Applied Energy 260 (2020) 114251

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Optimal offer-bid strategy of an energy storage portfolio: A linear quasi-relaxation approach

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HIGHLIGHTS

- Stochastic disjunctive program model of a strategic merchant storage port-folio.
- A Specialized Branch-and-Bound solution algorithm.
- Superior computational performance of the proposed solution algorithm.
- Simulations reveal various abilities of storage to exercise unilateral market power.

ARTICLE INFO

Keywords:
Merchant storage
Offer-bid strategy
Bilinear program
Disjunctive program
Linear quasi-relaxation

ABSTRACT

This paper proposes a model of the behavior of an expected profit-maximizing merchant storage owner with the ability to exercise unilateral market power. The resulting non-linear bilevel optimization problem is transformed into a single-level stochastic bilinear program using the Karush-Kuhn-Tucker conditions of the lower-level Independent System Operator dispatch problem. By discretizing the offers and bids of the merchant storage owner, the problem is formulated as a stochastic disjunctive program. Using the disjunctive nature of the derived program, a specialized branch-and-bound algorithm that applies a linear quasi-relaxation of the merchant storage problem is proposed. Our solution algorithm is able to solve the problem in an efficient manner; returning the charge and discharge strategies for the merchant storage owner that yield the highest expected profits. Simulations of test systems reveal the various abilities of the merchant storage owner to exercise unilateral market power. Those include *demand withholding*, *generation withholding* and *under-use* which result in an increased congestion in both space and time when compared to the welfare-maximizing use of storage. Factors such as uncertain bids by other players, final state-of-charge requirements and arbitrage by other storage players are investigated. Moreover, numerical results demonstrate the superior computational performance of the proposed solution algorithm when benchmarked against current practices in the literature.

1. Introduction

Energy storage systems have the potential to significantly improve the operation of the power system of today [1], especially because of the ever-increasing generation from intermittent renewable resources [2]. The applications of energy storage systems are diverse and include voltage support [3], frequency regulation and synchronous/non-synchronous reserve [4] as well as spatio-temporal energy arbitrage, i.e. storing surplus energy from renewable sources for later use by loads [5]. In this manner, storage systems are suitable for integrating wind power [6] as well as solar power [7] and can participate in both day-ahead [8] and real-time [9] energy markets.

Several references cover applications of energy storage in microgrids such as real-time management for renewable integration [10], capacity optimization considering cogeneration and electric vehicle scheduling [11] as well as optimal sizing [12] and placement in multi-energy microgrids [13]. Demand response can also take advantage of the many attractive properties of energy storage [14]. In [15], the authors investigate the sizing of additional distributed generation and energy storage systems to be applied in smart households. In a similar fashion, [16] presents a control algorithm for joint demand response management and thermal comfort optimization in microgrids equipped with renewable energy

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https://doi.org/10.1016/j.apenergy.2019.114251
Received 24 May 2019; Received in revised form 20 November 2019; Accepted 23 November 2019

Nomenclature

Indices and sets

i	Index of units
n, m	Indices of buses
t	Index of time periods
l	Index of discrete offer-bid values
w	Index of stochastic scenarios
$\mathcal{I}^S$	Set of strategic storage units
$\mathcal{I}^{NS}$	Set of units of non-strategic players

Variables

$\widehat{p}_{it}$	Generation offer of players
$\widehat{d}_{it}$	Demand bid of players
$\widehat{c}_{it}$	Offered price of players
f_{nmtw}	Real power transmission line flow
p_{itw}	Real power generation of players
d_{itw}	Real power demand of players
s_{itw}	State of charge of storage unit
θ_{ntw}	Voltage angle at bus n
λ, μ	Dual variables
$\lambda_{ntw}^{(2)}$	Electricity price at bus n
$x_{itlw}^P, x_{itlw}^D, x_{itlw}^C$	Binary variables in MILP
$z_{itlw}^P, z_{itlw}^D, z_{itlw}^C$	Bilinear term intermediate values
$\pi_{itw}^P, \pi_{itw}^D, \pi_{itw}^C$	Bilinear term intermediate values
$y_{it}^P, y_{it}^D, y_{it}^C$	Spanning variables

