF LINEAR COMPLEMENTARITY SYSTEMS

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This talk deals with linear complementarity systems, i.e. linear dynamical systems that are coupled to complementarity characteristics. These systems can be seen as dynamical complementarity problems. We first summarize the results on the existence and uniqueness of solutions to these problems. These results show that, similar to the linear complementarity problem, certain positivity-type conditions play a key role in the study of solvability issues. Under such conditions, one can further show that the solution trajectories have certain regularities such as continuous dependence on the initial data. One step further, one can ask whether the trajectories are directionally differentiable with respect to the initial data. We will demonstrate that the directional differentiability holds in case there is only one complementarity pair. As a side result, our treatment allows us to compute the derivative explicitly. The talk ends with a discussion on the possible extensions and limitations of our approach.

**TuM2** RECENT ADVANCES IN THE THEORY OF PROJECTED DYNAMICAL SYSTEMS (PDS)

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We present here some recent advances in the theory of PDS related to the existence of periodic solutions for such systems. We investigate the relation between the existence of periodic cycles, their stability properties and the existence of equilibrium points. Finally, we make some remarks about some recent applications of PDS in the double layered dynamics theory (DLD).

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**Banquet** LEGENDS AND LEGACIES OF CARLTON E. LEMKE AND GEORGE B. DANTZIG

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Our field owes much to Carlton E. Lemke and George B. Dantzig who passed away in April 2004 and May 2005, respectively. This talk is meant to be a tribute to these two men and a brief review of their enduring contributions to mathematical programming and complementarity.

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**WeM2** A COMPARISON OF ELECTRICITY MARKET DESIGNS IN NETWORKS

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In the US and Europe, two basic market designs are being debated, to address congestion in a looped transmission network. Because of high concentration in regional markets for electricity, one important criterion of evaluating the designs is their performance in the presence of market power. Emerging mathematical programming paradigms such as Equilibrium Programs with Equilibrium Constraints (EPEC) allow us to model these two market designs. We provide a systematic numerical comparison on a model of the Belgium, Dutch, German and French network and rank the designs according to their performance under market power.

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**WeM2** OPTIMIZING ENERGY BIDS FOR A SINGLE SUPPLIER IN A SPOT ELECTRICITY MARKET

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We present methodology for formulating and solving equilibrium problems with piecewise-linear supply and demand functions under perfect market information. This is a relaxation of

typical assumptions in the most-used set of equilibrium models, which require such functions to be linear. This relaxation allows for the direct comparison between Competitive Equilibria and Agent-Based Simulation Equilibria that utilize piecewise-linear bid functions in their formulations. The ability to make such a comparison helps establish a beneficial range for the agent-based results, since the competitive prices are seen to be at the lower range of possibilities and the prices resulting from perfect market information are seen to be at the upper range of possibilities.

We also describe a three-step process of formulating a so-called single-supplier problem, devising an algorithm for solving it, and then solving for the equilibrium among a multitude of such suppliers using a round-robin process of individual improvement. Through a simple example, the authors reveal that this round-robin process can demonstrate behaviors reminiscent of actual markets.

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**WeA2** SOME COMPUTATIONAL LEGACIES OF LEMKE AND DANTZIG IN COMPLEMENTARITY

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Pivoting methods, such as those championed by Lemke and Dantzig, have enjoyed remarkable success in the past 50+ years. While their use in linear programming is widely known, and has been refined significantly in both computational and theoretical settings, the adaptation of the technology to complementarity problems has remained a niche area until recently. We outline these strategies, explain some of their more recent enhancements and generalizations in the context of complementarity, and highlight some directions for future research and exploration.

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**WeA1** CONVERGENCE RESULTS FOR LEVENBERG-MARQUARDT TYPE ALGORITHMS

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Levenberg-Marquardt (LM) type subproblems and related techniques turned out to be an important tool for solving problems with non-isolated solutions if they satisfy a certain error bound condition. In this paper some new results on the local fast convergence of LM type algorithms will be presented. In particular, LM subproblems for solving nonsmooth equation-based reformulations of Karush-Kuhn-Tucker systems will be dealt with.

