Designing Nonlinear Price Schedules for Urban Water Utilities to Balance Revenue and Conservation Goals*

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Abstract

This paper formulates and estimates a household-level, billing-cycle water demand model under increasing block prices that accounts for the impact of monthly weather variation, the amount of vegetation on the household's property, and customer-level heterogeneity in demand due to household demographics. The model utilizes US Census data on the distribution of household demographics in the utility's service territory to recover the impact of these factors on water demand. An index of the amount of vegetation on the household's property is obtained from NASA satellite data. The household-level demand models are used to compute the distribution of utility-level water demand and revenues for any possible price schedule. It can be used to design nonlinear pricing plans that achieve competing revenue or water conservation goals, which is crucial for water utilities to manage increasingly uncertain water availability yet still remain financially viable. Knowledge of how these demands differ across customers based on observable household characteristics can allow the utility to reduce the utility-wide revenue or sales risk it faces for any pricing plan. Knowledge of how the structure of demand varies across customers can be used to design personalized (based on observable household demographic characteristics) increasing block price schedules to further reduce the risk the utility faces on a system-wide basis. For the utilities considered, knowledge of the customer-level demographics that predict demand differences across households reduces the uncertainty in the utility's system-wide revenues from 22 to 84 percent. Further reductions in the uncertainty in the utility's system-wide revenues, in the range of 10 to 79 percent, are possible by re-designing the utility's nonlinear price schedules to minimize the revenue risk it faces given the distribution of household-level demand in its service territory.

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1 Introduction

There is a growing need for urban water utilities to manage periods with limited water supplies, particularly in arid parts of the United States. Because more that 85 percent of the total cost of a typical urban water utility does not vary with the volume of water produced, this has led to an increasing frequency of revenue shortfalls for these entities. According to the California Public Utilities Commission (CPUC), over the past 15 years as high as 50 percent of the largest water utilities it regulates have annual revenue shortfalls as large as 20 percent of their annual revenue requirement. These revenue shortfalls have resulted in a far greater use of ex post revenue adjustment mechanisms that increase water prices after periods with limited water availability to recover these revenue shortfalls. This has led to an increasing temporal mismatch between the retail price consumers are charged and their need to reduce to water consumption.[1]

Despite rapidly growing populations in the western states over the past 30 years, there has been no major water storage or delivery infrastructure investment west of the Continental Divide since the early 1970s. For example, the population of California in 1970, around the time the State Water Project was completed, was roughly half of the current value of 38.9 million. This hiatus in water infrastructure investments is partially responsible for the increasing frequency of shortfalls in water availability to urban water utilities in the West.

This set of circumstances suggests two possible approaches to meet the West's future water demand: (1) manage existing water resources, primarily through pricing, or (2) build and pay for additional water storage and/or transportation infrastructure. Both approaches argue for a significantly enhanced understanding of the customer-level demand for water. This argument is strengthened by the fact that nonlinear pricing is the standard approach used by water utilities to balance the competing goals of managing limited water resources and achieving sufficient revenues to recover their costs. Customers typically face schedules where the price charged for each additional unit, the marginal price, rises with the customer's monthly consumption. The marginal price is fixed for a block or range of values of monthly consumption, but it increases across these blocks with increases in the value of monthly consumption. For this reason these nonlinear price schedules are called increasing block price schedules.

[1]For example, the Water Revenue Adjustment Mechanism (WRAM) set by CPUC to recover past revenue shortfalls has temporarily increased future monthly water bills for the same level of consumption by more than 40%.

The form of the increasing block price schedule set by the utility impacts how much water each customer purchases and the revenues the utility receives from that customer. The form of the nonlinear price schedule also impacts the amount of uncertainty the utility faces in the quantity of water it sells and the revenues it receives from each customer. This uncertainty in customer-level water sales and revenues to the utility is aggregated across customers to create uncertainty in the utility-level water sales and revenues. If a utility can accurately predict the customer-level demand for water for any possible nonlinear price schedule it can design increasing block price schedules to achieve any conservation or revenue goal while minimizing utility-level water sales or revenue risk. Increased information about the distribution of customer-level demand directly translates into reduced water sales and revenue risk associated with any rate design goal.

This paper formulates and estimates a household-level demand model for water under increasing block prices that accounts for the impact of weather variation within the household's billing cycle and customer-level heterogeneity in demand due to observable demographic characteristics and other unobserved factors that differ across customers. This model is applied to billing cycle level data from two urban water utilities, one serving Cobb (Cobb) County, Georgia and the other serving the Valley of the Moon (VoM) in Napa County, California.

Our model accounts for the fact that the demand for water, electricity, and natural gas is derived from the household's demand for services these products provide and the relationship between the demand for these services and the consumption water, electricity or natural gas is uncertain. For example, in the case of water, the customer may have a demand for drinking, bathing, and other household activities, but it is typically impossible for the customer to control precisely their consumption of water. The household might run the tap for an uncertain length of time before filling their glass for drinking, run the shower until the water becomes warm enough to shower, or stay in the shower longer than normal. Similar logic applies to the case of electricity. How much electricity many appliances consume depends on temperature conditions. The household may forget to turn off the lights or the television when they leave a room.

Previous models of demand under nonlinear pricing such as Hewitt and Hanemann (1995) and Olmstead et al. (2007) account for this uncertainty in consumption of water by introducing an error term that is unobservable to the econometrician and customer that impacts their consumption. Specifically, these models assume that the household chooses where to consume along their nonlinear budget set, but this "perception error" can subsequently cause them to consume at any point along the customer's nonlinear budget set. Our model of consumer demand takes a different approach to modeling this uncertainty between the demand for water services and the customer's consumption of water by assuming that the household knows that an error term will be realized during the billing cycle that impacts their consumption of water. The household optimizes against the realization of this error by choosing to consume according to the price step on the nonlinear price schedule that maximizes the expected utility associated the distribution of the error subsequently realized during the billing cycle. We estimate both models of consumer demand under nonlinear pricing and find that our model yields a statistically superior description of the joint density of billing cycle level consumption levels across customers and time using a Vuong (1989) non-nested hypothesis test.

There is some debate among economists whether customers understand and are able to respond to nonlinear prices. Ito (2014) and Borenstein (2009) argue that consumers respond to the average price they face rather than to the nonlinear price schedule. We use the Vuong (1989) non-nested hypothesis test to compare our model of household-level demand with nonlinear pricing to four competing models of household-level demand that embody alternative price measures that that household responds to. Two of the alternative models of demand are based the assumption that the household's demand is a function of the average price. For both utilities considered, the non-nested hypothesis tests find that our model of water demand with nonlinear pricing provides a statistically superior description of the observed pattern of the household-level demand relative to each of the four alternative models, with the exception of one average-price model for VoM, where the null hypothesis of an equal average log-likelihood value cannot be rejected.

Our model can be used to construct an estimate of the distribution (because of unobservable demographic factors and other unobservables) of each customer's monthly demand and total amount paid for water for any arbitrary nonlinear price schedule. Combined with data on the distribution on observable customer-level heterogeneity in the utility's service territory, these household-level demand models can be used to compute the distribution of aggregate water demand for any possible price schedule. This process also yields an estimate of the distribution of total utility-level revenues for any arbitrary nonlinear price schedule or set of nonlinear price schedules, which implies that the modeling results can be used to measure both the household-level and aggregate willingness to pay for a proposed water infrastructure investment. Specifically, it can be used to determine if there exists a nonlinear price schedule consistent with the utility's water pricing goals that recovers sufficient revenues to recover the cost of a given water infrastructure investment. In general, the estimated household-level water demand model can be used by the utility to design nonlinear prices for water to achieve a wide range of system-wide policy goals.

Our model assumes that a household's water demand depends on the nonlinear price schedule, the characteristics of household (such as household income, the size of the dwelling, size of the property, and number of adults living in the dwelling), weather conditions (such as average daily temperature and rainfall) during the customer's billing cycle, and a measure of the amount of outdoor vegetation on the customer's property. The United States (US) Bureau of Census Public Use Microdata Sample (PUMS) of American Community Survey provides data on the distribution of household demographic characteristics in each United States Postal Service (USPS) Zip Code. The National Oceanic and Atmospheric Administration (NOAA) provides data on daily weather conditions in that Zip Code during the billing cycle. Information on the amount of outdoor vegetation for each customer is obtained from satellite data compiled by the National Atmospheric Information Administration (NASA) on a bi-monthly basis.

Several computations are performed using the model to assess the impact of resolving uncertainty about customer-level demand and the distribution of these demands throughout utility's service territory on the system-wide sales and revenue uncertainty faced by the utility. The difference in the system-wide revenue risk between a scenario that assumes the utility only knows the prior distribution of demographic characteristics in the household's zip code and the scenario that assumes the utility knows each customer's demographic characteristics provides a metric for assessing the revenues and sales risk reduction benefits to the utility from collecting demographic information from each of its customers.

Counterfactual nonlinear price schedules are computed that yield no more than the same expected system-wide water sales and at least as much system-wide revenues as the utility's existing price schedules, but also minimize the uncertainty in the utility's annual revenues from water sales. Price schedules that reduce system-wide water consumption by 25 percent with a 95 percent probability while still obtaining at least a much expected sales, are also computed. These counterfactual price schedules are constructed under the assumption that the utility knows the demographic characteristics for each household. The demographic characteristics are assigned based on the posterior distribution of demographics in the household's zip code and the household's observed vector of billing cycle-level water consumption.

These experiments demonstrate several sources of economic benefits to the utility from having more detailed knowledge of individual customers. First, knowledge of the customer-level demand can be used by the utility to design increasing block pricing plans that achieve any revenue or sales goals with less revenue or sales risk. Second, knowledge of how these demands differ across customers based on observable demographic characteristics can allow the utility to significantly reduce the utility-wide revenue or water sales risk it faces for any pricing plan. Third, knowledge of how the structure of demand varies across customers can be used to design personalized (based on observable household demographic characteristics) increasing block price schedules to further reduce the risk the utility faces on a system-wide basis.[2] Finally, with detailed knowledge of how demands differ across customers based on observable demographic characteristics, the utility can more accurately assess the likely impact to water sales and revenue from changes in the number and types of customers in their service territory.

For the two utilities considered, knowledge of the customer-level demographics that predict demand differences across households reduced the uncertainty in the utility's system-wide revenues by 22 and 84 percent. Further reductions in the uncertainty in the utility's system-wide revenues, in the range of 10 to 79 percent, are possible by re-designing the utility's nonlinear price schedules to minimize the revenue risk it faces given the distribution of household-level demand in its service territory. This household-level demand information is also particularly important for assessing the economic benefits of proposed water infrastructure projects and designing the price schedules necessary raise the revenue needed to pay for them with the least amount of water sales or revenue risk to the utility.

The remainder of the paper proceeds as follows. Section 2 discusses the design of nonlinear pricing plans. Section 3 describes the datasets used to estimate the demand model. Section 4, 5, 6 present the econometric model of demand, the estimation results, and the specification tests performed, respectively. Section 7 describes how the model can be used to estimate the distribution of household-level and system-wide water sales and revenues. Section 8 presents the counterfactual experiments performed using the model results. Section 9 concludes.

[2]Because it is relatively straightforward for the utility to prevent resale of residential water service, utilities can set different increasing block price schedules for each customer based on its observable demographic characteristics.

2 Rate Design with Nonlinear Pricing

A major rationale for increasing block pricing by water utilities is that this form of nonlinear pricing balances two competing public policy goals. The first is to provide the "essential" amount of water a household needs for drinking, cooking, bathing, and other indoor use at a price that is affordable for virtually all households in the utility's service territory. The second goal is to provide a financial incentive for households using more than the "essential" amount to reduce their demand for water. By this logic, the higher-priced steps in the increasing block price schedule beyond the initial baseline or essential consumption level are designed to discourage less essential water consumption. For example, the second price step might be intended for the demand to fill the household's swimming pool. The third price step might be intended for the demand for watering the household's outdoor trees, bushes, and shrubs. The fourth price step might be intended for the demand for watering the household's lawn.

Another argument in favor of increasing block pricing of water is that it recovers an increasing amount of the utility's revenue from high demand customers, which are also likely to be the high income customers. Because higher income consumers typically consume more water, the highest marginal price they pay is typically greater than the highest marginal price low income consumers pay. For this reason, increasing block pricing implies that high income consumers that purchase more water pay a higher average price (total monthly payments divided by total monthly consumption) for their water consumption than low income consumers.

Increasing block pricing can also create revenue adequacy challenges for the water utility if the utility makes the length of the baseline level of demand too large. High demand households might consume along the baseline marginal price step as opposed to consuming at a higher marginal price step. Figure 1 illustrates case with $D_L(p)$, the demand curve for low-demand consumers, and $D_H(p)$, the demand curve for high income consumers. Both curves intersect the increasing block price schedule on the first price block, which raises significantly less revenue for the utility than would be the case if $D_H(p)$ intersected the price schedule on the higher-priced block. If the first block of the price schedule is too short, this can impose an excessive financial burden on low-demand, low-income consumers by charging them the marginal price intended for high demand consumers. Figure 2 illustrates this case where both demand curves intersect the increasing block price schedule on the higher-priced block.

From the perspective of achieving enough revenues to recover the utility's cost and while selling no more than a certain amount of water to all customers, the design of a nonlinear price schedule amounts to choosing the length of each step to separate customers into distinct groups based on their willingness to pay for water. Moreover, if the utility has some uncertainty about the location and shape of each customer's demand, then reducing this uncertainty could help the utility determine where to set the baseline demand level, q_B, shown in Figure 3. This figure shows the range of possible uncertainty (from the perspective of the utility) in $D_L(p)$ and $D_H(p)$. This is indicated by the dotted lines to the left and right of each demand curve. Note that q_B has been chosen so that regardless of the realization of $D_L(p)$ and $D_H(p)$, each type of customer will continue to consume along the same step of the increasing block price schedule. Choosing the value of q_B in this manner limits the amount of revenue variability facing the utility due to its uncertainty about the realized values of $D_L(p)$ and $D_H(p)$. The second part of the increasing block pricing design process must choose the levels of the first marginal price and second higher marginal price to recover sufficient revenues to cover the utility's costs, while still achieving the goal of limiting the economic burden placed on low-income consumers to purchase their essential water needs.

If the utility is able to sort households into different categories based on observable demographic characteristics, then is possible to assign different increasing block price schedules to different households based on their observable demographic characteristics. In this case, the utility would like to achieve the outcome in Figure 3 for each set of observable demographic characteristics that predict differences in the form of the demand.

One possible set of counterfactual pricing experiments would use the estimated household-level demand model to determine the extent to which it is possible for the utility to re-design its increasing block price schedule to achieve at least as much expected revenue and expected water sales no larger than it does under the current rate schedule while facing less risk to its total revenues. A second set of counterfactual pricing experiments could set separate increasing block prices schedules for households with different observable demographic characteristics to achieve at least as much expected revenue and no larger expected water sales than with the current increasing block price schedules used by the utility while facing the utility with less risk to its total revenues. Both sets of counterfactual pricing experiments demonstrate that if a utility has more information about the demand for water of individual customers, it can significantly reduce the revenue or sales risk it faces in meeting a set of pricing goals.

3 Data Used in Analysis

Four datasets are used to estimate the customer-level demand model for each utility service territory. The first is billing cycle-level monthly water consumption data for a sample of households for at least one year in duration. The second dataset is composed of daily weather variables at the Zip Code level obtained from the National Oceanic and Atmospheric Administration (NOAA) for the utility's service territory. The third dataset is the distribution of household-level demographic characteristics within each Zip Code in the utility's service territory obtained from the US Bureau of the Census. The fourth dataset is composed of the value of the Normalized Vegetation Difference Index (NDVI) compiled by NASA for each household's property.

Monthly household-level water consumption is available from two utilities at the billing cycle-level, along with the customer's zip code, the form of the nonlinear price schedule faced by household, and other information necessary to compute customer's monthly water bill. Although utilities typically bill their customers on a monthly basis, customers receive their bills at different times during the month. The time between consecutive billing dates is called the customer's billing cycle and it depends on when the meter reader shows up at the customer's premises to read the meter each month. For example, one customer might be billed on the third day of every month, whereas another customer might be billed on a twentieth day of the month.

Having data available on each customer's billing cycle level is important for accurately modeling the impact of weather conditions on a household's demand for water. In terms of the above example, it might be the case that first two weeks of July are extremely hot so the water demand is extremely high, whereas the last two weeks of July are mild so that water demand is significantly lower. The customer with a billing cycle that starts on the third day of the month will have much higher weather-related demand than customer whose billing cycle begins on the 20th day of the month. Only by knowing the customer's billing cycle is it possible to properly account for differences across customers in their weather-related demand for water.

The NOAA provides daily measures of rainfall and the maximum daily temperature at the Zip Code level for each utility service territory. The average value of the maximum daily temperature is computed as the average of the daily maximum temperature across all days in the billing cycle. The total amount of rainfall in that Zip Code during the billing cycle is also computed from this data. The inter-quartile range of the maximum daily temperatures and inter-quartile range of daily rainfall in the zip code during the billing cycle are also compiled.[3]

The distributions of household-level demographic variables for each Zip Code in each utility's service territory are obtained from the US Bureau of Census Public Use Microdata Sample (PUMS) of American Community Survey. The demographic characteristics for each household surveyed in each Public Use Microdata Area (PUMA) are compiled along with the sampling weight for that household. These PUMAs can be matched to zip codes so that a distribution of household-level demographic variables in the Zip Code is available for all Zip Codes in the service territory.