Parameters

b_{nm}	Susceptance of a transmission line
D_{ntw}	Net real power demand at bus n
c_i	Variable cost of units
ρ_w	Probability of scenario w
$\underline{S}_i, \overline{S}_i$	Charge limits of merchant storage
S_i^0	Initial state of charge
$\underline{F}_{nm}, \overline{F}_{nm}$	Real flow limits of a transmission line
$\underline{\Theta}_n, \overline{\Theta}_n$	Voltage angle limits on bus n
$\underline{P}_i, \overline{P}_i$	Real power generation limits of units
$\underline{D}_i, \overline{D}_i$	Real power demand limits of units
C_{ni}	Node-unit connection matrix
$\widehat{P}_l, \widehat{D}_l, \widehat{C}_l$	Discrete offer-bid values
n_P, n_D, n_C	Number of possible discrete values
M^P, M^D, M^C	Sufficiently large constants
$\underline{p}_{it}, \overline{p}_{it}$	Lower/upper bounds of $\widehat{p}_{it}$
$\epsilon_i^C, \epsilon_i^D$	Charging/discharging efficiencies

Operator

$\mathbf{I}(condition)$	"If" operator

sources and energy storage units. Vehicle-to-grid enables plug-in electric vehicles to have bi-directional power flows once they are connected to the grid. Grid-tied applications of electric vehicle storage have recently been gaining ground where the operation of electric vehicles can for example be coordinated with volatile renewable energy sources [17] or used to supply a distributed spinning reserve according to the frequency deviation at the plug-in terminal [18]. In [19], the authors propose a game-theoretic model to understand the interactions among electric vehicles and aggregators in a vehicle-to-grid market. Moreover, [20] presents a control strategy for large-scale electric vehicles, battery energy storage stations and traditional frequency regulation resources involved in automatic generation control.

Energy storage includes a variety of different technologies, such as hydro, chemical, hydrogen and battery. Battery storage can be further divided into two types: (i) large-scale transmission-level battery storage and (ii) small-scale distribution-level battery storage. This paper focuses on a merchant battery storage portfolio that might include large-scale transmission-level battery storage units, aggregated small-scale battery storage units or even a mixture of both.[1]

In the literature, battery storage systems are often considered to be price takers due to their lower installed capacity compared to traditional generators. An example of this is [23] which analyzes two models for the hourly scheduling of centralized and distributed Electric Energy Storage (EES) systems. The results compare the impacts of utilizing the two EES models on system operations and quantify the operation benefits of EES. The authors of [24] consider the case where a group of investor-owned independently-operated storage units seek to offer energy and reserve in the day-ahead market and energy in the hour-ahead market. The particular system investigated has a significant portion of the power generated from intermittent renewable energy resources and therefore a stochastic programming framework is used to account for the fluctuating nature of the market prices. Reference [25] looks at how battery storage could increase its profitability by providing fast regulation service which takes advantage of the battery's fast ramping capability. The authors of [26] study the participation of small energy-storage units in electricity markets through aggregators which are modelled as price-takers in the market.

It is expected that a decrease in the capital cost of energy storage systems will eventually spur the deployment of large amounts of energy storage [27]. This raises the issue of market power. Exercise of unilateral market power[2] is a concern in today's electricity markets and numerous mathematical models have been developed to study such behavior [28]. The author of [29] quantifies the impact of the exercise of unilateral market power by a large hydroelectric generation facility in the western U.S. The author develops a model which solves for a subgame perfect equilibrium of a multi-period Cournot game between strategic producers, one of which owns hydroelectric capacity with the ability to store water. The model results find that the large hydroelectric producer reduces output in peak demand hours and shifts this output to the off-peak demand hours, relative to the case in which the hydroelectric producer behaves as a price-taker. Reference [30] models the offer behavior of a plant owner maximizing its expected profit and with the ability to exercise unilateral market power. The plant owner is assumed to choose its price and quantity offer pairs to maximize the expected value of the realized profits that it would earn across a distribution of scenarios for system demand and the offer behavior of its competitors. These are the two major sources of uncertainty the plant owner faces when it submits its offers into the short-term market. In [31], the framework from [30] is used to test the assumption of

[1] Merchant battery storage portfolios of both the aforementioned types currently exist in practice where several countries have installed large-scale batteries for their grid. In November 2017, Tesla installed a 100 MW, 129 MWh battery system in South Australia and the UK had a 50 MW lithium-ion grid-battery installed in Hertfordshire in 2018. According to a 2019 market insight from IHS Markit, deployments of grid-connected energy storage in the United States this year are expected to amount to 712 MW, representing a near-doubling from 376 MW in in 2018. Moreover, IHS Markit expects over 2 GW of energy storage to be paired with utility-scale solar photovoltaic (PV) systems from 2019 to 2023 in the United States [21]. In addition to storage in conjunction with utility-scale solar, aggregating small-scale distribution-level battery storage is currently an accepted concept [22].