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**WeM1** TOLLS FOR HETEROGENEOUS, SELFISH USERS OF A MULTICOMMODITY NETWORK AND GENERALIZED CONGESTION GAMES

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In traffic networks, transit delay is a function of congestion. To provide a sufficient quality of service to customers, one goal of a network owner may be to have traffic that minimizes total, or average, system delay. On the other hand, if customers can choose their own routes, they would naturally seek routes that minimize their individual delay. The traffic pattern that results from customers acting “selfishly” may result in overall higher system-wide delay. A natural question is: how can a network owner influence selfish network users to independently choose a traffic pattern that minimizes system-delay?

One natural approach is to charge network users for use of each link in the network via tolls. In this setting, each user selects a path that minimizes some function of delay plus tolls. Each user may have a different preference for delay versus toll. In such a setting, we give a complete characterization of traffic patterns that can be induced via tolls. These include patterns that minimize mean delay, mean weighted delay, and maximum delay. Our proof is constructive—yielding a polynomial time algorithm—and extends to general congestion games.

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**WeM2 DYNAMIC OLIGOPOLISTIC ELECTRIC POWER NETWORK COMPETITION WITH RAMPING COSTS AND JOINT CONSTRAINTS**

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In this talk we describe dynamic oligopolistic competition among producing firms who participate in an electric power network market. The electric power network is represented explicitly, and the producing firms are modeled as Cournot-Nash agents. Furthermore, both ramping costs and joint constraints arising from regional sales caps are considered. Most previous Cournot-Nash models of competition among electricity generators take a static perspective, resulting in finite dimensional variational and quasi-variational inequality formulations. In particular, these finite dimensional models typically do not explicitly consider a generating company’s ramping costs and constraints and, thereby, fail to capture an important dynamic feature of power networks that influences both pricing and generation. In contrast to such antecedent literature, this talk presents a model that includes (i) explicit intra-day dynamics that describe the market’s evolution from one Generalized Cournot-Nash Equilibrium to another for a 24 hour planning horizon, (ii) ramping costs for changing the power output of generating units, and (iii) profit maximizing behaviors of the generating firms expressed as coupled continuous time optimal control problems subject to the joint regional sales cap constraints mentioned above when these are expressed in terms of both own and non-own control (sales) variables. Such joint caps can be used to represent transmission constraints in, for instance, radial transmission systems.

The coupled optimal control problems with joint constraints describe a dynamic generalized Nash equilibrium problem (GNEP), which may be articulated as a differential quasi-variational inequality (DQVI). Because static GNEPs are well known to have multiple so-

lutions and to pose computational challenges that can cause traditional algorithms for variational inequalities to stall or fail, the DQVI we present poses significant computational challenges. We discuss an algorithmic approach for DQVIs based on a restricted formulation for which the underlying instantaneous GNEPs are expressed as partitioned variational inequalities which we solve using an implicit fixed point algorithm. We provide a numerical example based on a 3-node, 3-arc network with 3 firms. Each firm owns 2 generation units (usually one with lower generation capacity but faster ramping capability, and the other with higher generation capacity but lower ramping capability) at each of the nodes. Each of these nodes represents a regional market with an exogenous sales cap. The total number of variables following time discretization is over 1600.

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**TuA2** THE Q-PROPERTY OF COMPOSITE TRANSFORMATIONS AND THE P-PROPERTY OF STEIN-TYPE TRANSFORMATIONS ON SELF-DUAL AND SYMMETRIC CONES

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Motivated by the Q-property of nonsingular M-matrices, Lyapunov and Stein transformations (corresponding to positive stable and Schur stable matrices, respectively) and their products, in the first part of the talk we present a unifying result on the product of Q-transformations defined on self-dual closed convex cones. The second part deals with the P-property of the linear transformation I-S on a Euclidean Jordan algebra where S leaves the corresponding symmetric cone invariant and the spectral radius of S is less than one. We describe the P-property for the Lorentz cone and present some partial results in the general case.