The NDVI data is compiled by NASA from satellite data taken from the using NOAA's Advanced Very High Resolution Radiometer (AVHRR). An algorithm is applied to the wavelengths and intensity of visible and near-infrared light reflected by the land surface back up into space to quantify the concentrations of green leaf vegetation for 30 meter by 30 meter quadrants of the earth's surface. The NDVI lies on the interval [-1,1], with higher values indicating more green vegetation. Values close to -1 correspond to water, whereas values close to zero (-0.1 to 0.1) correspond to rock, sand, or snow. Small positive values, generally between 0.2 and 0.4, represent shrub and grassland, and values close to 1 indicate temperate and tropical rainforests.

Household-level, billing-cycle data is available from the VoM and Cobb water utilities. Daily weather data has also been compiled for the time period that the customer-level billing cycle data is available for each utility service territory. The Zip Code-level distribution of household demographic data has also been compiled for the time period that the customer-level billing cycle data is available for each utility service territory. The NDVI data is available on a bi-monthly basis for each household in both utility service territories. All nominal prices are converted to 2012 dollars using the Federal Reserve Economic Database Gross Domestic Product (GDP) deflator from St Louis Federal Resource Bank. [4]

[3] The inter-quartile range is the difference between the 75^{th} percentile of the daily variables in the billing cycle and 25^{th} percentile of this same distribution.

[4] Data available from https://research.stlouisfed.org/fred2/series/GDPDEF

4 Econometric Model

This section describes the specification of two econometric models of the billing cycle-level and household-level water demand under increasing block prices that account for the weather facing that household during its billing cycle, differences in demographic characteristics across households, differences in the NDVI value for the property over time and across households, and uncertainty in the household's demand for water services and its consumption of water.

The first model assumes that the household knows that it will be subject to a consumption shock during the billing cycle, and therefore the household chooses to consume along the price step the maximizes its expected utility given the distribution of this consumption shock. The second model is based on Hewitt and Hanemann (1995) model of demand under nonlinear pricing where a *perception* shock is realized after the household chooses an unobserved water service consumption level and this can lead the household to consume water at any point along its nonlinear budget set. This *perception* shock is unobserved and unanticipated by the household.

For both econometric models, we derive the joint density of all billing cycle-level consumption choices for each household during the sample period conditional on the nonlinear price schedule the household faces, its demographic characteristics, the value of NDVI on its property, and the temperature and rainfall distributions it was exposed each billing cycle during the sample period. Because the demographic characteristics of each household are unobserved, to arrive at the likelihood function used to estimate the parameters of both demand models, the conditional distribution of the household's monthly billing cycle-level consumption choices given its unobserved demographic characteristics is integrated with respect to the distribution of demographic characteristics in the Zip Code that contains that household. This yields a likelihood function that depends on observable data—the household's vector of monthly water consumption choices, the values of the household's property vegetation index, the vector of billing cycle-level monthly weather variables and the distribution of demographic characteristics for that household's Zip Code.

4.1 Water Demand with Nonlinear Pricing

Both models deal with the fact that consumers can only imperfectly predict which step of the increasing block price schedule they will end up on during their billing cycle because of the uncertain relationship between the demand for water utility services and their consumption of water during the billing cycle. Accounting for this fact is necessary for the household-level demand model to match the distribution of actual water consumption across households and billing cycles. Similar to other studies of water and electricity demand under nonlinear pricing, this distribution does not show bunching at the endpoints of steps of the nonlinear price schedule.[5] The alternative demand model under nonlinear pricing presented in Section 4.4 that only allows for unobserved heterogeneity in consumer preferences yields a distribution of billing cycle-level consumption that bunches at the endpoints of the steps of the nonlinear price schedule.[6]

The first model assumes that consumers follow a two-step process that explicitly recognizes this uncertain relationship between the demand for water services and water consumption when choosing to consume along a nonlinear price schedule. At the start of the billing cycle, the household is assumed to choose the price step, p_i $(i = 1, 2, ..., N)$, that yields the highest expected utility given the distribution of demand shocks within the billing cycle. Throughout the billing cycle, the household consumes electricity assuming it will end up on that marginal price step. However, during the month the household experiences demand shocks which may cause it to end up on a different price step from the one that it found optimal at the beginning of the month.

It is virtually impossible for a household to manage its water consumption within the billing cycle with sufficient precision to always end up on its expected utility-maximizing price step. Uncertainty in water consumption during the billing cycle may move the household's marginal price from one price tier to another price tier. As discussed earlier, virtually all household uses of water involve uncertainty in the actual amount of water consumed for the water services demanded by the household. A household member demands water services such as taking bath or shower, filling their swimming pool, washing their car, or watering their plants or lawn. Despite the individual's best intentions to use only a certain amount of water, each water service demand has technological uncertainty in the exact amount of water consumed. For example, running water for a hot shower on a cold day takes longer than on a warm day and therefore uses more water. For this reason, we have the stochastic unobservable, ϵ, as the demand shock to account for uncertainty in actual amount of water consumed by the household relative to their intended water service consumption level.

[5] See Figures 11 and 15 for a plot of these distributions for VoM and Cobb.

[6] The model of household-level electricity demand under nonlinear pricing presented in Reiss and White (2005) only allows for unobserved heterogeneity. Their analysis avoids this issue the authors only observe the household's annual demand for electricity, which is sum of the household's billing cycle-level demands for the year.

Virtual income is an important concept in modeling demand under increasing block pricing. It accounts for the fact that initial units of consumption in the billing cycle take place at a lower marginal price and accounting for this fact in consumer's choice problem yields the same utility-maximizing choice as assuming the customer has this virtual income and all consumption takes place at a higher marginal price. Let the price steps of the increasing block pricing be defined from lowest to the highest $p_1 < p_2 < ... < p_N$ and let q_i equal the endpoint of price step p_i. Let FC equals the household's fixed charge for the billing cycle. Define d_k as the difference between the cost of consumption level w (in the kth block of the increasing block price schedule) when all units are purchased at price p_k, the marginal price for this block, and the actual cost of purchasing w under increasing the block price schedule, so that $d_k = -FC - \sum_{j=1}^{k-1}(p_j - p_{j+1})q_j$. For example, if the household is purchasing along the first price tier, then $d_1 = -FC$ and if the household is purchasing along the second price tier, then $d_2 = -FC–(p_1–p_2)q_1$. A household with vector of demographic characteristics, A, purchasing along the kth price tier has virtual income, $V_k(A) = I(A) + d_k$, where $I(A)$ is the household's actual income. We write income, $I(A)$, as a function of the vector of demographic characteristics to denote the fact that monthly income is one of the elements of A, the vector demographic characteristics obtained from PUMS data. Nonlinear pricing creates an income effect for consumers that face marginal prices higher than first marginal price during the billing cycle.

Define $V(p_k, d_k, A_i, Z_i, G_i, \epsilon_i, \beta) + \eta_k$ to be the conditional indirect utility of the household i being on price step k, where d_k is defined as above to illustrate virtual income; A_i is the household i's demographic characteristics; Z_i is the vector of weather variables faced by household i; G_i is the value of the NDVI index for household i. Note that Z_i can change over time for the same household. For now we will skip the subscript t for time for notational convenience. Assume the ϵ_i is sum of all demand shocks during the billing cycle and is realized after the household makes its price step choice for the billing cycle. We assume that η_k is a Type I extreme value random variable that is in the indirect utility associated with price step k. This random variable is assumed to be observed by the household, but not the researcher. This utility function is parameterized by the vector β.

From this conditional (on the price step chosen) indirect utility function $V(p_k, d_k, A_i, Z_i, G_i, \epsilon_i, \beta)+ \eta_k$, we can derive the ex post demand for electricity during the billing cycle conditional on price step k being the marginal price step during the billing cycle. We can then calculate the expected utility value conditional on choosing price step k as $E_{\epsilon_i}[V(p, d, A_i, Z_i, G_i, \epsilon_i, \beta) \mid p_k]+$

η_k where $E_{\epsilon_i}(\cdot)$ means that expectation is taken with respect to the distribution of ϵ_i. Note that here we don't have p_k and d_k anymore because the realization of ϵ_i might move the consumers to a different price step other than price step k. Then we are able to decide the probability of the consumer choosing each price tier based on the expected utility values and the distribution of η_k. With the added demand shock ϵ_i, we completely characterize the water demand for household facing a nonlinaer price schedule with known values of A_i, Z_i, and G_i.

Because of the demand uncertainty that household faces during the billing cycle, our model assumes that at the beginning of the model, the household chooses the price tier that it will end the month on to maximize the expected utility associated with consuming at the marginal price during the month, rather than the usual approach of choosing the quantity to consume during the billing cycle. The household's realized demand accounts for the fact that it faces an increasing block price schedule, because of the virtual income that household receives from consuming at that price tier during the month in the expression for total expenditure for the billing cycle. Because of the demand uncertainty faced by the household during the month, it can end up on a different price tier from the one it expected to consume along at the end of the month. Because the customer does not see its consumption for the billing cycle until it receives its bill for that month, it is unable to learn which price on the increasing block price schedule it ultimately ends up on.

We specify the details of the computation of the log-likelihood function once we specify the functional form for $V(p_k, d_k, A_i, Z_i, G_i, \epsilon_i, \beta)$ and the distribution of ϵ_i. With this information we can derive the conditional density of the household's observed water consumption $f(w_i|P_i, A_i, Z_i, G_i, \beta)$, which is the conditional (on P_i, A_i, Z_i, and G_i) likelihood function for a single observation of monthly billing cycle-level consumption for household i.

4.2 Log-Likelihood Function

Let the subscript t denote the value of a variable for billing cycle t and $T(i)$ equal the number of monthly billing cycle observations for household i in the sample and N is the total number of households in the sample. Let $W_i = (w_{i1}, w_{i2}, \ldots, w_{iT(i)})'$ equal the $T(i)$ dimensional vector of realized monthly water consumption observations for household i. Let $W = (W_1', W_2', \ldots, W_N')'$ equal the vector of the N vectors of monthly water consumption observations for all households in the sample. The first step in computing the likelihood function for the econometric model is to compute the joint density of W_i for each household in the sample conditional on the household's demographic characteristics and the $T(i)$ realizations of monthly weather conditions that they faced. In terms of the above notation, this joint density takes the form:

$$\prod_{t=1}^{T(i)} f(w_{it}|P_{it}, A_n, Z_{it}, G_{it}, \beta) \tag{1}$$

where P_{it} denotes the overall price schedule composed of marginal prices, $(p_1, p_2, ..., p_N)$, corresponding quantity cutoffs, $(q_1, q_2, ..., q_{N-1}0$ and monthly fixed charge, FC. The PUMS data from American Community Survey can be used to compute the probability density functions for the vector of demographic characteristics for each Zip Code in the utility's service territory. This dataset provides the sampling weights for each household in the American Consumer Survey and the vector of their demographic characteristics for each 5-digit Zip Code in the utility's service territory. Let $(wt(i, n), A_n)$ for $n = 1, \ldots, L(i)$ equal the values of these sampling weights and associated vector of demographic characteristics for each sampled household in the Zip Code that contains household i. In terms of this notation, the log-likelihood function for single customer is equal to:

$$\ell(W_i|\beta) = \ln\Big[\sum_{n=1}^{L(i)} wt(i, n) \prod_{t=1}^{T(i)} f(w_{it}|P_{it}, A_n, Z_{it}, G_{it}, \beta)\Big] \tag{2}$$

Summing over all N households in the sample yields log-likelihood function for the entire sample:

$$L(W|\beta) = \sum_{i=1}^{N} \ell(W_i|\beta) = \sum_{i=1}^{N} \ln\Big[\sum_{n=1}^{L(i)} wt(i, n) \prod_{t=1}^{T(i)} f(w_{it}|P_{it}, A_n, Z_{it}, G_{it}, \beta)\Big] \tag{3}$$

Note that the joint distribution of $(w_{i1}, w_{i2}, \ldots, w_{iT(i)})$ is integrated with respect to density of the vector of demographic characteristics, A_n, rather than the density of each w_{it} individually, to account for the persistence in household i's billing cycle level demand over time. If the consumption of household i is unexpectedly high in billing cycle t relative to what would be predicted based on observable characteristics of this household, then it is likely that its consumption would be unexpectedly high in all other billing cycles. Integrating with respect to the density of A_n as is done in equation 3 is consistent with that logic.

4.3 Functional Forms

In order to implement the model empirically, it is necessary to choose functional forms for the household's indirect utility function, $V(p, d, A_i, Z_i, G_i, \epsilon_i, \beta)$, which yields the realized water demand function for the billing cycle, $g(A_i, Z_i, G_i, p_k, d_k)$, and density function for observed consumption in the billing cycle, $f(w_i|P_i, A_i, Z_i, G_i, \beta)$. Because the distribution of monthly water consumption across households for the same month and the distribution for the same household over time are both positively skewed in the sense that there are many observations just below mean, but a few observations far above the mean, the appropriate variable to model is the logarithm of the household's monthly demand for water.

Assume the conditional indirect utility of the household being on price step k for household i is equal to:

$$\begin{aligned} &V(p_k, d_k, \epsilon_i, \beta) + \eta_k = \\ &\qquad - exp(\beta_1{}'A_i + \beta_2{}'Z_i + \beta_3 G_i + \epsilon_i)\frac{p_k^{1-\alpha(A_i,G_i)}}{1-\alpha(A_i,G_i)} + \frac{(I(A_i)+d_k)^{1-\rho(A_i,G_i)}}{1-\rho(A_i,G_i)} + \eta_k \end{aligned} \tag{4}$$

where $\alpha(A, G) = -\exp(A'\beta_4 + G\beta_5)$ and $\rho(A, G) = -\exp(A'\beta_6 + G\beta_7)$. Define $\beta = (\beta_1', \beta_2', \beta_3', \beta_4', \beta_5', \beta_6', \beta_7')'$ as the vector of parameters of the demand/utility function. We assume that ϵ, sum of all demand shocks during the billing cycle, is a $N(0, \sigma^2)$ random variable that is realized after the household makes its price step choice. We also assume that η_k is a Type I extreme value random variable that is in the conditional indirect utility function associated with price step k. This random variable is assumed to be observed by the household, but not the researcher.

Applying Roy's Identity to the indirect utility associated with consuming on price step k,

$$q = -\frac{\partial V}{\partial p} / \frac{\partial V}{\partial M} \tag{5}$$

yields the logarithm of ex post demand for electricity conditional on consuming along price step k:

$$ln(q) = \beta_1{}'A_i + \beta_2{}'Z_i + \beta_3 G_i - \alpha(A_i, G_i)ln(p_k) + \rho(A_i, G_i)ln(I(A_i) + d_k) + \epsilon_i \tag{6}$$

where $I(A_i) + d_k$ is the virtual income associated with consuming along price step k defined above.

This functional form implies that the coefficient determining the price responsiveness of demand, $\alpha(A, G)$, is minus 1 times the exponential function of a linear combination of some of the elements of the vector of the household's demographic variables and its vegetation index, and income responsiveness of demand, $\rho(A, G)$, is an exponential function of a linear combination of some of the elements of the vector of the household's demographic variables and its vegetation index.

This functional form allows for substantial differences in both the price responsiveness and income responsiveness of water demand across households in each utility service territory. Both the price and income coefficients depend on the value of the vegetation index for the household's property and a subset of the vector of demographic characteristics to allow for differences in both the income and price elasticities across households and over time for the same household.

The expected utility conditional on choosing price step k is then equal to:

$$E_{\epsilon_i}[V(p, d, A_i, Z_i, G_i, \epsilon_i, \beta) \mid p_k] + \eta_k =$$
$$- exp(\beta_1{}' A_i + \beta_2{}' Z_i + \beta_3 G_i) \frac{E_{\epsilon_i}[exp(\epsilon_i) p^{1-\alpha(A_i,G_i)} \mid p_k]}{1 - \alpha(A_i, G_i)} + \frac{E_{\epsilon_i}[(I(A_i) + d)^{1-\rho(A_i,G_i)} \mid p_k]}{1 - \rho(A_i, G_i)} + \eta_k \tag{7}$$

Define $g(A_i, Z_i, G_i, p_k, d_k) = \beta_1{}' A_i + \beta_2{}' Z_i + \beta_3 G_i - \alpha(A_i, G_i) ln(p_k) + \rho(A_i, G_i) ln(I(A_i) + d_k)$. With this notation we can compute the following probabilities of ending up on each price step conditional on facing price step k:

$$prob(Step = 1 \mid p_k) = prob(ln(q) \leq ln(q_1) = \Phi([ln(q_1) - g(A_i, Z_i, G_i, p_k, d_k)]/\sigma) \tag{8}$$

$$prob(Step = j \mid p_k) = prob(ln(q_{j-1}) < ln(q) \leq ln(q_j)) = \Phi([ln(q_j) - g(A_i, Z_i, G_i, p_k, d_k)]/\sigma)$$
$$- \Phi([ln(q_{j-1}) - g(A_i, Z_i, G_i, p_k, d_k)]/\sigma]) \text{ for } 2 \leq j \leq N - 1 \tag{9}$$

$$prob(Step = N \mid p_k) = prob(ln(q_N) < ln(q)) = 1 - \Phi([ln(q_N) - g(A_i, Z_i, G_i, p_k, d_k)]/\sigma) \tag{10}$$

These results imply the following expected values given p_k:

$$E_{\epsilon_i}[(I(A_i)+d)^{1-\rho(A_i,G_i)} \mid p_k] = \sum_{j=1}^{N} prob(Step=j \mid p_k)(I(A_i)+d_j)^{1-\rho(A_i,G_i)} \tag{11}$$

$$E_{\epsilon_i}[exp(\epsilon_i)p^{1-\alpha(A_i,G_i)} \mid p_k] = \sum_{j=1}^{N} p_j^{1-\rho(A_i,G_i)} \int_{q_{j-1}/exp(g(A_i,Z_i,G_i,p_k,d_k))}^{q_j/exp(g(A_i,Z_i,G_i,p_k,d_k))} sf(s)ds \tag{12}$$

where $f(s)$ is the density of $exp(\epsilon)$, where ϵ_i is $N(0,\sigma^2)$, d_k is the virtual income value associated with price step k. Note that $q_0 = 0$ and $q_N = \infty$. We can evaluate equation 11 by substituting equation 8, 9 and 10 into the right-hand side.