[2] This term is generally used in the economics literature while the terms *strategic* or *strategic behavior* are more common in the engineering literature.

expected profit-maximizing offer behavior in a short-term electricity market and the results show no evidence against it.

The issue of unilateral market power is especially interesting in the case of energy storage because storage units can act both as generators and loads. A portfolio of storage units is able to influence the market price in multiple ways; by creating congestion in both space and time, by withholding generation as well as by withholding demand. Looking at the literature, relatively little focus has been put on studying large, price-maker storage portfolios. However, recent work such as [32] evaluates the impact of strategic behavior of an independent trader operating energy storage systems while the authors of [33] assess the various consequences, including those of market power, of storage via a complementarity model of a stylized Western European power system. Reference [34] studies how storage, operating as a price maker, may be optimally operated over an extended period of time. The authors of [35] explore the implications of different bid structures on the strategic behavior of a storage owner that participates in the wholesale market and is able to influence market prices. Four bid structures are analyzed: (i) simple quantity-based bids, (ii) simple price-based bids, (iii) price-quantity pairs bids and (iv) complex bidding in which the energy storage system owner provides full information of its technical characteristics through the bids. In [36], the authors explore the integration of large-scale, grid-level energy storage into wholesale electricity markets and conduct a comparative analysis on three natural market mechanisms. Reference [37] addresses the optimal bidding strategy problem of a commercial virtual power plant which includes battery storage systems. In [38], the author proposes an optimization framework to coordinate the operation of large, price-maker, geographically dispersed storage systems in a nodal transmission-constrained market.

In line with the above references, this paper studies the operation of a profit-maximizing merchant storage owner with the ability to exercise unilateral market power. The paper is motivated by the recent FERC order to allow medium to large scale storage resources to directly participate in the wholesale market (either through direct bidding or self-scheduling). Worldwide there has also been a growing trend for more storage resources to participate in wholesale markets. The point of the model is to understand what could happen in the future when storage capacity is expected to increase and the potential of one firm to own a significant amount of storage is likely. The problem formulation is a bilevel one where in the upper-level problem, the merchant storage owner makes offers and bids to the market in order to maximize its expected profit, subject to the lower-level optimal dispatch problem of the Independent System Operator (ISO) for a variety of possible scenarios. The main contributions of this paper are:

1. A derivation of a stochastic disjunctive program model for finding the optimal offer-bid strategy of a merchant storage portfolio which maximizes the expected profit over several possible market scenarios. The derivation paves the way for linearization of the original bilevel, non-linear model via discretization of the offer-bid values. Unlike the existing bilevel models found in the literature for capturing the behavior of price-making storage, this novel derivation allows a linear quasi-relaxation of the problem, which is required for solving the problem in an efficient manner.
2. A Specialized Branch-and-Bound (SBB) solution algorithm that applies a linear quasi-relaxation which significantly reduces the computational requirements when solving the merchant storage problem. Moreover, the proposed disjunctive formulation and solution algorithm avoid the computational complexity tied to the traditional Big-M linearization of the complementary slackness conditions of the lower level problem.
3. Both the proposed stochastic disjunctive programming model and the SBB solution algorithm are benchmarked against current practices in the literature for modeling and solving these types of problems.

The rest of the paper is organized as follows. Section 2 covers the mathematical model where the physical constraints of the system are listed and the market players of the system are introduced. A stochastic bilevel merchant storage problem is then derived and using the Karush-Kuhn-Tucker (KKT) conditions as well as bid discretization, it is reformulated as a single-level stochastic disjunctive program. Section 3 derives a specialized solution algorithm that applies a linear quasi-relaxation of the merchant storage problem and solves the problem in a computationally efficient way. Sections 4 and 5 show an illustrative example and numerical simulations, respectively, of test systems that confirm the performance of the proposed method. Further technical analysis is carried out in Section 6, where factors such as uncertain bids by other players, final state-of-charge requirements and arbitrage by other storage players are investigated. Section 7 concludes the paper.