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**ThA1** SOLVING STOCHASTIC MATHEMATICAL PROGRAMS WITH COMPLEMENTARITY CONSTRAINTS USING SIMULATION

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We consider a class of stochastic mathematical programs with complementarity constraints, in which both the objective and the constraints involve limit functions or expectations that need to be estimated or approximated. Such programs can be used for modeling “average” or steady-state behavior of complex stochastic systems. Recently, simulation-based methods have been successfully used for solving challenging stochastic optimization problems and equilibrium models. In this talk, we report some of the recent progress we had in broadening the applicability of so-called the sample-path method to include the solution of certain stochastic mathematical programs with equilibrium constraints. We first describe the class of stochastic mathematical programs with complementarity constraints that we are interested in solving and compare it with different stochastic mathematical programs with equilibrium constraints

from the literature. We then describe the method and provide sufficient conditions under which appropriate approximating problems will have solutions and these approximate solutions will converge to a solution of the original problem almost-surely. We also illustrate an application of the method to solving a toll pricing problem in transportation networks. These developments also make it possible to solve certain stochastic bilevel optimization problems and Stackelberg games, involving expectations or steady-state functions, using simulation.

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**ThM1** COMPLEMENTARITY EMERGES IN IMAGING

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In this talk, novel applications of complementarity concepts and algorithms in image sciences are highlighted. The focus will be on variational methods in image restoration based on the TV-paradigm. The TV-regularization, which is responsible for the desirable property of preserving edges in images, is nondifferentiable and it necessitates a non-reflexive Banach-space setting. Both aspects burden algorithmic developments (such as design and analysis). In a first part, a pre-dualization process is introduced which results in a Hilbert space problem which can be solved by complementarity techniques. It is also shown that certain (pre)dual quantities act as edge detectors. In a second part, an alternative approach is addressed which uses a local smoothing technique rather than pre-dualization. For both approaches numerical results are discussed.

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**ThA1** STOCHASTIC APPROXIMATION APPROACHES TO THE STOCHASTIC VARIATIONAL INEQUALITY PROBLEM

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Stochastic approximation methods have been extensively studied in the literature for solving systems of stochastic equations and stochastic optimization problems for which function values and first order derivatives are not observable but can be approximated through simulation. We consider stochastic approximation methods for solving stochastic variational inequality and stochastic complementarity problems. Applications of stochastic approximation methods in economics and revenue management are discussed.

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**FrM1** EQUILIBRIUM ANALYSIS FOR A NETWORK MARKET MODEL

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In this paper we model and analyze a market equilibrium structure working on a network. The model is motivated by competition in electricity power generation markets, where consumers

and producers are located in different nodes connected by power transmission lines. We analyze two different equilibrium concepts, namely, the Walrasian and the noncooperative Nash outcomes. By using concepts coming from Variational Analysis and Game Theory, we prove that both equilibria exist. Our existence proofs rely on fixed point theorems and epi-convergence stable approximations.

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**TuA1** A GAME THEORETIC VIEW OF EFFICIENCY LOSS IN RESOURCE ALLOCATION

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Motivated by resource allocation problems in networked systems, we consider the design of market mechanisms for such settings which are robust to gaming behavior by market participants. Recent results in this work will be reviewed, including: (1) efficiency loss guarantees for a data rate allocation mechanism first proposed by Kelly, both when link capacities are fixed and when they are elastic; (2) characterization of mechanisms that minimize the efficiency loss, within a certain class of “simple” mechanisms; (3) extensions to general networks; and (4) mechanism design for supply function bidding in electric power systems (if time permits).