Using results from Jawitz (2004), the equation (12) can be re-written as:

$$E_{\epsilon_i}[exp(\epsilon_i)\ p^{1-\alpha(A_i,G_i)} \mid p_k] = \sum_{j=1}^{N} p_j^{1-\rho(A_i,G_i)} \frac{1}{2} exp(\frac{\sigma^2}{2})$$
$$\times\left[erf\left[\frac{ln(q_j)-g(A_i,Z_i,G_i,p_k,d_k)\ -\ \sigma^2}{\sigma\sqrt{2}}\right]\ -\ erf\left[\frac{ln(q_{j-1})-g(A_i,Z_i,G_i,p_k,d_k)\ -\ \sigma^2}{\sigma\sqrt{2}}\right]\right] \tag{13}$$

where $erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t exp(-t^2)$.

We can evaluate equation (7) by substituting equation (11) and (13) into the right-hand side. Now under our distributional assumptions, the density of w_i given A_i, Z_i, G_i is equal to (here we emit the subscript t for notation convenience):

$$f(w_i|A_i,Z_i,G_i;\theta) = \sum_{j=1}^{N} Prob(p=p_j|\theta)\frac{1}{\sigma}\phi([ln(w_i)-g(A_i,Z_i,G_i,p_j,d_j|\theta)]/\sigma) \tag{14}$$

where $\theta = (\beta_1', \beta_2', \beta_3, \beta_4', \beta_5, \beta_6', \beta_7, \sigma)'$, $\alpha(A_i,G_i) = exp(A_i'\beta_4 + G_i\beta_5)$, $\rho(A_i,G_i) = exp(A_i'\beta_6 + G_i\beta_7)$, and

$$prob(p=p_j|\theta) = \frac{exp(E_{\epsilon_i}[V(p,d,A_i,Z_i,G_i,\epsilon_i,\beta) \mid p_j]}{\sum_{k=1}^{N} exp(E_{\epsilon_i}[V(p,d,A_i,Z_i,G_i,\epsilon_i,\beta) \mid p_k])} \tag{15}$$

This likelihood function accounts for the fact that the econometrician does not know which price step that household planned to consume on at the start of the billing cycle and it can only observe the household's ex post consumption and ex post price step. Consequently, this observed consumption could be arrived at from consumption at any planned price step, which why (15) is written at the sum of step-probabilities and conditional density of $ln(q)$ given that price step.

Maximizing this likelihood with respect to θ yields the maximum likelihood estimates of this parameter vector. Two sets of standard errors for the parameter estimates are computed. The first set uses the inverse of the matrix of the sum of the outer products of the observation-by-observation gradient of the log-likelihood for each household evaluated at the maximum likelihood parameter estimates. The second set uses the White (1982) quasi-maximum likelihood estimate covariance matrix which is equal to the inverse of the matrix of second partial derivatives evaluated at the maximum likelihood parameter estimates pre- and post-multiplied by the matrix of the sum of the outer products of gradient of the log-likelihood function.

4.4 Alternate Nonlinear Pricing Demand Model

This section describes an alternate model of demand under nonlinear pricing that take a different approach to account for the uncertain relationship between the demand for water services and the consumption of water. This model was first presented and estimated in Hewitt and Hanemann (1995). The superscript o is used to distinguish the notation for this model from the previous one.

In this model, households are assumed to choose their consumption of water to maximize utility subject to the nonlinear budget set created by the nonlinear price schedule. However, after they make this choice the *perception* error is realized and this is added to the logarithm of their optimal choice subject to the nonlinear budget constraint to yield the logarithm of these observed water consumption.

We choose the following functional form for the observed portion of the water demand function for household i conditional on consuming on price step k, and other variables A_i, Z_i, and G_i:

$$ln(w_k^o) = \beta_1^{o\prime} A_i + \beta_2^{o\prime} Z_i + \beta_3^o G_i - \alpha^o(A_i, G_i) ln(p_k) + \rho^o(A_i, G_i) ln(I(A_i) + d_k) \tag{16}$$

where $\alpha^o(A, G) = -\exp(A'\beta_4^o + G\beta_5^o)$ and $\rho^o(A, G) = -\exp(A'\beta_6^o + G\beta_7^o)$. Define $\beta^o = (\beta_1^{o\prime}, \beta_2^{o\prime}, \beta_3^{o\prime}, \beta_4^{o\prime}, \beta_5^{o\prime}, \beta_6^{o\prime}, \beta_7^{o\prime})'$ as the vector of parameters of the demand/utility function for this model. Note that this equation is very similar to Equation 6 of our new model.

There are two sources of unobservables in this model, $\epsilon^o = (\eta^o, \nu^o)'$, where $\eta^o \sim N(0, \sigma^2_{\eta^o})$ and $\nu^o \sim N(0, \sigma^2_{\nu^o})$ are independent random variables distributed independently across households and over time for the same household. The elements of $\epsilon^o = (\eta^o, \nu^o)'$ are called the unobserved household-level heterogeneity, η^o, and the household-level *perception* error, ν^o. Both are unobserved to researcher. The former is assumed to be observed by the household, but the latter is assumed to be unobserved by the household. Moreover, the household does not take any action to mitigate the impact of ν^o in its realized consumption. Note that ν^o accounts for the fact that despite the individual's best intentions to use only a certain amount of water, each water service demand has technological uncertainty in the exact amount of water it consumes.

The mapping from the realized values of the unobservables (η^o, ν^o) to the observed value of the logarithm of the household's monthly billing cycle-level consumption, $\ln(q)$, for a K-step increasing block price schedule takes the form

$$\ln(q) = \begin{cases} \ln(w_1^o) + \eta^o + \nu^o & \text{if } \eta^o < \ln(q_1) - \ln(w_1^o) \\ \ln(q_1) + \nu^o & \text{if } \ln(q_1) - \ln(w_1^o) < \eta^o < \ln(q_1) - \ln(w_2^o) \\ \ln(w_2^o) + \eta^o + \nu^o & \text{if } \ln(q_1) - \ln(w_2^o) < \eta^o < \ln(q_2) - \ln(w_2^o) \\ \ln(q_2) + \nu^o & \text{if } \ln(q_2) - \ln(w_2^o) < \eta^o < \ln(q_2) - \ln(w_3^o) \\ \dots & \\ \ln(q_{K-1}) + \nu^o & \text{if } \ln(q_{K-1}) - \ln(w_{K-1}^o) < \eta^o < \ln(q_{K-1}) - \ln(w_K^o) \\ \ln(w_K^o) + \eta^o + \nu^o & \text{if } \ln(q_{K-1}) - \ln(w_K^o) < \eta^o \end{cases} \tag{17}$$

Where $q_1, q_2, \dots, q_{K-1}$ are quantity cutoffs of the nonlinear price schedule, and $w_1^o, w_2^o, \dots, w_K^o$ are defined in Equation 16. This model embodies that the logic that given the realized value of ν^o the household chooses where to consume along the nonlinear price schedule, by maximizing utility subject to the nonlinear budget set. The realization of $exp(\eta^o)$ is multiplied by this optimal choice to determine where on this nonlinear price schedule the household ultimately ends up.

Given the mapping, the likelihood function for the water demand for this model is the following:

$$f(w_i|A_i, Z_i, G_i) = \sum_{k=1}^{K} \frac{\phi(s_k)}{\sqrt{\sigma^2_{\eta^o} + \sigma^2_{\nu^o}}}(\Phi(r_k) - \Phi(n_k)) + \sum_{k=1}^{K-1} \frac{\phi(u_k)}{\sigma^2_{\nu^o}}(\Phi(m_k) - \Phi(t_k)) \tag{18}$$

where

$$t_k = [\ln(q_k) - \ln(w^o_k)]/\sigma_{\eta^o}$$

$$r_k = (t_k - \rho s_k)/\sqrt{1-\rho^2}$$

$$\rho = \frac{\sigma^2_{\eta^o}}{\sqrt{(\sigma^2_{\eta^o})(\sigma^2_{\eta^o} + \sigma^2_{\nu^o})}}$$

$$s_k = (\ln(w_i) - \ln(w^o_k)/\sqrt{\sigma^2_{\eta^o} + \sigma^2_{\nu^o}}$$

$$n_k = (m_{k-1} - \rho s_k)/\sqrt{1-\rho^2}$$

$$m_k = (\ln(q_k) - \ln(w^o_{k+1})/\sigma_{\eta^o}$$

$$u_k = (\ln(w_i) - \ln(q_k))/\sigma_{\nu^o}$$

Note that this model and our model both contain an error term that accounts for the fact that household cannot perfectly control for their water consumption. However in the alternate nonlinear price model, the exponential of the *perception* error is multiplied by the optimal consumption to obtained the observed consumption, and existence of this error does not affect the household's decision making. In our model, household takes into account the fact that there is uncertainty in the consumption throughout the billing cycle and chooses to behave as if it consuming along the price tier that maximizes its expected utility taken with respect to the distribution of these demand shocks within the billing cycle.

5 Estimation Results

This section first presents estimates of our model for both VoM and Cobb. This is followed by the estimates of the alternative model.

5.1 New Model Estimates

Table 1 contains the estimation results for Valley of the Moon (VoM). Table 2 contains the estimates for Cobb County. The coefficient estimates and the two sets of standard errors described above are reported for each region. The number of households in the sample is also reported for each region. There are different numbers of months of data for each household because of differences in billing cycles across households during the sample period for each utility.

The following variables make up Z_{it}, the vector of weather characteristics that customer i was exposed to during billing cycle t.[7]

- *Average high temperature*: The average of the daily maximum temperature values in household i's Zip Code during household i's billing cycle.

- *Inter-quartile range of maximum daily temperatures*: The 75$^{\text{th}}$ percentile of the daily maximum temperature values in household i's Zip Code during household i's billing cycle minus the 25$^{\text{th}}$ percentile of the daily maximum temperature values in household i's Zip Code during household i's billing cycle

- *Total precipitation in billing cycle*: Sum of daily precipitation in inches during the billing cycle for the Zip Code containing household i.

- *Interquartile range of daily precipitation*: The 75$^{\text{th}}$ percentile of the daily precipitation in household i's Zip Code during household i's billing cycle minus the 25$^{\text{th}}$ percentile of the daily precipitation in household i's Zip Code during household i's billing cycle

- *Vegetation*: Value of NDVI for household i as of the start of billing cycle t. Figure 4 and 5 present the histogram of the NDVI index for households in VoM and Cobb, respectively. Consistent with the hotter and wetter climate in Georgia versus Northern California, the average value of the NDVI in Cobb is higher than in VoM, and the spread of the distribution of the NDVI is significantly larger in Cobb relative to VoM.

The household-level demographics variables, the vector A, all come from the PUMS data set. A subset of the available demographic variables most likely to predict differences in water demand across households are included A.

- *Monthly income of household*: Monthly household income in 2012 dollars. (Annual number reported in PUMS data divided by 12)

- *Number of people over 18 years-old living in the household*

- *Number of people under 18 years-old living in the household*

[7] All of the Zip Code-level weather data for each utility was obtained from the www.wunderground.com.

- *House Size Indicators*: House acreage between 1 and 10 acres. House acreage above 10 acres.
- *Number of bedrooms in the house*

As discussed earlier, for each household sampled by the US Bureau Census in that Zip Code, this demographic information is reported along with a sampling weight which gives the number of households in the Zip Code estimated to have this same vector of demographic characteristics as the sampled household. Dividing each sampling weight by the sum of the sampling weights for all households sampled in that zip code yields the weight, $wt(i,n)$, used in the construction of the likelihood function.

The price coefficient differs across households in the utility service territory, because the coefficient on the logarithm of price depends on A_{in} and G_{it}. Nonlinear pricing of water and the assumed stochastic structure described in Section 4.3 that gives rise to the joint density of W_i, the vector of billing cycle-level consumption values for household i, implies that the coefficient on the logarithm of price for a given household cannot be interpreted as a price elasticity of demand. The same logic applies to the coefficient on logarithm of household-level income. Nevertheless, as shown in Section 7, analogues to price and income elasticities can be computed with respect to the expected water demand of the household.

Parameter estimates of the model can be used to compute the posterior probability that household i has the vector of demographics A_{in} given its vector of billing cycle-level consumption W_i as:

$$p(A_{in}|W_i) = \frac{wt(i,n)\prod_{t=1}^{T(i)} f(w_{it}|A_{in}, Z_{it}, G_{it}, \theta)}{\sum_{j=1}^{N(i)} wt(i,j)\prod_{t=1}^{T(i)} f(w_{it}|A_{ij}, Z_{it}, G_{it}, \theta)} \tag{19}$$

For each household in the sample, the value of A_{in} that has the highest posterior probability for that household is assigned that vector of demographics for the purposes of computing the distribution of system-wide sales and revenues, and the counterfactual pricing, assuming that the utility knows each household's demographic attributes presented in Section 8.

5.2 Alternate Demand Model Results

Tables 3 and 4 present the parameter estimates and two sets of standard errors the alternate nonlinear pricing model for VoM and Cobb.

6 Specification Tests for Single Price Models

This section presents the results of specification tests for the household-level demand model subject to nonlinear pricing. These tests uses four alternative single price models of the household-level demand for water. We also test our demand model against the alternative nonlinear pricing demand model. From the results of Vuong (1989), the appropriately normalized difference between these optimized log-likelihood functions has an asymptotic $N(0,1)$ distribution under the null hypothesis that both models are equidistant (according to the Kullback-Leiber criteria) from the true unknown data generation process. The direction of rejection of the two-sided test indicates which of the two competing models provides a statistically superior description of the distribution of the observed endogenous variables given the observed conditioning variables.

Four alternative single price demand models are considered for the same functional form and distribution of unobservables. The functional form for each of the four demand functions is:

$$\ln(w^*(p^*, I^*, A, G, Z, \beta^*)) = A'\beta_1^* + Z'\beta_2^* + G'\beta_3^* + \alpha^*(A,G)\ln(p^*) + \rho^*(A,G)\ln(I^*) \quad (20)$$

where $\alpha^*(A,G) = -\exp(A'\beta_4^* + G\beta_5^*)$ and $\rho^*(A,G) = -\exp(A'\beta_6^* + G\beta_7^*)$. The four models differ only in terms of what variables are substituted for the price p^* and income I^* in equation 20. Given the assumed distribution $N(0, \sigma^{*2})$ for the error term ϵ^*, each of these models gives rise to a log-likelihood function which is then optimized with respect to (β^*, σ^*).

The four single price models considered are:

- 1) Actual price tier: p^* = tier price at actual consumption level and I^* = actual income less the fixed connect charge
- 2) Average variable price: p^* = (Variable Cost of Bill)/(Actual Consumption) and I^* = actual income less the fixed connect charge
- 3) Alternative actual price tier: p^* = tier price at their actual consumption and I^* = actual income less the fixed connect charge plus additional income due to nonlinear price schedule
- 4) Total Average Price: p^* = (Total Bill)/(Actual Consumption) and I^* = actual income

Note that the each of the above prices is calculated using data from the previous billing cycle instead of the current billing cycle. The primary reason for using values from the previous billing cycle is that consumers do not know their average price or their realized marginal price until the end of the current billing cycle, after they make their consumption decision. Researchers argue against consumers responding to nonlinear price schedules, instead argue that consumers respond to the average or realized marginal price from the previous billing cycle. See for example, Ito (2014), Fan and Hyndman (2011), Charney and Woodard (1984). Using the realized average price from the current period creates a logical inconsistency that the consumer does not know this price until billing cycle is complete.