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**TuM1** OPTIMAL INCOME TAXATION WITH MULTIDIMENSIONAL TAXPAYER TYPES

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Beginning with Mirrlees, the optimal taxation literature has generally focused on economies where individuals are differentiated by only their productivity. In this paper we examine models where individuals are differentiated by two or more characteristics. For example, we examine the case where individuals also have different tastes for labor supply. We find that the extra dimensionality produces substantively different results, such as negative marginal tax rates for some high productivity taxpayers. This is a difficult nonlinear optimization problem. We evaluate the performance of alternative NLP solvers, including SNOPT, MINOS, IPOPT, and Filter.

## FrM2 ON THE MINIMUM TOLL BOOTH PROBLEM

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In this talk, we address a toll or congestion pricing problem whose objective is to reduce congestion or travel delay in a transportation network by imposing tolls on roads and highways. Because each tolled road requires a collection facility (such as a toll booth and an electronic sensor) and the public is generally not eager to accept the idea of congestion pricing, our goal is to find a solution with the least number of tolled roads, a problem we refer to as the minimum toll booth problem or MINTB. We show that MINTB is NP-complete and can be formulated either as an integer programming problem or a mathematical program with complementarity constraints (MPCC). As an integer program, one heuristic algorithm for MINTB is the dynamic slope scaling procedure (DSSP), an algorithm proposed by Kim and Pardalos in 1999 and appeared to be unrelated to other optimization techniques. However, we show that DSSP can be viewed as solving the MPCC version of MINTB via Lagrangian relaxation.

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## TuA1 SOLVING MULTI-LEADER-FOLLOWER GAMES

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Multi-leader-follower games arise when modeling competition between two or more dominant firms and lead in a natural way to equilibrium problems with equilibrium constraints (EPECs). We examine a variety of nonlinear optimization and nonlinear complementarity formulations of EPECs. We demonstrate the practical viability of our approach by solving a range of medium-sized test problems.

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## WeM2 ON THE COLLUSIVE GAMES IN TRANSMISSION-CONSTRAINED ELECTRICITY MARKETS

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In deregulated electricity markets, market power abuse by generators is a serious concern among regulators and consumers. This issue has been discussed extensively in the literature under the context of static games. However, when a game is repeated infinitely, non-

cooperative players may behave as if they collectively maximize their joint profits, a phenomenon known as tacit collusion. It is well known in industrial organization literature that collusion can impede competition and significantly reduce consumer surplus, yet the effects of collusive games among generators in electricity markets are not well studied. In this talk, we first discuss a general type of collusive games among asymmetric players that can be modeled as MPECs. The MPEC structure arises because of the presence of incentive compatibility constraints that state that collusive profit for each player must be no less than the present worth of profit from cheating and subsequently being punished. These constraints require characterization of every players optimal deviations from the collusive equilibrium, yielding a set of equilibrium conditions for each player to be imbedded in the collusive optimization. To find the globally optimal solutions, we propose a branch-and-bound type algorithm tailored to the specific problem and to the complementarity feature of MPEC constraints. Then the model and the algorithm are extended into the case of electricity markets with linearized DC transmission networks. Test problems and numerical results will be presented. Finally, we will explore some extensions of our model, including models with endogenized capacity decisions in a two-stage game constraints.

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#### **WeA1** REGULARIZATION VIA PENALTY METHODS

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We study a regularization approach, via exact penalization, for both active-set and interior methods. The goal is to efficiently solve large-scale problems that violate standard constraint qualifications at the solution (e.g., MPCs and switch-off problems). We propose a practical updating strategy to handle the penalty parameter that promotes feasibility at every iteration. The method improves on the infeasibility detection capabilities of practical algorithms and can be coordinated with a barrier scheme. We demonstrate the features of the algorithm with numerical results.