To illustrate how we perform the specification test, let $\ln(f(Y|X,\theta))$ denote the log-likelihood function for an observation from the demand model with non-linear pricing and $\ln(g(Y|X,\gamma)$ the log-likelihood function for one of five competing price response models. Vuong (1989) proposed the following non-nested test between two competing parametric models for the conditional density of Y given X

$$\text{H: } E(\ln(f(Y|X,\theta^*))) = E(\ln(g(Y|X,\gamma^*) \text{ versus K: } E(\ln(f(Y|X,\theta^*))) \neq E(\ln(g(Y|X,\gamma^*))) \tag{21}$$

where $E(.)$ is expectation with respect to true joint distribution of Y and X, θ^* and γ^* are probability limits of the maximum likelihood estimates of θ and γ. The null hypothesis is that the expected value of the log-likelihood functions for both models with respect to $h(Y,X)$, the true joint density of Y and X, are equal versus the alternative that expected value for one model is greater than the other. Failure to reject the null hypothesis implies that both models are equidistant from the true data generation process, whereas a rejection implies that one model has a statistically superior average log-likelihood function value than the others, with the direction of rejection indicating which model is superior.

To implement the hypothesis test, we need to first estimate the models that we test against. For the four single price alternative demand models, because we are using data from the previous billing cycle to calculate the price variable, we lose the first billing cycle observation for all the households. However, to implement the hypothesis test, we need to ensure that the two model we compare have the same observations. To include the first billing cycle, we use the household's average price across all the billing cycles as the price variable for the first billing cycle. The parameter estimates of the four alternative models for Valley of Moon are presented in Table 5, 6, 7 and 8. The parameter estimates of the four alternative models for Cobb are presented in Table 9, 10, 11 and 12.

For each of the five alternative models, we compute $H_j = \ln(f(Y_j|X_j,\hat{\theta})) - \ln(g(Y_j|X_j,\hat{\gamma}))$, the difference between maximized log-likelihood function value for j^th^ observation for each model where $\hat{\theta}$ is maximum likelihood estimate of θ^* and $\hat{\gamma}$ is the maximum likelihood estimate of γ^*. Vuong (1989) shows that under null hypothesis,

$$L = \sqrt{N}\overline{H}/S \tag{22}$$

is asymptotically $N(0,1)$ where N is the number of customers, $\overline{H} = \frac{1}{N}\sum_{i=1}^{N} H_i$ and $S = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(H_i - \overline{H})^2}$.

Table 13 shows the results of these hypothesis tests for VoM and Cobb for each of the five alternative price response models–our alternative nonlinear pricing model and the four alternative single price models. In all five cases and both datasets except for the VoM total average price model, the null hypothesis is overwhelmingly rejected against the alternative hypothesis. Based on the direction of the rejection, we can see that our nonlinear price model has higher average log-likelihood value, which is consistent with the conclusion that it provides a statistically superior description of the conditional density of Y given X relative for the other models considered. For the case of VoM total average cost model, we cannot reject the null hypothesis.

7 Using Model to Reduce Revenue and Quantity Risk

The estimates of the parameters of the household-level demand model given in Table 1 and 2 make it possible to compute an estimate of the distribution of a household's water consumption and monthly bill for any nonlinear price schedule either conditional on the household's assigned demographic characteristics or without conditioning on the household's demographic characteristics.

The expected value and variance of these magnitudes can be computed as follows. For notational convenience, we assume that all the equations in this section are evaluated at the maximum likelihood estimates given in Table 1 and 2. Let A^* be the household demographic characteristics. Let P be the price schedule faced by the household, which may depend on the household's demographic characteristics A^*. The notation of p_j, q_j, d_j are the same as previous sections. Let $R(\cdot)$ be the bill amount function corresponding to P. Let $w^*(A^*, P)$ denote the household consumption and $R^*(A^*, P)$ denote the household bill amount. Note that the consumption function and bill function depend on variables other than demographic characteristics and price schedule. For notation convenience we do not expand for the other variables. The household has expected consumption and the variance in this consumption equal to:

$$E[w^*(A^*, P)] = exp(\frac{\sigma^2}{2}) \sum_{j=1}^{N} Prob(p = p_j) exp(g(A^*, Z, G, p_j, d_j)) \tag{23}$$

$$V[w^*(A^*, P)] = \sum_{j=1}^{N} Prob(p = p_j) \int_{-\infty}^{\infty} exp(g(A^*, Z, G, p_j, d_j) + s)^2 f^*(s) ds - E[w^*(A^*, P)]^2 \tag{24}$$

where $f^*(s)$ is the density function for the normal distribution $N(0, \sigma^2)$, $Prob(\cdot)$ is defined in equation 15. A^* is assigned by the rule based on equation (19). The expectation and variance are taken with the distribution of ϵ and η.

A household with assigned demographic characteristics A^* has an expected monthly water bill and the variance of its monthly water bill equal to:

$$E[R^*(A^*, P)] = \sum_{j=1}^{N} Prob(p = p_j) \int_{-\infty}^{\infty} R(exp(g(A^*, Z, G, p_j, d_j) + s)) f^*(s) ds \tag{25}$$

$$V[R^*(A^*, P)] = \sum_{j=1}^{N} Prob(p = p_j) \int_{-\infty}^{\infty} R(exp(g(A^*, Z, G, p_j, d_j) + s))^2 f^*(s) ds - E[R^*(A^*, P)]^2 \tag{26}$$

For the case that the household i's demographics are assumed to be unknown, the household's expected monthly water consumption and bill and the variance in its monthly water consumption and bill are equal to:

$$E[w^*(\cdot, P)] = \sum_{n=1}^{L(i)} wt(i, n) E[w^*(A_n, P)] \tag{27}$$

$$V[w^*(\cdot, P)] = \sum_{n=1}^{L(i)} wt(i, n)(V[w^*(A_n, P)] + E[w^*(A_n, P)]^2) \; - \; E[w^*(\cdot, P)]^2 \tag{28}$$

and

$$E[R^*(\cdot, P)] = \sum_{n=1}^{L(i)} wt(i, n) E[R^*(A_n, P)] \tag{29}$$

$$V[R^*(\cdot,P)] = \sum_{n=1}^{L(i)} wt(i,n)(V[R^*(A_n,P)] + E[R^*(A_n,P)]^2) \; - \; E[R^*(\cdot,P)]^2 \tag{30}$$

The expectations in equations (27) to (30) are taken with respect to the distribution of (ϵ, η) and the distribution of the demographic characteristics within the household's Zip Code. The expectations in equation (23) to (26) are taken with respect to the distribution of (ϵ, η) for the value of the household's demographic characteristics assigned using the vector of demographic characteristics with the highest posterior probability as determined by equation (19). Consequently, comparing the variance of water consumption and total revenues given the assigned value of A and the variance with respect to distributions of ϵ and A provides a measure of the value of demographic information to utility. It is also possible to substitute the posterior probabilities computed from equation (19) into equation (27) to (30) and compute the expected values and variances of sales and revenues based on these distributions of the demographic characteristics for household i.

These expressions in equation (23) and (27) can also be used to compute analogues to the price elasticity and income elasticity of the demand for water. For the case of the price elasticity this is computed as

$$\{E[w(P,A^*)] - E[w(P^+,A^*)]\}/\{0.05 \cdot E[w(P,A^*)]\} \tag{31}$$

where P is the actual nonlinear price schedule charged by the utility and P^+ is the actual nonlinear price schedule with each price step multiplied by 1.05. This "price elasticity" is the percent change in household i's expected water consumption as a result of a 5 percent increase in all prices on the nonlinear price function divided 0.05. Computing an "income elasticity" as the percent change in expected consumption from a 5 percent increase in household i's income divided by 5 percent yields the coefficient on logarithm of income. Since demographic characteristics and vegetation index are included in the income and price coefficients, we have different "price elasticity" and income "elasticity" for each household.

The "price elasticities" can be computed conditional on the vector of the household's demographic characteristics or unconditional on the household's vector of demographic characteristics. The only differences in the two "price elasticities" is whether the expectations in equation (31) are taken with respect to the distribution of A or assume a fixed value of A. Figure 8 and 9 compute the joint distribution of income and price elasticities for VoM and Cobb, respectively, using the posterior distribution of A, given in equation (19) for each observation in the sample. There is considerable heterogeneity in these elasticity estimates for both utilities.

It is also possible to compute the distribution of water consumption for all households in the utility's service territory and analogous aggregate demand elasticity estimates. Suppose there are J types of households, where households of type j have a vector of observed attributes, A_j, and H_j is the number of type j customers in the utility's service territory. This implies that the expected sales of water by the utility (summed across all customers) associated with rate schedule P is:

$$\text{Expected System-wide Water Sales} = \sum_{j=1}^{J} E[w^*(P, A_j)]H_j \tag{32}$$

$$\text{Variance in System-wide Water Sales} = \sum_{j=1}^{J} \mathrm{V}[w^*(P, A_j)]H_j \tag{33}$$

Following the same procedure for system-wide revenues yields:

$$\text{Expected System-wide Revenues} = \sum_{j=1}^{J} E[R^*(P, A_j)]H_j \tag{34}$$

$$\text{Variance in System-wide Revenues} = \sum_{j=1}^{J} \mathrm{V}[R^*(P, A_j)]H_j \tag{35}$$

The aggregate or system-wide "price elasticity of demand" can be computed by finding the percentage increase in expected system-wide demand as a result of a 5 percent increase in all price steps faced by all customers divided by 5 percent. The aggregate "income" elasticity is the percentage increase in expected system-wide demand as a result of a 5 percent increase in all customer incomes divided by 5 percent. Other functions of the distribution of system-wide sales and revenues can be computed. The water utility or its regulatory body might be interested in the probability that system-wide sales or revenues exceed or fall below a pre-specified value for a prospective rate schedule. The model estimates can be used to compute that probability.

8 Counterfactual Price Schedules

This section first quantifies the revenue risk reduction that is possible for the utility simply from gathering information on the demographic characteristics of its customers. It then reports on the computation of counterfactual price schedules to achieve two policy goals to demonstrate potential uses of our model of demand.

First, we illustrate the possible risk reduction by gathering information on customer demographic characteristics. For both Cobb and VoM, the distribution of sales and revenues for each household is computed under three different assumptions about the distribution of demographic characteristics in each zip code in order to quantify the impact of compiling information on the demographic characteristics of each household in the utility's service territory. The first case assumes the demographic characteristics of each household are unknown, but drawn from the prior distribution for that zip code obtained from the PUMS data. The second case assumes the household's demographic characteristics are unknown, but drawn from the posterior distribution for that zip code obtained from the model estimated and equation 19. The final set assumes the household's demographic characteristics are known and set equal to the value of A_n with the highest posterior probability from equation 19.

Table 14 and 15 report the expected revenue per month per customer, the standard deviation of system-wide revenues, expected consumption per month per customer, and the standard deviation of system-wide consumption per month for each of the three distributions of demographic characteristic for each household. Two conclusions emerge from this table. First, for both VoM and Cobb, the standard deviation in system-wide revenues for the case of known demographics is between $0.78 = (725/926)$ and $0.16 = (2100/13119)$ of the value for case that the demographics are drawn from the prior distribution of A_n. Second, using the posterior distribution instead of the prior distribution of A_n yields an estimate of the system-wide standard deviation of revenues that is slightly larger than the value for case that the vector of demographic characteristics is assumed to be known. Taken together these results, emphasize that even the imperfect knowledge of the value of A_n obtained from estimating the demand model and computing the posterior distribution of A_n can significantly reduce the sales and revenue risk faced by each utility.

Next, we compute counterfactual price schedules to achieve relevant policy goals to demonstrate potential uses of our model of demand. For VoM, we first compute a counterfactual price schedule that is consistent with California's current water demand reduction goals and is also unlikely to run afoul of Proposition 218, which requires that municipal water customers only pay for the cost of the water that they consume. Specifically, we compute a price schedule which yields 25 percent less system-wide water sales than the existing price schedule with 95 percent probability and achieves the system-wide expected revenue goals based on Proposition 218, while minimizing a measure of the financial burden of achieving these water consumption reduction goals across all classes of customers. The price schedule chosen minimizes the weighted sum of the squares of the difference between expected payments by each household under the counterfactual schedule and the current price schedule weighted by the inverse of that household's expected payments under the current price schedule. This objective function places the greatest burden to achieve water consumption reductions on households that currently have the largest water bills. We also compute a second counterfactual price schedule, with the same objective function and constraint, but instead offer households two price schedules for them to choose from.

For Cobb, we first compute a price schedule which yield the same or superior sales and revenue outcomes for the utility but minimizing aggregate revenue risk. This price schedule minimizes the standard deviation of utility-wide water revenues subject to the constraints that the utility expects to sell no more water than it does under the current price schedule and raises at least as much total revenue for the utility as the existing rate. We then solve the same optimization problem subject to the same constraints, but allow the utility to set two price schedules that depend on value of the household's NDVI. Specifically, the utility is allowed to set a schedule for households with an NDVI value less than 0.35 and one for households for an NDVI value greater than 0.35.

Three main conclusions emerge from this counterfactual price schedule design exercise:

- 1) The model of the household-level demand for a water utility can be used to reduce the system-wide revenue or sales risk associated with achieving any water pricing goal.
- 2) By compiling information on the demographic characteristics of their customers and building this information into the utility's model of household-level water demand, utilities can significantly reduce (up to 84% for two utilities considered) both the water sales and revenue risk associated with any expected water sales and revenue goals.
- 3) The customer-level model of demand incorporating demographic characteristics can be used to design a menu of price schedules that can be offered to households (that allows them to select which specific price schedule they would like to be) to achieve a given water pricing goal for the utility.

The price schedule optimization framework can readily incorporate constraints like the majority of customers have the same or lower monthly water bills under the optimal price schedules compared to the current schedules. For the rest of the counterfactual price design, we use the predicted demographics to compute consumption and revenue.

8.1 VoM — Predicted Demographics

This section considers a set of counterfactual price schedule choices that reflect policy goals and constraints relevant to California during the summer of 2015, the fourth consecutive summer of low water availability in the state. As a consequence, in the spring of 2015, Governor Jerry Brown issued an executive order requesting a 25 percent reduction in urban water consumption state-wide relative to 2013. A pre-existing legal constraint has further complicated the ability of municipal utilities to achieve this goal.

Proposition 218–*The Right to Vote on Taxes Initiative* requires that municipal utility consumers only pay what it costs to provide them with the water that they consume. AB 2882–*Allocation-based conservation water pricing*, signed into law in 2008, attempts to clarify how nonlinear pricing of water can be implemented to avoid running afoul of Proposition 218. However, a lawsuit filed by customers of the municipal utility in San Juan Capistrano and the resulting decision which struck down the utility's increasing block rate structure has led to considerable uncertainty over the use of nonlinear pricing of water in California, studied by Stephens (2015).

One possible solution to this problem is to determine a system-wide average cost of delivering a thousand gallons of water for the utility and then set a nonlinear price schedule so that the revenues recovered from each type of household (as determined by their demographic characteristics) equals this average cost times the amount of water they consume. Because this average cost information is not available for VoM, an aggregate revenue constraint is imposed that households in the utility service territory do not pay more under the new schedule than they pay under the existing price schedule. (The constraint implicitly assumes that the utility was only recovering the cost of the water supplied under the existing schedule.) The other constraint on the counterfactual price schedule is that it reduces system-wide water consumption by 25 percent relative to expected consumption under the existing schedule with at least a 95 percent probability. We impose no other constraints on the price schedule, except that it should be an increasing block tariff (the prices for each tier should be increasing). We also determine a uniform fixed cost for the counterfactual price schedule.

The objective function assumed for the optimal tariff design problem is to minimize the weighted sum of squared differences between each household's expected monthly bill under the current price schedule and the household's expected monthly bill under the counterfactual price schedule, where the weight applied to each household-level squared difference is the inverse of that household's expected monthly bill under the current price schedule. This objective function is designed to obtain the largest revenue increases from households with the largest current water bills and the smallest revenue increases from households with the smallest current water bills. Finding this price schedule requires solving the following optimization problem:

$$\begin{aligned} &\min_{P(w)} \sum_{h=1}^{H} [E(R_h(P(w))) - E(R_h(P_e(w)))]^2 / E(R_h(P_e(w))) \\ &\quad \text{subject to } \mathrm{Prob}\Big(\sum_{h=1}^{H} q_h(P(w)) < 0.75 \sum_{h=1}^{H} q_h(P_e(w))) \Big) \geq 0.95 \\ &\quad \text{and } E\Big[\sum_{h=1}^{H} R_h(P(w)) - R_h(P_e(w)) \Big] \leq 0 \end{aligned} \tag{36}$$

where $P(w)$ is the price schedule being solved for, $P_e(w)$ is the existing price schedule, $R_h((P(w)))$ is the revenue received from household h under the price schedule $P(w)$, $q_h(P(w))$ is the quantity demanded by household h under the price schedule $P(w)$, and $E(\cdot)$ is the expectation operator.

There are many other possible objective functions to optimize to obtain Governor Brown's desired 25 percent reduction in system-wide water consumption with a high probability. This one has the desirable property of putting less of the burden on households that are currently spending less on water.

Figure 10 plots the major current price schedule and price schedule that solves equation (36). Figure 11 plots the simulated distribution of household-level consumption under the current price schedule set by VoM and simulated distribution of household-level consumption under the counterfactual price schedule. We also present the expected revenue per month per customer, the standard deviation of system-wide revenues, expected consumption per month per customer, and the standard deviation of system-wide consumption per month under the current and counterfactual price schedule. We can see that under the counterfactual price schedule, expected consumption per month per customer drops significantly, and the expected revenue per month per customer also decreases. This is consistent with our policy goals.