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#### **WeM1** SENSITIVITY OF STATIC TRAFFIC USER EQUILIBRIUM

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This paper deals with sensitivity analysis of static traffic user equilibrium problems. We introduce perturbation parameters into the travel demand and the arc cost function of such a problem and then study the sensitivity of the equilibrium arc flow under those perturbations. By applying some recently developed sensitivity analysis techniques for general variational conditions, we show that under mild conditions the equilibrium arc flow is B-differentiable with respect to the perturbation parameters, and it is differentiable when additional conditions hold. We provide methods to calculate the B-derivative or the derivative when these exist. The sensitivity results do not rely on the choice of any particular route flow solution.

**FrM2** TWO PROBLEMS INVOLVING A BI-CRITERION NETWORK EQUILIBRIUM MODEL

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Let us consider a network equilibrium problem where each user's disutility is a linear combination of latency and cost. Assuming that the density of the value-of-time parameter across the population is known, an equilibrium can be characterized as a solution of an infinite-dimensional variational inequality. This talk addresses two aspects of the problem. First, we analyze an inexact variant of a linearly convergent algorithm proposed by Marcotte and Zhu at the first ICCP (Baltimore). Next we investigate, from the theoretical and numerical points of view, inverse problems that arise naturally in the modelling of two pricing problems. The first problem, considered by several authors, is about inducing system-optimal flow patterns through suitable tolls, while the second consists in maximizing the revenue raised from tolls set on the links of a transportation network.

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**TuA1** THE PRICE IS RIGHT

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In this talk, I will present current work on generalized Nash, Stackelberg, and multi-leader-identical-follower games, concentrating on the development of price-consistent formulations, which are restrictions of the original problem. Illustrative examples and numerical results will be provided.

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**ThM1** AN EPEC ALGORITHM FOR A TWO-SETTLEMENT ELECTRICITY MARKET EQUILIBRIUM MODEL

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This paper studies two-settlement electricity markets with horizontal market power, flow congestion, demand uncertainties and probabilistic system contingencies. The Nash equilibrium therein is modeled as an equilibrium problem with equilibrium constraints (EPEC), in which each Cournot generation firm solves a mathematical program with equilibrium constraints (MPEC). Under certain assumptions, these MPECs have quadratic objective functions and their shared inner problem forms a parametric linear complementarity problem (LCP). The size and complexity of this model present a computation challenge. An MPEC algorithm is developed based on sequential quadratic programs and parametric LCP pivoting. We further

develop an EPEC algorithm which iteratively invokes the MPEC algorithm. Numerical examples from the stylized Belgian electricity network show the effectiveness of these algorithms.

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**ThA2** QUASI-RIGID CONTACT WITH FRICTION

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Complementarity problems arise naturally in modeling contact in computer graphics, biomechanics, robotics, and other engineering fields, but much of the work to date has focused on rigid contact. In this talk, I will consider complementarity problems that arise in “quasi-rigid” contact models, that is, models in which elastic deformation can be separated from inertial dynamics. I will describe how complementarity problems arise in this setting, and show that they are generally easier to solve than in the rigid contact setting. I will also show how this approach extends to contact with friction.

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**TuA1** OLIGOPOLY COMPETITION: BERTRAND VERSUS COURNOT

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We study an oligopoly where firms compete on differentiated products. Each firm proposes a contract that specifies what information it wishes to communicate to the market. Special cases include the well-known Bertrand (price) and Cournot (quantity) competition models. This analysis allows us to study the open problem of comparing Bertrand and Cournot total payoffs. We characterize cases where the total payoff under Cournot competition is at least as high as under Bertrand competition. We also present counterexamples, where the reverse conclusion holds. Furthermore, we focus on special cases of demand functions in order to obtain insights into the factors affecting the difference between the two problems. Finally, we discuss an extension of this model that incorporates a multi-period setting over a finite time horizon as well as allows finite capacities.

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**ThA1** SELF-ENFORCING AGREEMENTS AND INTERNATIONAL TRADE IN GREENHOUSE GAS EMISSION RIGHTS

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The success of any international climate change agreement depends on abatement targets and

the incentives for countries to participate. We demonstrate that international emission trading is effective in making headway on both issues despite the assumption that countries choose their permit endowments non-cooperatively. Developing countries are lured into a trading system by the prospective rents from permit sales. Developed countries benefit from the reduced cost of emission abatement. Using a calibrated representation of the global economy in seven sectors and six regions, we find that the most effective permit-trading agreements are sub-global and yield abatement at approximately twice the level achieved in a world without permit trading.

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## **TuA2** LINEAR SYSTEMS OF INTERVAL EQUATIONS AND THE LCP

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A linear system of interval equations is defined by a linear system of equations, say  $Ax=b$ , where the components of  $A$  and  $b$  are not explicitly known but can be enclosed in intervals. These uncertainties arise, for example, from roundoff errors and measurements with given tolerances, respectively. They also arise from other applied mathematical problems such as bounding the solution to certain two-point boundary-value problems for ordinary differential equations and enclosing the zero of a function, respectively. In 1989, Jiri Rohn proved as a byproduct that the LCP arises when a linear system of interval equations is to be solved in some sense. We want to emphasize this application and give some further developments.

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## **FrM1** FINDING ALL EQUILIBRIA IN STATIC AND DYNAMIC GAMES

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Static and dynamic games are often used to analyze strategic interactions. We consider static and dynamic games in continuous strategies and show that if a game reduces to solving a system of polynomial equations, algorithms for finding all solutions to polynomial systems can be effectively used. We first illustrate this for a static Bertrand game. While most dynamic games are far too large for a direct application of this approach, we study a common type of dynamic games where equilibrium can be analyzed as a sequence of small games and apply an all solutions algorithm to each such game. We apply this to an R and D race, a cost-reducing investment game, and a learning curve game to show that this approach is practical given current computational technology.

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## **TuM2** DYNAMICS OF EQUILIBRIA

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Equilibrium in the multiple-agent sense and equilibrium in the dynamical-system sense are two different concepts, which however may be connected. A classical link between the two notions lies in the study of dynamic stability of economic equilibria. In this talk a different connection is discussed, which is based on a separation between time scales. Economic (multiple-agent) equilibrium is supposed to be reached quickly. On a longer time scale, the parameters of the economic equilibrium problem change; therefore, the equilibrium moves in the course of time. The laws of evolution of the environmental parameters depend in general on the location of the economic equilibrium, so that there is a two-way interaction between the time scales. For the purpose of a simplified description, the short-term equilibrium is taken to be reached instantaneously. We then obtain a system that is always in economic equilibrium and that may or may not eventually reach dynamic equilibrium. In the course of the system's evolution, inequality constraints can become active or inactive and so the dynamics is typically nonsmooth. A general model for this type of "dynamics of equilibria" can be written down as a mixed complementarity system, that is, as a system of ordinary differential equations with additional variables that are connected by a mixed complementarity problem. Examples are shown of problems in industrial organization and in user-resource interaction that can be modeled in this way.

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**TuM2** NON-ZENONESS, OBSERVABILITY AND STABILITY OF COMPLEMENTARITY SYSTEMS

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This talk is concerned about certain analysis and control issues of complementarity systems. The first issue is the non-Zeno behavior of complementarity systems, which has many interesting consequences for numerical and system/control analysis of complementarity systems. We will introduce several non-Zeno concepts and show the non-Zenoness in strong or weak sense for strongly regular differential variational inequalities and a class of linear cone complementarity systems respectively. We then show the local observability conditions derived based upon the non-Zeno results. If time permits, we will also talk about our recent work on Lyapunov stability of complementarity systems with  $C^1$  state trajectories.