In our second scenario, we have the same objective function to minimize and the same constraints on revenue and consumption. Instead of offering one single price schedule to every household, we now offer two price schedules and the household can choose whichever they want. We assume that the choice of price schedule happens after their realization of η_K, the Type I extreme value random variable that is in the conditional indirect utility function associated with price step k. Thus, households can choose the price tier that yields the highest expected utility among price tiers from both price schedules.

Figure 12 plots the major current price schedule and the two optimal counterfactual price schedules that households can choose from. The two counterfactual price schedules usually charge a higher marginal price than the current price schedule in order to reduce system-wide consumption. To compare between the two counterfactual price schedules, schedule 1 usually charges a higher marginal price than schedule 2, but has a lower fixed cost than schedule 2. Thus it makes sense for households with high consumption to choose schedule 2 to enjoy the low marginal price, and households with low consumption to choose price 1 to avoid the higher fixed cost. Figure 13 presents the histogram of consumption distribution for households under current price schedule, households choosing schedule 1, and households choosing schedule 2. Consistent with our prediction, households choosing schedule 1 consume less water compared to households choosing schedule 2. Households choosing schedule 2, despite facing a higher price schedule, consume more water than households under the current schedule. This suggests that households with high water consumption are more likely to choose schedule 2. This is again consistent with our intuition.

8.2 Cobb — Predicted Demographics

We compute multiple counterfactual price schedules for Cobb, with different price constraints and price schedule structure. The objective function is always to minimize the standard deviation of system-wide revenue. To maintain the structure of the 5-tier price schedule, we impose the following constraints on all the counterfactual computation: the first quantity cutoff of the counterfactual must be the same as the first quantity cutoff of the current schedule; the other quantity cutoffs of the counterfactual must not be bigger than the corresponding quantity cutoff of the current schedule; the price tiers must be increasing with at least \$0.1; the fixed cost cannot be bigger than \$15.

Figure 14 plots the actual price schedule faced by customers in Cobb County. This figure also plots two optimal counterfactual price schedules. Both price schedules are computed by minimizing the standard deviation of system-wide revenues subject to the constraints that expected revenues are at least as large as under the current price schedule and expected water sales are no larger than under the current price schedule. The only difference is that one has no additional constraints besides what mentioned in the previous paragraph, and the other has two additional price constraints to make sure the marginal prices are not too high: the lowest marginal price cannot be higher than the lowest marginal price in the actual price schedule, and the highest marginal price cannot be higher than the highest marginal price in the actual price schedule. Without the additional two price constraints, the counterfactual schedule reduces the standard deviation of system-wide revenues by 79%. With the two additional price constraints, we still see a 10.3% decrease in the system-wide revenue risk.

Figure 15 plots the simulated distribution of household-level consumption under the current price schedule set by Cobb and the simulated distribution under the two optimal counterfactual price schedules described in Figure 14. We also present the expected revenue per month per customer, the standard deviation of system-wide revenues, expected consumption per month per customer, and the standard deviation of system-wide consumption per month for all the scenarios. For the scenario "Counterfactual with price constraints", we can see that the consumption constraint in the optimization problem is binding, which means that the system-wide aggregated consumption level remains the same under the counterfactual compared to the system-wide aggregated consumption level under the current schedule. Consumption distribution doesn't change much as well. We do see increase in the expected revenue per month per customer, and decrease in the standard deviation of system-wide revenues. Without any constraints on price, we see a much higher price schedule in the scenario "Counterfactual without price constraint". As a result, we see significant drop in consumption, both from the consumption histograms and the expected consumption per month per customer. Revenue constraint in the optimization problem is binding now, thus the expected revenue per month per customer remains the same. We also see a huge drop in standard deviation of system-wide revenue, which is our objective function in the optimization problem.

Figure 16 plots the optimal NDVI-based price schedules when we can differentiate household based on their vegetation index and charge them with different price schedules. Similarly, the optimal NVDI-based price schedule minimizes the standard deviation of system-wide revenues subject to achieving at least as much expected revenues and no larger expected water sales than the actual price schedule. What is different is that the NDVI-based price schedules consist of two schedules, one assigned to customers with a value of NDVI less than 0.35 and the other assigned to customers with an NDVI value more than 0.35. Both schedules are subject to the price constraints described in the first paragraph of this subsection and also the two additional price constraints in the previous paragraphs. Figure 17 plots the simulated distribution of household-level consumption under the current price schedule set by Cobb and the simulated distribution under the two different price schedules based on vegetation index. We also present the expected revenue per month per customer, the standard deviation of system-wide revenues, expected consumption per month per customer, and the standard deviation of system-wide consumption per month for the current and counterfactual price scenario.

Notice that besides the differentiation based on vegetation index, this scenario is otherwise the same as the "Counterfactual with price constraints" in Figure 14 and 15. Through differentiated pricing, we see another 1% decrease in the standard deviation of system-wide revenues. We again see that the expected consumption per month per customer remains the same as the current level, while the expected revenue per customer per month increases compared to the current level.

Looking at the two counterfactual price schedules based on vegetation index in Figure 16, we notice that households with vegetation index$<$0.35 face a higher price schedule than households with vegetation index ≥ 0.35. This is consistent with Figure 18, where we plot the histograms of price elasticities for household with vegetation index$<$0.35 and with vegetation index ≥ 0.35 separately. In general, households with vegetation index$<$0.35 are more price inelastic, while households with vegetation index ≥ 0.35 have more elastic demand. It makes sense to set higher price for households with more inelastic demand, namely households with vegetation index$<$0.35. We also notice from Figure 17 that households with vegetation index ≥ 0.35 consume more than households with vegetation index < 0.35. This is consistent with our intuition that more vegetation requires more water consumption, and the fact that households with vegetation index ≥ 0.35 face lower price.

A number of other counterfactual price schedules can be computed that depend on demographic characteristics, elements of Z, and combinations of these variables. Further reductions in the standard deviation on system-wide revenues are possible with greater differentiation of price schedules based on household characteristics.

9 Conclusion

Model of demand can be used to simulate the distribution of the customer-level billing cycle level household demand for water for any increasing block price schedule. This model can then be used to simulate the distribution of the system-wide demand for water for any nonlinear price schedule. Consequently, we can use this model to design price schedules that achieve a wide range of water supply risk or revenue risk management goals in the utility's rate design process.

An important implication of this modeling and simulation exercise is to demonstrate the tremendous reduction in revenue risk facing the utility if it has the information on the demographic characteristics of its households. For the case of Cobb, the measure of the variance of system-wide revenue conditioning on the assumed knowledge of the vector of demographic characteristics was roughly 16% of the measure of the variance of system-wide revenues assuming that only the distribution of demographic in each Zip Code in the utility's service area was known.

The model was used to show that further revenue variance reductions could be achieved by demographics-based price schedules. The household-level water demand model was used to solve for the optimal (minimum system-wide revenue variance) demographic-based price schedules. Again, significant variance reductions were possible when we have more demographic information. The model can even be used to assist the utility in managing water shortfall and potential revenue shortfalls.

The results presented here demonstrate that there is significant value to utility from understanding distribution of household level demand to design price schedules to achieve competing policy goals. In particular, by compiling demographic data from customers and using in customer-level models of demand, utilities can significantly reduce the variance in both the system-wide revenues and the amount of water sold in any price schedule design process. This results implies up to roughly 84% reduction in the revenue risk that the utility faces from knowledge of the demographic characteristics of its customers is possible, suggests significant economic benefits to water utilities from collecting demographic data on its customers and formulating household-level demand models for price schedule design.

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Tables

Table 1: Model Parameter Estimates, Valley of The Moon, California, Nonlinear Model

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-3.70677	0.07458	0.21342
Constant in the price elasticity formula	-0.88211	0.02658	0.11121
Std.dev. of demand shock	0.27934	0.00236	0.00435
Constant	3.14291	0.10957	0.14946
Average high temp in billing cycle	0.01787	0.00110	0.00131
75th - 25th percentile of temperature in billing cycle	-0.00579	0.00150	0.00094
Total precipitation in billing cycle	-0.00303	0.00420	0.00325
75th - 25th percentile of precipitation in billing cycle	-0.09926	0.32078	0.30925
Number of people over 18 in house	-0.87256	0.01470	0.03795
Number of people under 18 in house	0.99133	0.00916	0.02708
House acreage above 1 acre	0.41858	0.00886	0.02095
Number of bedrooms in house	-0.40277	0.00668	0.01136
Price*temp	-0.00784	0.00057	0.00125
Price*precip	-0.00035	0.00179	0.00248
Price* # of adults	-0.09592	0.00993	0.03001
Price* # of children	0.74159	0.00963	0.03904
Price* # of bedrooms	-1.02665	0.01484	0.08791
Income* # of bedrooms	-1.30210	0.03244	0.11138
Vegetation Index	0.06638	0.06928	0.08475
Income*Vegetation Index	0.02277	0.08146	0.13313
Price*Vegetation Index	0.01531	0.04590	0.08007
Number of customers	2001		

Table 2: Model Parameter Estimates, Cobb County, Georgia, Nonlinear Model

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant Term in income elasticity formula	-0.47228	0.01263	0.03522
Constant Term in price elasticity formula	0.48235	0.01288	0.02028
Std.dev. of demand shock	0.21543	0.00247	0.00594
Constant	-1.44246	0.12488	0.32257
Average high temp in billing cycle	0.00259	0.00058	0.00093
75th - 25th percentile of temperature in billing cycle	0.00166	0.00128	0.00114
Total precipitation in billing cycle	0.01142	0.00504	0.00647
75th - 25th percentile of precipitation in billing cycle	-0.20471	0.12739	0.11624
Number of people over 18 in house	0.39160	0.01033	0.02420
Number of people under 18 in house	-0.48318	0.00721	0.01843
House acreage above 1 acre	-0.00053	0.02260	0.03944
Number of bedrooms in house	0.32012	0.01763	0.04764
Price*temp	-0.00117	0.00012	0.00029
Price*precip	0.00028	0.00101	0.00185
Price* # of adults	0.02143	0.00240	0.00494
Price* # of children	-0.13161	0.00227	0.00658
Price* # of bedrooms	-0.71587	0.00941	0.00853
Income* # of bedrooms	-0.64361	0.00898	0.01221
Vegetation Index	-0.40642	0.04940	0.05576
Income*Vegetation Index	0.40519	0.02532	0.05125
Price*Vegetation Index	0.40937	0.02872	0.05442
Number of customers	1004		

Table 3: Model Parameter Estimates, Valley of The Moon, California, Alternate Nonlinear Price Model

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.85194	0.02877	0.06931
Constant in the price elasticity formula	0.05170	0.07406	0.11288
Std.dev. of household heterogeneity, η^o	0.28244	0.00962	0.01733
Std.dev. of perception error, ν^o	0.23489	0.01100	0.01899
Constant	-8.60166	0.19820	0.35696
Average high temp in billing cycle	0.06809	0.00142	0.00271
75th - 25th percentile of temperature in billing cycle	-0.01786	0.00181	0.00138
Total precipitation in billing cycle	-0.00679	0.00454	0.00613
75th - 25th percentile of precipitation in billing cycle	-1.46303	0.35349	0.40253
Number of people over 18 in house	1.49165	0.04932	0.07654
Number of people under 18 in house	-0.53010	0.03370	0.06262
House acreage above 1 acre	0.30189	0.02093	0.04124
Number of bedrooms in house	0.32636	0.01632	0.03145
Price*temp	0.01510	0.00099	0.00148
Price*precip	-0.00996	0.00207	0.00286
Price* # of adults	0.41921	0.01314	0.03324
Price* # of children	-0.79405	0.03575	0.04886
Price* # of bedrooms	-0.55449	0.01134	0.03011
Income* # of bedrooms	-0.35649	0.01319	0.01633
Vegetation Index	0.89537	0.20289	0.38153
Income*Vegetation Index	-0.16842	0.07123	0.13055
Price*Vegetation Index	0.07914	0.05887	0.10221
Number of customers	2001		

Table 4: Model Parameter Estimates, Cobb County, Georgia, Alternative Nonlinear Price Model

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-1.50618	0.04998	0.09037
Constant in the price elasticity formula	-0.04314	0.08634	0.20551
Std.dev. of household heterogeneity, η^o	0.05313	0.04708	0.08209
Std.dev. of perception error, ν^o	0.37147	0.00605	0.01237
Constant	0.49737	0.08503	0.13644
Average high temp in billing cycle	-0.00762	0.00120	0.00164
75th - 25th percentile of temperature in billing cycle	0.00302	0.00157	0.00143
Total precipitation in billing cycle	-0.00146	0.00668	0.01043
75th - 25th percentile of precipitation in billing cycle	-0.34150	0.17086	0.14969
Number of people over 18 in house	0.69489	0.03076	0.08129
Number of people under 18 in house	-0.36883	0.01261	0.04468
House acreage above 1 acre	-0.06528	0.03710	0.04857
Number of bedrooms in house	0.15980	0.04141	0.06387
Price*temp	-0.00638	0.00044	0.00102
Price*precip	-0.00175	0.00152	0.00391
Price* # of adults	0.14059	0.00931	0.01899
Price* # of children	0.03184	0.00261	0.00906
Price* # of bedrooms	0.41575	0.02009	0.02969
Income* # of bedrooms	0.50085	0.01929	0.02963
Vegetation Index	-0.08881	0.06354	0.09905
Income*Vegetation Index	0.44660	0.06923	0.13087
Price*Vegetation Index	0.35808	0.06957	0.14615
Number of customers	1004		

Table 5: Model Parameter Estimates, Valley of The Moon, California, Alternative 1

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.47265	0.02150	0.03702
Constant in the price elasticity formula	-3.36721	0.33528	0.66109
Std.dev. of error term	0.36656	0.00133	0.00718
Constant	-3.15137	0.20755	0.31736
Average high temp in billing cycle	0.01430	0.00118	0.00152
75th - 25th percentile of temperature in billing cycle	-0.01634	0.00179	0.00135
Total precipitation in billing cycle	0.00491	0.00402	0.00488
75th - 25th percentile of precipitation in billing cycle	-0.65539	0.34797	0.38098
Number of people over 18 in house	-0.42495	0.02643	0.04996
Number of people under 18 in house	-0.06685	0.01060	0.01658
House acreage above 1 acre	-0.64191	0.02203	0.04327
Number of bedrooms in house	0.11507	0.01443	0.02945
Price*temp	-0.05502	0.00198	0.00361
Price*precip	0.01083	0.00317	0.00754
Price* # of adults	0.55557	0.02382	0.04258
Price* # of children	-3.36712	0.49235	1.01796
Price* # of bedrooms	-0.96219	0.02155	0.05693
Income* # of bedrooms	-0.16105	0.00520	0.01177
Vegetation Index	-0.94390	0.31134	0.43796
Income*Vegetation Index	0.23268	0.06210	0.08509
Price*Vegetation Index	0.35340	0.11652	0.29195
Number of customers	2001		

Table 6: Model Parameter Estimates, Valley of The Moon, California, Alternative 2

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.33191	0.01733	0.04288
Constant in the price elasticity formula	-2.06277	0.15052	0.26083
Std.dev. of error term	0.36447	0.00129	0.00701
Constant	-4.25562	0.17069	0.44819
Average high temp in billing cycle	0.01186	0.00137	0.00152
75th - 25th percentile of temperature in billing cycle	-0.01557	0.00177	0.00133
Total precipitation in billing cycle	0.00397	0.00435	0.00459
75th - 25th percentile of precipitation in billing cycle	-0.76789	0.33990	0.39514
Number of people over 18 in house	-0.87397	0.02253	0.02785
Number of people under 18 in house	-0.38895	0.00925	0.01391
House acreage above 1 acre	0.45748	0.02080	0.07069
Number of bedrooms in house	0.46729	0.01425	0.03532
Price*temp	-0.04331	0.00173	0.00324
Price*precip	0.00954	0.00318	0.00496
Price* # of adults	-1.00047	0.04438	0.08447
Price* # of children	-1.63913	0.16855	0.20597
Price* # of bedrooms	-0.00185	0.00480	0.01422
Income* # of bedrooms	-0.17174	0.00395	0.00651
Vegetation Index	-0.06406	0.24490	0.66623
Income*Vegetation Index	0.02648	0.03965	0.11430
Price*Vegetation Index	-0.11463	0.07805	0.20003
Number of customers	2001		

Table 7: Model Parameter Estimates, Valley of The Moon, California, Alternative 3

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.45971	0.02081	0.03161
Constant in the price elasticity formula	-1.69362	0.08841	0.08164
Std.dev. of error term	0.36864	0.00136	0.00727
Constant	-3.25872	0.19225	0.28066
Average high temp in billing cycle	0.01344	0.00127	0.00157
75th - 25th percentile of temperature in billing cycle	-0.01648	0.00181	0.00135
Total precipitation in billing cycle	0.00833	0.00415	0.00510
75th - 25th percentile of precipitation in billing cycle	-0.62495	0.34948	0.38411
Number of people over 18 in house	-0.42408	0.02246	0.07900
Number of people under 18 in house	-0.05106	0.01306	0.04248
House acreage above 1 acre	-0.57542	0.02470	0.06323
Number of bedrooms in house	0.13650	0.01231	0.01886
Price*temp	-0.04968	0.00205	0.00316
Price*precip	0.01377	0.00307	0.00703
Price* # of adults	0.56883	0.02682	0.05046
Price* # of children	-0.79591	0.05206	0.06941
Price* # of bedrooms	-0.97832	0.02689	0.04440
Income* # of bedrooms	-0.16968	0.00541	0.01265
Vegetation Index	-0.73120	0.27397	0.45455
Income*Vegetation Index	0.18243	0.05531	0.08939
Price*Vegetation Index	0.24278	0.11045	0.29257
Number of customers	2001		