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**ThM1** THE ROLE OF LONG TERM CONTRACTS FOR STRATEGIC INVESTMENT DECISIONS IN OLIGOPOLISTIC ELECTRICITY MARKETS

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There is growing concern about insufficient investment in generation in the European elec-

tricity market. Long term contracts are known to favor investment and could be a possible remedy. However, the European competition authorities dislike long term contracts because they foreclose the markets. We compare two numerical investment models for electricity generation: The first model simulates an investment mode where plants sell all production in the moment of investment, using long term contracts. The second model represents merchant investment plants in which the generators make their decisions in anticipation of the outcome of the spot market. We give computational results for the North-West European electricity market and analyze the impact of existing capacities and the organization of transmission.

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**ThA2** IMPACT PROBLEMS FOR ELASTIC BODIES

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Impact problems for elastic bodies are dynamic infinite dimensional problems that are represented by partial differential equations. The author (in collaboration with Jeongho Ahn) has been able to prove not only existence of solutions, but also the convergence of time and space discretizations, for a class of these problems. The talk will outline the essential issues in these results, and show some numerical results. The question of what happens to the energy for these solutions will also be discussed, with some general principles presented which have the potential to be applicable to these problems.

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**FrM1** MPEC APPROACHES TO MORAL-HAZARD PROBLEMS

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We introduce MPEC approaches to moral-hazard problems, including deterministic contracts as well as contracts with action and/or compensation lotteries. To avoid local solutions for the MPEC formulation, we propose a hybrid procedure that combines the best features of the MPEC approach and the linear programming (LP) approach with lotteries. The hybrid procedure obtains a solution that is, if not global, at least as good as an LP solution. The numerical results on an example show that the hybrid procedure outperforms the LP approach in both computational time and solution quality in terms of the optimal objective value.

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**TuA2** A DUAL OPTIMIZATION APPROACH FOR INVERSE QUADRATIC EIGENVALUE PROBLEMS WITH PARTIAL EIGENSTRUCTURE

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The inverse quadratic eigenvalue problem (IQEP) arises in the field of structural dynamics.

It aims to find three symmetric matrices, known as the mass, the damping and the stiffness matrices, respectively such that they are closest to the given analytical matrices and satisfy the measured data. The difficulty of this problem lies in the fact that in applications the mass matrix should be positive definite and the stiffness matrix positive semidefinite. Based on an equivalent dual optimization version of the IQEP, we present a quadratically convergent Newton-type method whose high efficiency is confirmed by our numerical experiments.

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**TuM1** CONNECTIONS BETWEEN DYNAMIC AND QUASISTATIC MODELS OF MULTI-RIGID-BODY SYSTEMS WITH UNILATERAL CONTACT

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A family of pseudo-dynamic models of multi-rigid-body systems with unilateral contacts is formed by scaling the time variable of the Newton-Euler equation. When the scale parameter is set to one, the Newton-Euler equation is unchanged. As the scaling parameter increases to infinity, the speed of the system goes to zero, thus yielding a quasistatic system. The properties of this family of models are explored, especially the convergence of solution trajectories obtained by solving discretized versions. A theorem for solution uniqueness of several time-stepping methods is presented along with simulation results for a simple illustrative example.

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**ThM2** RECENT APPROACHES TO COMPLEMENTARITY IN FUNCTION SPACES

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Recently, it was discovered that modern techniques for finite-dimensional complementarity and related problems allow for elegant and computationally efficient extensions to a function space setting. This includes important problem classes, e.g., constrained optimal control, variational inequalities and optimization problems governed by variational inequalities (i.e., infinite-dimensional MPECs). In this talk, we discuss the state of the art of semismooth Newton methods for complementarity problems in function spaces, in particular recent achievements and future challenges. Both, theoretical and numerical aspects are addressed. Fur-

thermore, we plan to report on work in progress on infinite-dimensional MPECs and their numerical solution.