Table 8: Model Parameter Estimates, Valley of The Moon, California, Alternative 4

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-1.06894	0.04600	0.12908
Constant in the price elasticity formula	-0.25119	0.03023	0.10559
Std.dev. of error term	0.38464	0.00153	0.00768
Constant	-2.93365	0.24219	0.53549
Average high temp in billing cycle	0.06400	0.00144	0.00252
75th - 25th percentile of temperature in billing cycle	-0.00882	0.00170	0.00151
Total precipitation in billing cycle	-0.02410	0.00508	0.00806
75th - 25th percentile of precipitation in billing cycle	-0.35662	0.37857	0.38281
Number of people over 18 in house	-0.89945	0.02518	0.07362
Number of people under 18 in house	-0.47474	0.01126	0.02177
House acreage above 1 acre	0.25359	0.03194	0.04717
Number of bedrooms in house	0.21446	0.01994	0.04988
Price*temp	0.02068	0.00087	0.00148
Price*precip	-0.00223	0.00250	0.00391
Price* # of adults	-0.17142	0.01656	0.04233
Price* # of children	-0.84181	0.02932	0.11172
Price* # of bedrooms	-0.15807	0.01453	0.03207
Income* # of bedrooms	-0.23436	0.01217	0.03275
Vegetation Index	0.89180	0.31415	0.39601
Income*Vegetation Index	-0.17431	0.11570	0.18084
Price*Vegetation Index	0.01313	0.04898	0.10758
Number of customers	2001		

Table 9: Model Parameter Estimates, Cobb County, Georgia, Alternative 1

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.47346	0.01992	0.06681
Constant in the price elasticity formula	-3.66710	0.27189	0.26784
Std.dev. of error term	0.37927	0.00126	0.00619
Constant	-2.52731	0.10800	0.22752
Average high temp in billing cycle	0.00302	0.00067	0.00081
75th - 25th percentile of temperature in billing cycle	0.00174	0.00149	0.00138
Total precipitation in billing cycle	0.00860	0.00538	0.00590
75th - 25th percentile of precipitation in billing cycle	-0.41771	0.15956	0.13863
Number of people over 18 in house	0.35078	0.02544	0.03352
Number of people under 18 in house	-0.16692	0.01076	0.01507
House acreage above 1 acre	-0.03980	0.03392	0.08367
Number of bedrooms in house	-0.58047	0.03239	0.13300
Price*temp	-0.03324	0.00250	0.00326
Price*precip	-0.01598	0.00751	0.02242
Price* # of adults	1.23847	0.08965	0.10416
Price* # of children	-0.06708	0.02388	0.04352
Price* # of bedrooms	0.36360	0.03125	0.04763
Income* # of bedrooms	0.12932	0.00458	0.01124
Vegetation Index	-0.70405	0.13742	0.24799
Income*Vegetation Index	0.12867	0.02636	0.04715
Price*Vegetation Index	-0.29063	0.11312	0.23816
Number of customers	1004		

Table 10: Model Parameter Estimates, Cobb County, Georgia, Alternative 2

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.42983	0.01773	0.09463
Constant in the price elasticity formula	-1.44802	0.15342	0.24818
Std.dev. of error term	0.37611	0.00137	0.00616
Constant	1.25484	0.24788	1.23824
Average high temp in billing cycle	0.02263	0.00095	0.00190
75th - 25th percentile of temperature in billing cycle	0.00249	0.00149	0.00142
Total precipitation in billing cycle	0.00726	0.00707	0.02870
75th - 25th percentile of precipitation in billing cycle	-0.47332	0.15853	0.14164
Number of people over 18 in house	0.18768	0.02118	0.40274
Number of people under 18 in house	-0.21237	0.01360	0.04238
House acreage above 1 acre	-0.35848	0.03938	0.36731
Number of bedrooms in house	-1.83027	0.06737	0.11055
Price*temp	0.02149	0.00186	0.00263
Price*precip	-0.00641	0.00765	0.02689
Price* # of adults	-0.20770	0.03428	0.27076
Price* # of children	-0.08056	0.02348	0.04081
Price* # of bedrooms	-0.59325	0.02841	0.09604
Income* # of bedrooms	0.22549	0.00380	0.02329
Vegetation Index	-2.53845	0.12222	0.45988
Income*Vegetation Index	0.32370	0.01553	0.05101
Price*Vegetation Index	-0.99495	0.10591	0.26858
Number of customers	1004		

Table 11: Model Parameter Estimates, Cobb County, Georgia, Alternative 3

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.47201	0.01991	0.06773
Constant in the price elasticity formula	-3.66402	0.27188	0.26742
Std.dev. of error term	0.37922	0.00126	0.00619
Constant	-2.52625	0.10798	0.22774
Average high temp in billing cycle	0.00301	0.00067	0.00081
75th - 25th percentile of temperature in billing cycle	0.00174	0.00149	0.00138
Total precipitation in billing cycle	0.00857	0.00538	0.00589
75th - 25th percentile of precipitation in billing cycle	-0.41796	0.15954	0.13861
Number of people over 18 in house	0.35046	0.02545	0.03356
Number of people under 18 in house	-0.16676	0.01078	0.01518
House acreage above 1 acre	-0.03902	0.03392	0.08357
Number of bedrooms in house	-0.58288	0.03244	0.13540
Price*temp	-0.03323	0.00250	0.00327
Price*precip	-0.01599	0.00751	0.02242
Price* # of adults	1.23683	0.08960	0.10369
Price* # of children	-0.06748	0.02393	0.04380
Price* # of bedrooms	0.36358	0.03124	0.04762
Income* # of bedrooms	0.12946	0.00457	0.01125
Vegetation Index	-0.70389	0.13729	0.25017
Income*Vegetation Index	0.12842	0.02630	0.04744
Price*Vegetation Index	-0.29156	0.11337	0.23812
Number of customers	1004		

Table 12: Model Parameter Estimates, Cobb County, Georgia, Alternative 4

Parameter Name	Estimate	Standard Error (Outer Product of Gradients)	Standard Error (White (1982) Formula)
Constant in the income elasticity formula	-0.72914	0.02782	0.14899
Constant in the price elasticity formula	-0.36119	0.05708	0.21913
Std.dev. of error term	0.37954	0.00136	0.00611
Constant	-4.68268	0.25160	1.09953
Average high temp in billing cycle	0.03137	0.00103	0.00245
75th - 25th percentile of temperature in billing cycle	0.00150	0.00147	0.00140
Total precipitation in billing cycle	0.02696	0.00985	0.01537
75th - 25th percentile of precipitation in billing cycle	-0.43953	0.16596	0.14502
Number of people over 18 in house	0.02226	0.02872	0.08939
Number of people under 18 in house	-0.85098	0.02625	0.08457
House acreage above 1 acre	-0.13625	0.04505	0.22650
Number of bedrooms in house	0.62558	0.03437	0.25571
Price*temp	0.01155	0.00054	0.00137
Price*precip	0.00728	0.00406	0.00625
Price* # of adults	0.05633	0.01267	0.02721
Price* # of children	-0.71261	0.03440	0.10205
Price* # of bedrooms	0.04260	0.01016	0.07189
Income* # of bedrooms	-0.08357	0.01050	0.08444
Vegetation Index	0.31517	0.34412	1.28492
Income*Vegetation Index	0.03159	0.08563	0.31804
Price*Vegetation Index	0.19165	0.03019	0.05016
Number of customers	1004		

Table 13: Non-Nested Test of New Nonlinear Pricing Model Versus Alternative Price Model

Utility	Alternative Model				
	Actual	Average	Alt. Actual	Total Average	Alt. Nonlinear Price
VoM	5.77	4.63	6.24	-1.62	4.72
Cobb	6.32	6.86	6.30	5.22	5.28

Note: Test statistic is asymptotically distributed as a $N(0,1)$ random variable under null hypothesis.

Table 14: Revenue Uncertainty and Demographic Information, VoM, New Model

	E[Revenue per month per bill]	sd[System wide revenue]	E[Consumption per month per bill]	sd[System-wide consumption]
Demographics drawn from prior	$28.97	$926	6.07 TGAL	233 TGAL
Demographics drawn from posterior	$29.23	$740	6.15 TGAL	179 TGAL
Demographics predicted	$29.32	$725	6.18 TGAL	175 TGAL

Note: TGAL = Thousands of gallons

Table 15: Revenue Uncertainty and Demographic Information, Cobb, New Model

	E[Revenue per month per bill]	sd[System wide revenue]	E[Consumption per month per bill]	sd[System-wide consumption]
Demographics drawn from prior	$95.58	$13119	8.65 TGAL	987 TGAL
Demographics drawn from posterior	$63.18	$2265	6.14 TGAL	184 TGAL
Demographics predicted	$62.79	$2100	6.11 TGAL	172 TGAL

Note: TGAL = Thousands of gallons

Figures

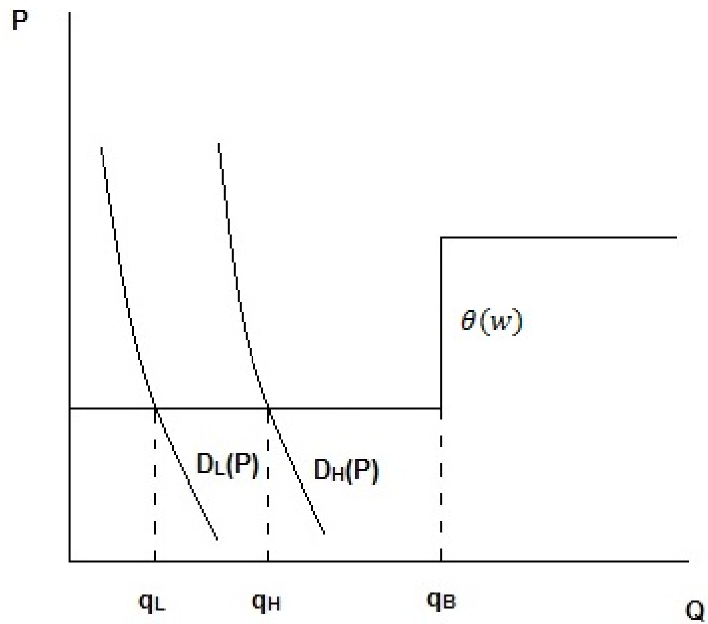

Figure 1

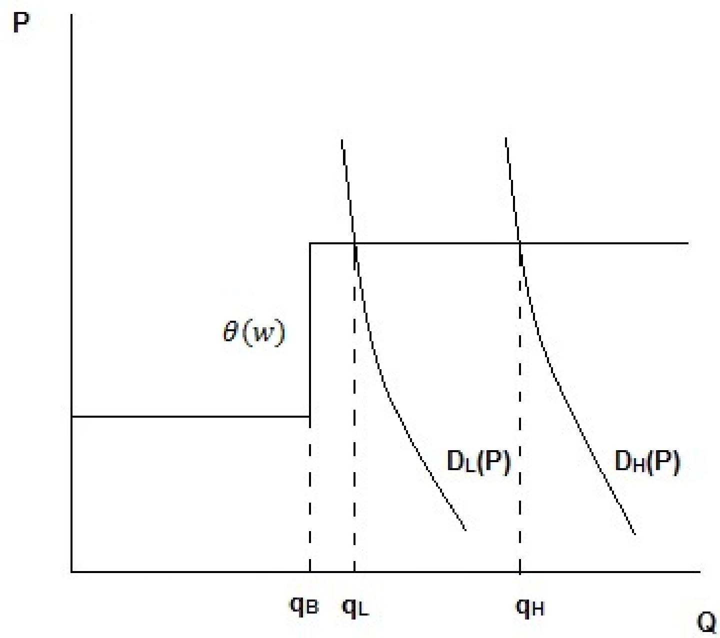

Figure 2

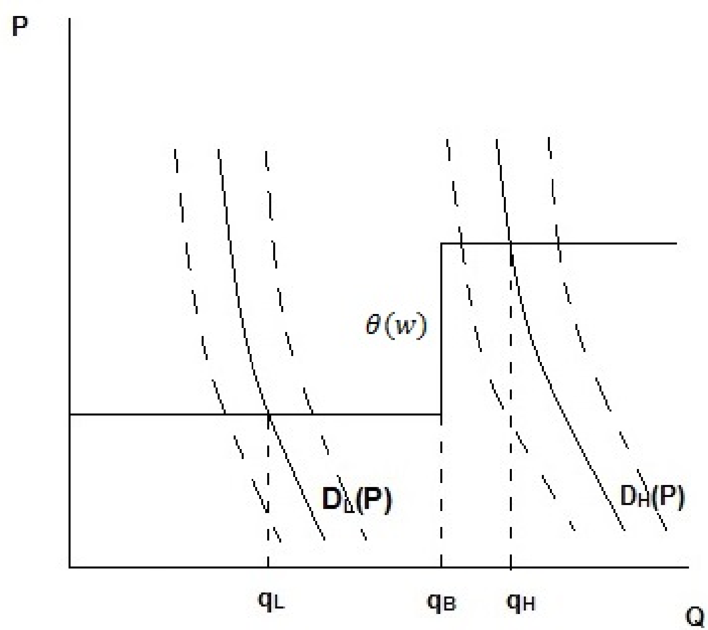

Figure 3

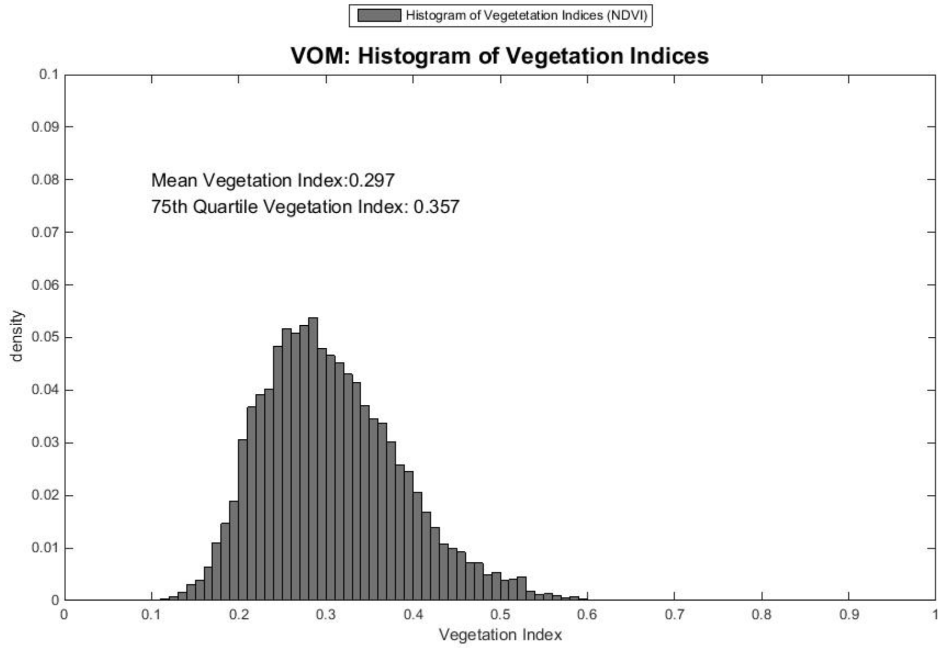

Figure 4: Histogram of NDVI for Valley of the Moon

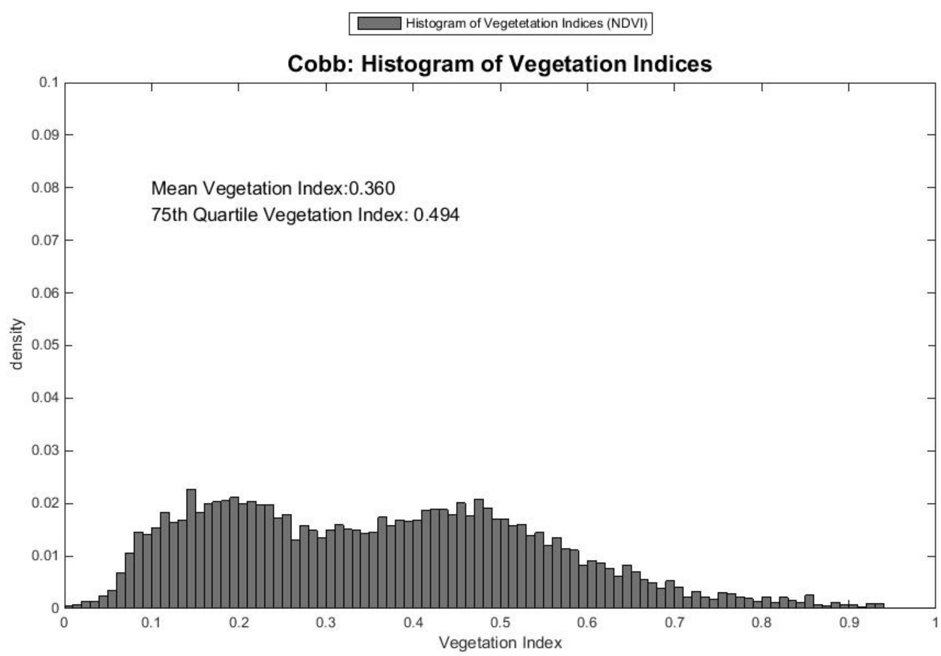

Figure 5: Histogram of NDVI for Cobb County, Georgia

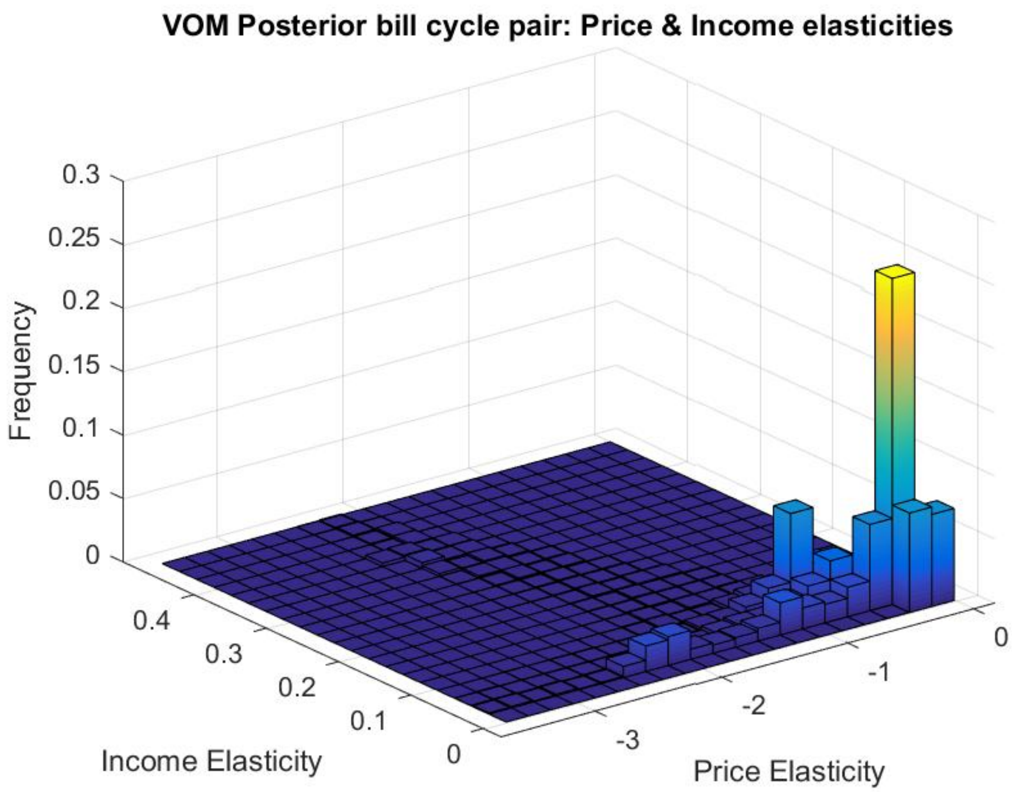

Figure 6: Histogram of Price and Income Elasticities for VoM, New Model

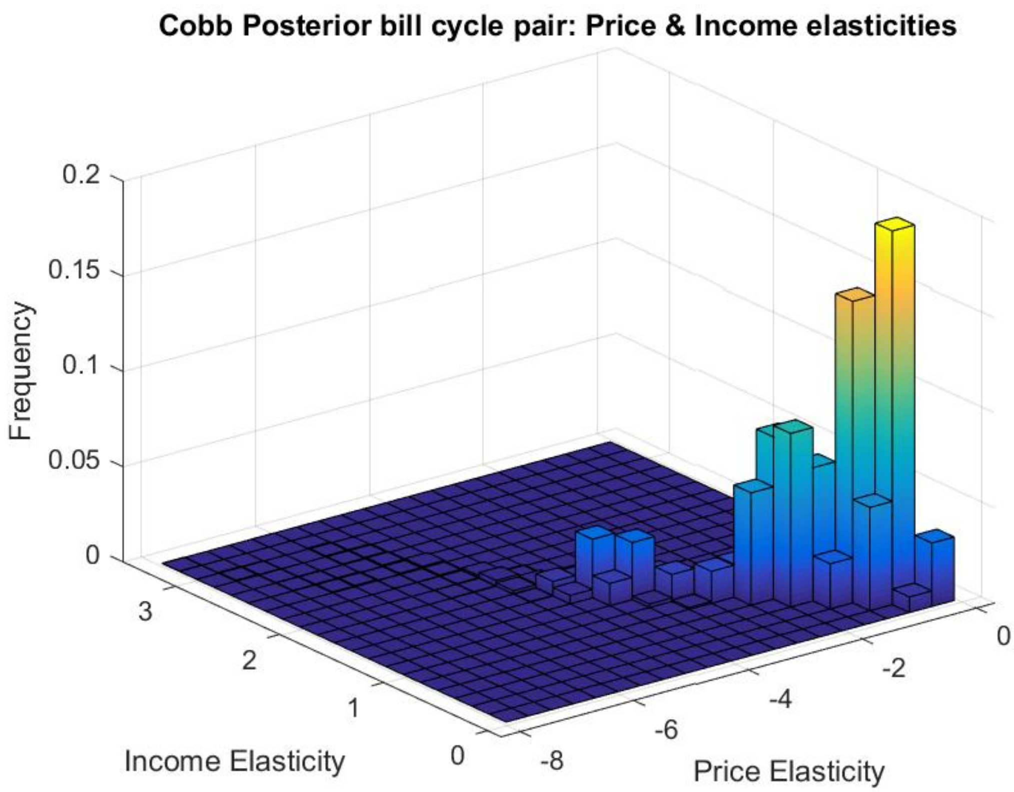

Figure 7: Histogram of Price and Income Elasticities for Cobb, New Model

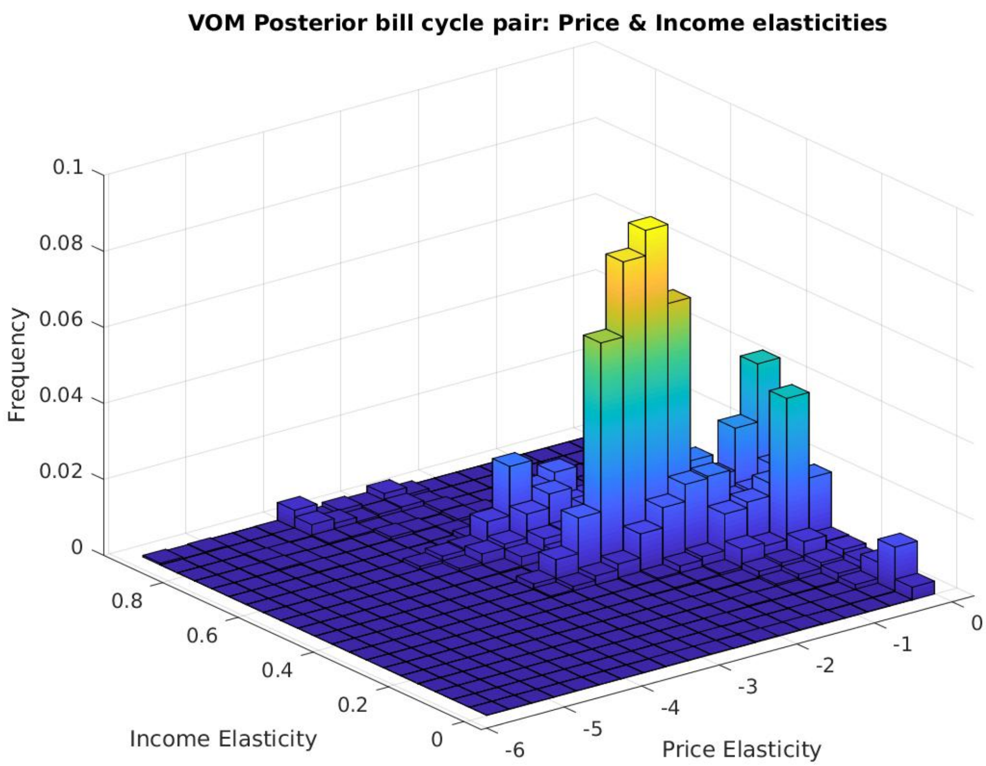

Figure 8: Histogram of Price and Income Elasticities for VoM, Alternate Nonlinear Price Model

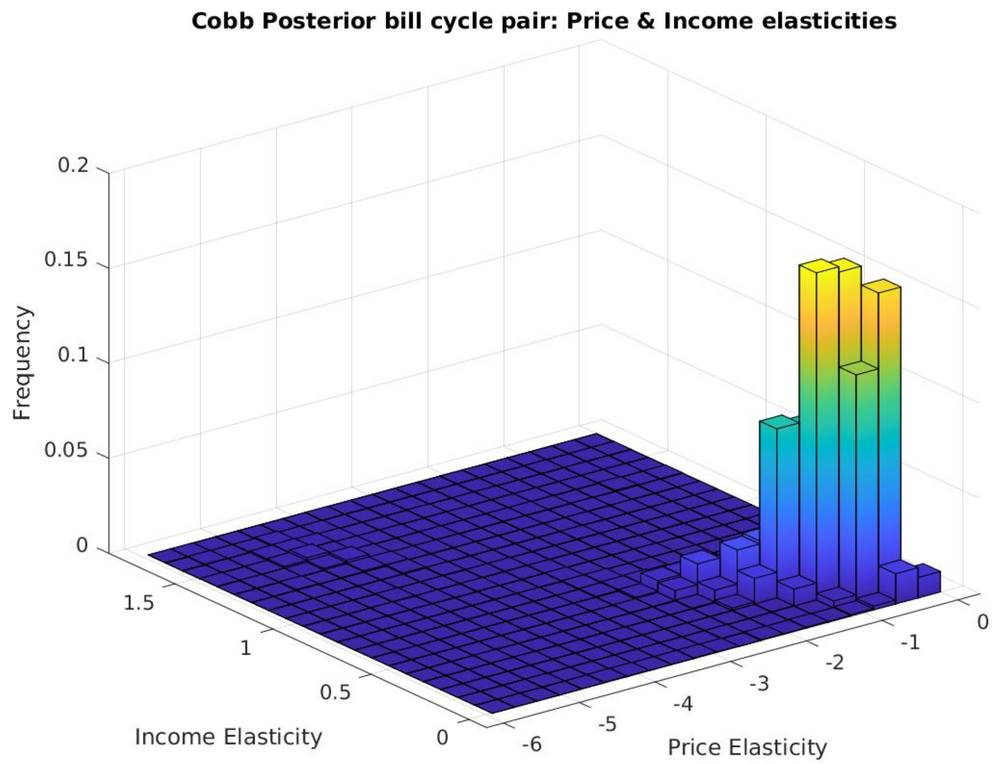

Figure 9: Histogram of Price and Income Elasticities for Cobb, Alternate Nonlinear Price Model

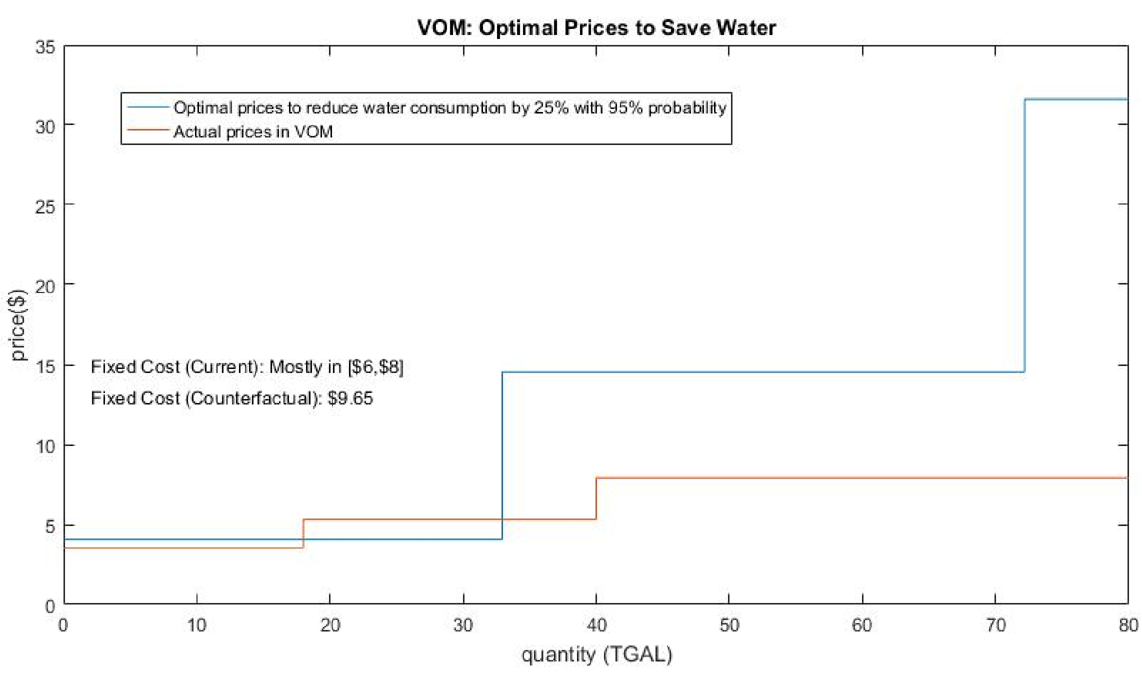

Figure 10: Optimal Price Schedule to Save Water, VoM

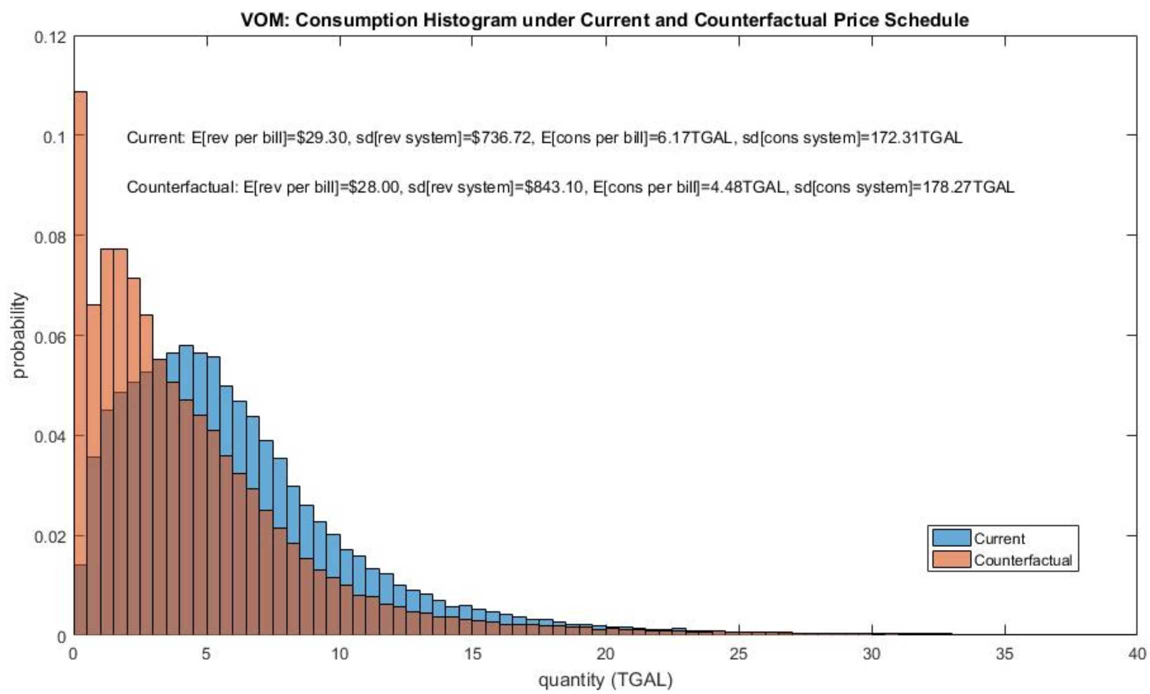

Figure 11: Histogram of Consumption Under Current and Counterfactual Price Schedule, VoM