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**ThM2** PRIMAL-DUAL INTERIOR POINT MULTIGRID METHODS FOR COMPLEMENTARITY PROBLEMS IN FUNCTION SPACE WITH APPLICATIONS TO PDE-CONSTRAINED OPTIMIZATION

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We present recent results on primal-dual interior point methods for complementarity problems in function space, where the complementarity condition is imposed almost everywhere between a function in  $L^p$  and a function in the dual space. In particular, optimality systems for control constrained optimization problems with PDEs as well as regularized obstacle and contact problems lead to this class of problems. As in the finite-dimensional case the basic concept of the proposed interior point method consists in following the central path by using damped Newton steps inside a neighborhood of the central path. However, the algorithmic details and the convergence analysis are strongly influenced by the infinite-dimensional nature of the problem. We present recent results on the adequate formulation, convergence theory and numerical implementation of interior point methods for infinite-dimensional complementarity problems arising in particular in PDE-constrained optimization. Results on global and superlinear local convergence are presented. The convergence analysis differs significantly from the classical finite dimensional setting and leads to convergence results that are independent of the dimension of the discretization. Moreover, we show how multigrid methods can be used for the fast solution of the arising primal-dual Newton system. We illustrate the efficiency of the resulting interior point multigrid method by numerical results.

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**ThM2** CONTROL REDUCED INTERIOR POINT METHODS IN FUNCTION SPACE

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The talk discusses interior point methods applied to elliptic PDE constrained optimization problems. Existence and convergence of the central path in the infinite dimensional function space setting are established. Primal-dual and control reduced primal variants of interior point algorithms are directly applied to the infinite dimensional problem, involving discretization only in the innermost loop when solving linear equations in order to obtain inexact Newton corrections. The talk addresses linear and superlinear convergence of short step pathfollowing methods as well as adaptive stepsize control and accuracy matching for actual implementation. In particular, the order of discretization errors is discussed.

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**TuM1** ON THE STABILITY OF EQUILIBRIUM POINTS

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The stability of equilibrium points in Economics and Mechanics, as well as the stability of

the solutions of variational inequalities, fixed points, the solutions of inclusions, etc. can be studied in the following framework: Consider sequences of bivariate functions, defined on a product space, and analyze the stability of their (one-sided) maxinf-points. A convergence notion for bivariate functions is introduced that guarantees the convergence of their maxinf-points. These results are then exploited to the various applications mentioned earlier.

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**FrM1** A DECOMPOSITION ALGORITHM FOR N-PLAYER GAMES

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From a game with  $N$  players we construct a game with  $N+1$  players whose equilibria yield approximate equilibria of the original game. In the new game the  $N$  players interact bilaterally with the new player (the coordinator) but not with each other. In the resulting linear complementarity problem, decentralized calculation for each player separately, and generating columns only as needed, enables efficient calculation of an equilibrium. Experience with an implementation of the algorithm will be reported.

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**WeA1** CONVERGENCE OF NONLINEAR PROGRAMMING FORMULATIONS OF MPECs

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MPECs can be formulated as nonlinear programs by using regularization and penalization schemes, which account for the complementarity constraint in a fashion that avoids the lack of constraint qualification inherent in the obvious formulation. This talk describes the relationship between solutions of the nonlinear programming formulations and solutions of the MPEC, and describes local and global convergence properties of algorithms based on these formulations. We focus in particular on the exact penalty (elastic-mode) formulation.

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**ThA1** CONVERGENCE ANALYSIS OF SAMPLE AVERAGE APPROXIMATION METHODS FOR A CLASS OF STOCHASTIC MATHEMATICAL PROGRAMS WITH EQUALITY CONSTRAINTS

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In this talk we discuss Sample Average Approximation (SAA) method for a class of stochas-

tic programs with equality constraints. We use the Strong Law of Large Numbers for random compact sets due to Artstein and Vitale (Z. Artstein and R.A. Vitale, *The Annals of Probability*, Vol. 3, pp. 879-882, 1975) to investigate the convergence of generalized Karush-Kuhn-Tucker points of SAA programs as sample size increases. We also study the exponential convergence rate of the sample average approximations to global (local) minimizers of the true problem. The convergence analysis is extended to a smoothed SAA program. Finally we apply the established results to a class of stochastic mathematical programs with complementarity constraints.