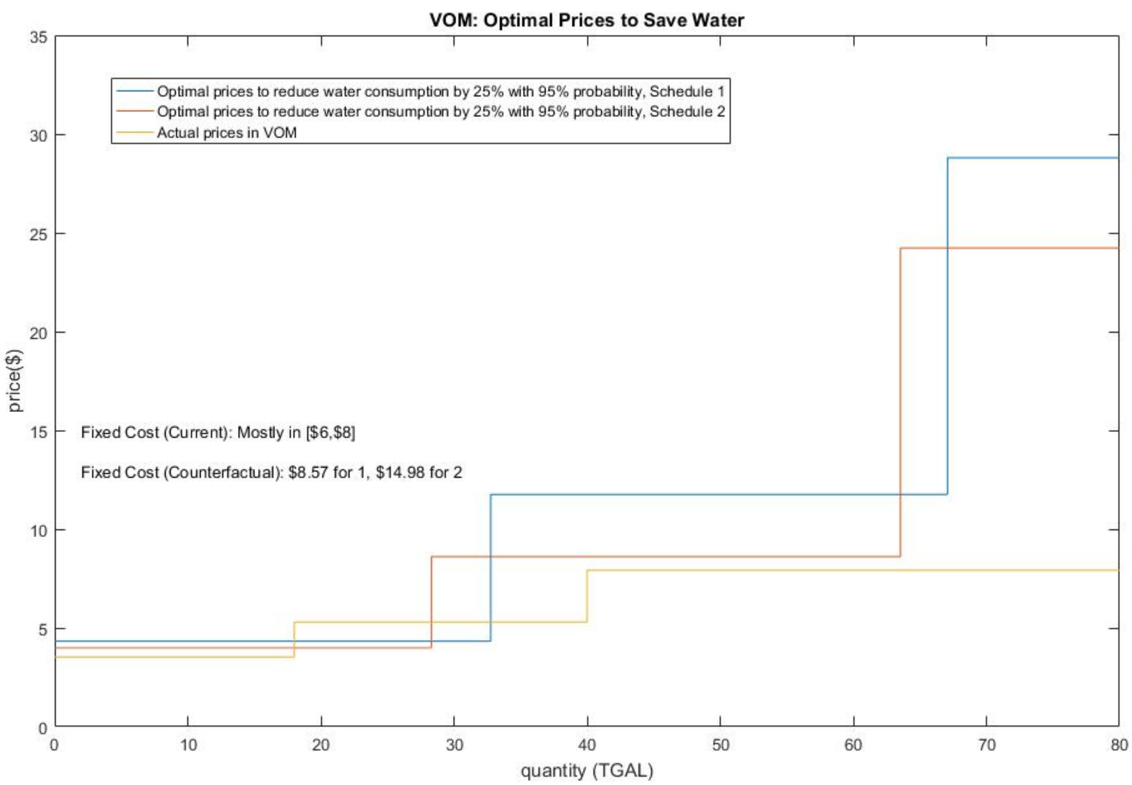

Figure 12: Optimal Price Schedule to Save Water, Self-selected, VoM

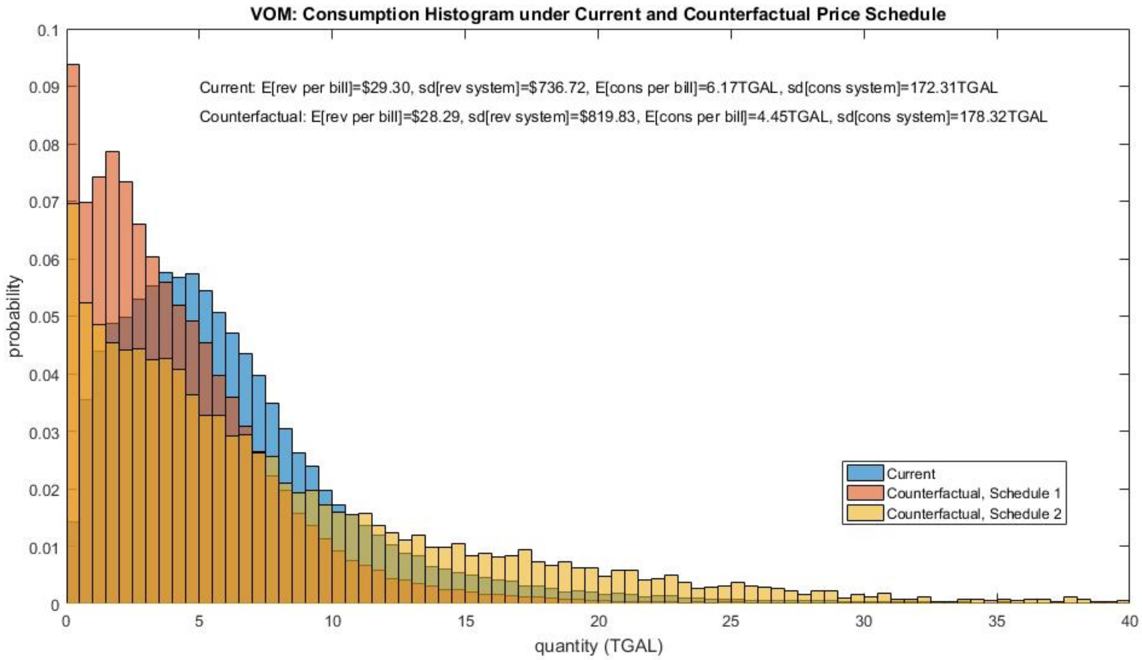

Figure 13: Histogram of Consumption Under Current and Counterfactual Price Schedule, Self-selected, VoM

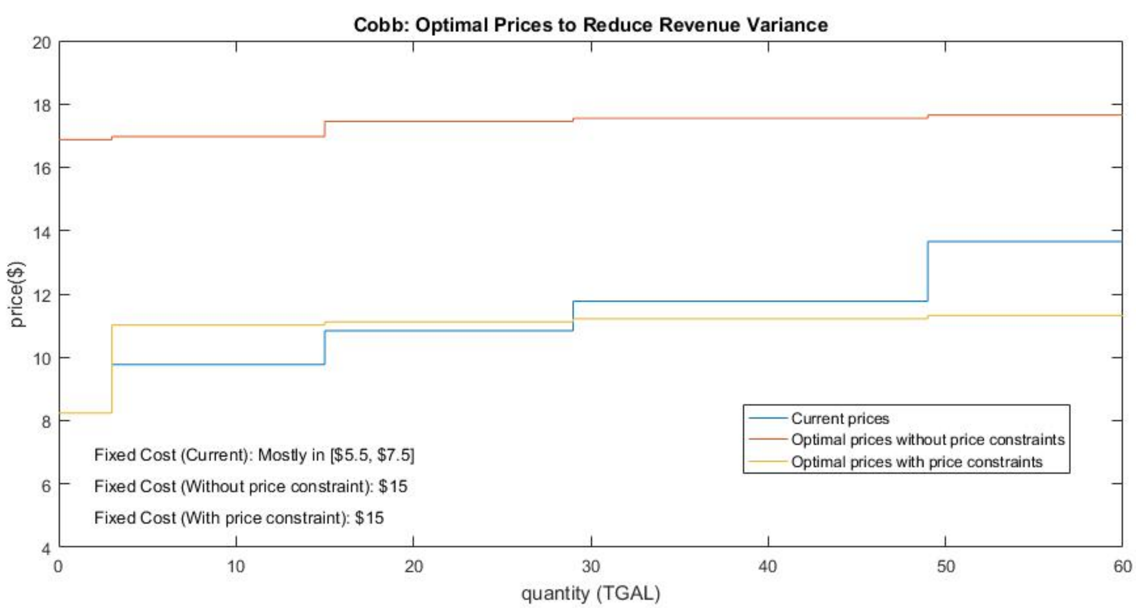

Figure 14: Optimal Price Schedules to Minimum System-wide Revenue Variation, Cobb

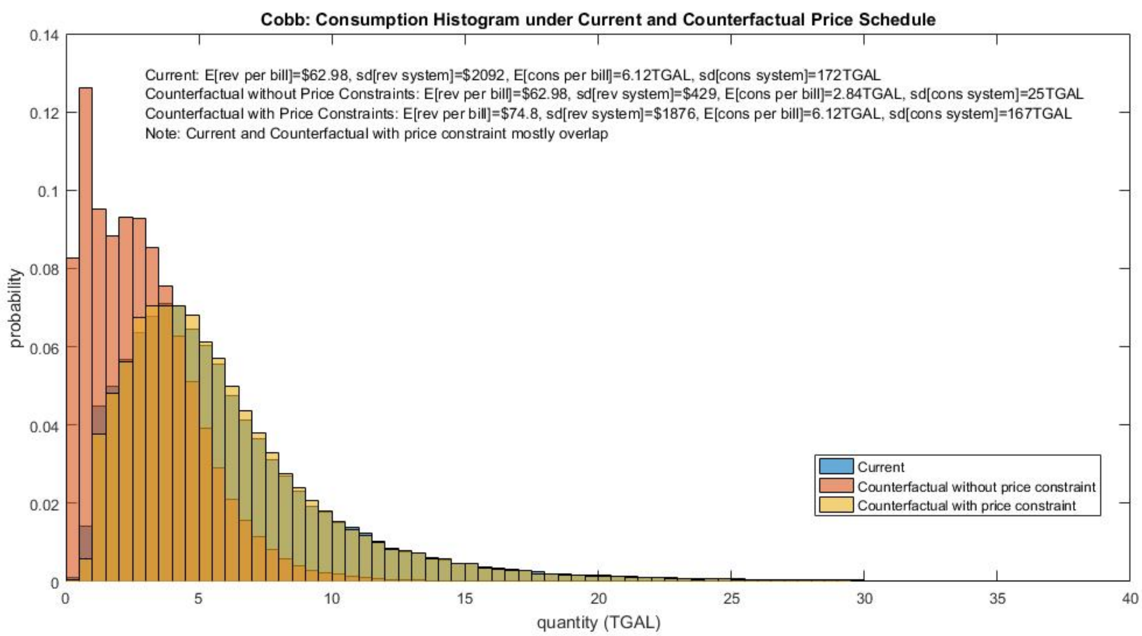

Figure 15: Histogram of Consumption Under Current and Counterfactual Price Schedule, Cobb

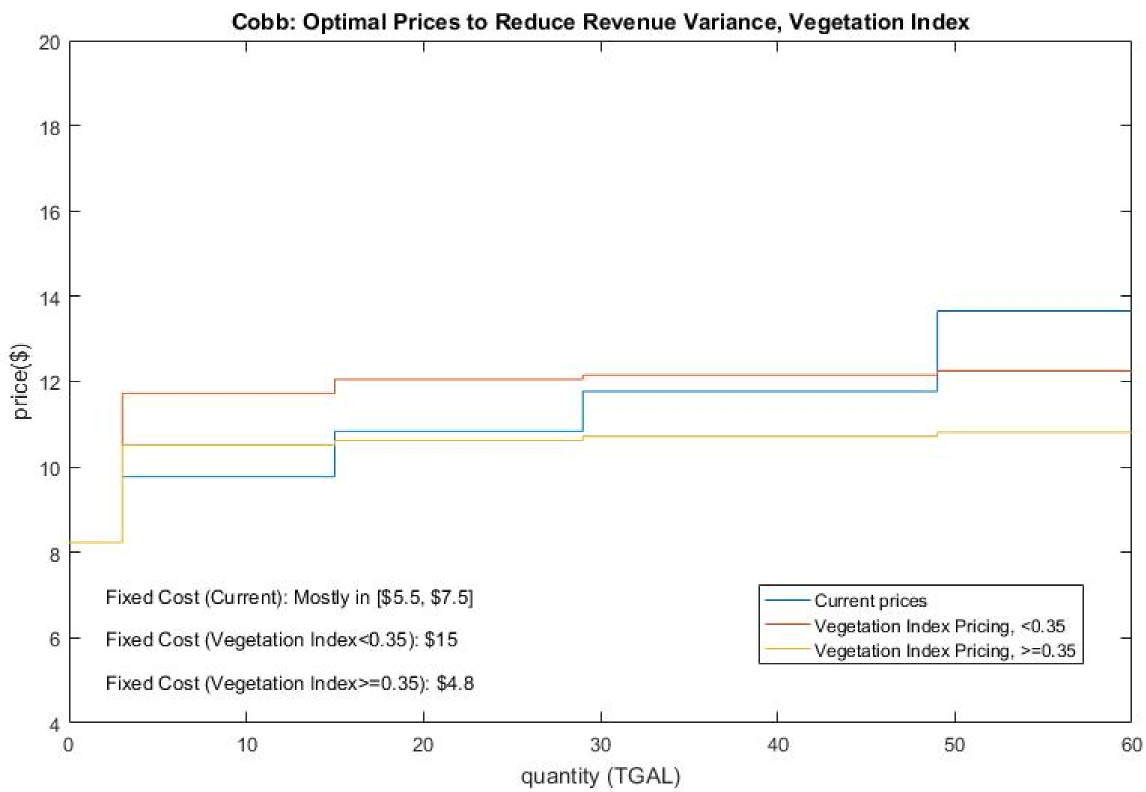

Figure 16: Optimal Price Schedule to Minimum System-wide Revenue Variation, Vegetation Index, Cobb

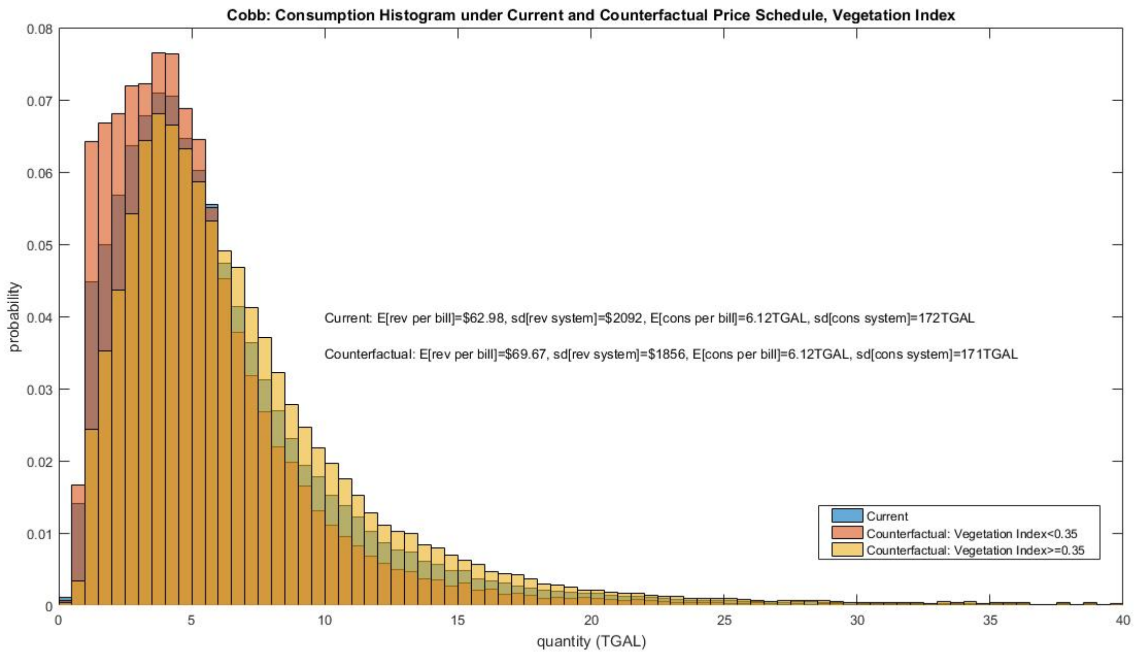

Figure 17: Histogram of Consumption Under Current and Counterfactual Price Schedule, Vegetation Index, Cobb

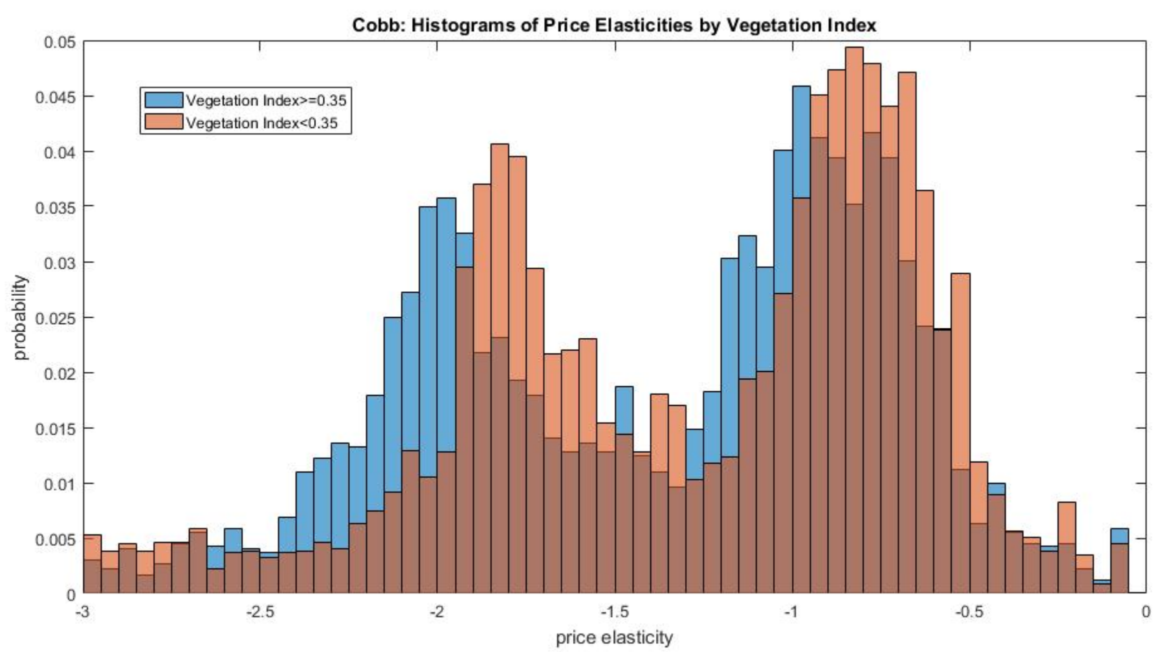

Figure 18: Histogram of Price Elasticities, Vegetation Index, Cobb