2. Mathematical model

The market is composed of two types of market players; a single large merchant storage owner whose units are in the set $\mathcal{I}^S$ and traditional generators whose units are in the set $\mathcal{I}^{NS}$. Index i represents all units in the market. The production of each player at time t, scenario w is represented by p_{itw} and the demand of each player is represented by d_{itw}. The production and demand are limited by the lower and upper limits ($\underline{P}_i$, $\overline{P}_i$) and ($\underline{D}_i$, $\overline{D}_i$), respectively. The charging power of a storage unit is therefore represented by d_{itw} (unit acts as a load) and the discharging power of a storage unit is represented by p_{itw} (unit acts as a generator).[3] The storage level is represented by s_{itw} and the storage capacity is limited by the lower and upper limits ($\underline{S}_i$, $\overline{S}_i$). The initial state of charge is represented by S_i^0. The storage owner chooses its offer price and quantity pairs to maximize its expected profits. The storage owner computes the expected profits associated with a combination of offer price and quantity pairs by solving a lower-level ISO market equilibrium several times for a variety of possible scenarios. The expected profits are the probability weighted sum of these realized profit outcomes. The ISO is assumed to solve an optimization problem minimizing the as offered cost of generation and therefore the complete optimization problem that the merchant storage owner solves is a bilevel one.

2.1. Stochastic bilevel program

The bilevel merchant storage problem is given in (1).

$$\underset{\hat{p}_{it},\hat{d}_{it},\hat{c}_{it},i\in\mathcal{I}^S}{\text{maximize}} \sum_{i\in\mathcal{I}^S,t,w} \rho_w \sum_n C_{ni}\lambda_{ntw}^{(2)}\left(p_{itw} - d_{itw}\right) \tag{1a}$$

subject to:

$$\hat{p}_{it}\hat{d}_{it} = 0, \forall\ (i\in\mathcal{I}^S)t, \tag{1b}$$

$$\underline{P}_i \leqslant \hat{p}_{it} \leqslant \overline{P}_i,\ \underline{D}_i \leqslant \hat{d}_{it} \leqslant \overline{D}_i, \forall\ (i\in\mathcal{I}^S)t, \tag{1c}$$

$$s_{itw} = s_{i,t-1,w} + S_i^0\mathbf{I}\left(t=1\right) + \epsilon_i^C\, d_{itw} - \frac{1}{\epsilon_i^D}p_{itw}, \forall\ \left(i\in\mathcal{I}^S\right)tw, \tag{1d}$$

$$\underline{S}_i \leqslant s_{itw} \leqslant \overline{S}_i, \forall\ (i\in\mathcal{I}^S)tw, \tag{1e}$$

$$S_i^0 \leqslant s_{iTw}, \forall\ (i\in\mathcal{I}^S)w, \tag{1f}$$

[3] Note that storage is merely treated as a power system element that can both produce and consume electricity. Therefore the production and demand of storage are represented with the same symbols as any other unit in the system, p_{itw} and d_{itw}. This is done in order to simplify the lower level problem, that is introduced in the next subsection, where both the objective as well as the constraints can be written much more compactly. The different kind of players in the market are nevertheless differentiated using the sets $\mathcal{I}^S$ for strategic storage and $\mathcal{I}^{NS}$ for traditional generators.

where $\{\lambda^{(2)}_{ntw}, p_{itw}, d_{itw}\} \in$

$$\arg\left\{ \underset{p_{itw}, d_{itw}, f_{nmtw}, \theta_{ntw}}{\text{minimize}} \sum_{i,t} \left(p_{itw} - d_{itw} \right) \hat{c}_{it} \right. \tag{1g}$$

subject to:

$$f_{nmtw} = -b_{nm}(\theta_{ntw} - \theta_{mtw}): \lambda^{(1)}_{nmtw}, \forall\ nmtw, \tag{1h}$$

$$\sum_{i} \left(p_{itw} - d_{itw} \right) C_{ni} - \sum_{m}^{n \neq m} f_{nmtw} = D_{ntw}: \lambda^{(2)}_{ntw}, \forall\ ntw, \tag{1i}$$

$$\underline{F}_{nm} \leqslant f_{nmtw} \leqslant \overline{F}_{nm}: \mu^{(1)}_{nmtw}, \mu^{(2)}_{nmtw}, \forall\ nmtw, \tag{1j}$$

$$\underline{\Theta}_{n} \leqslant \theta_{ntw} \leqslant \overline{\Theta}_{n}: \mu^{(3)}_{ntw}, \mu^{(4)}_{ntw}, \forall\ ntw, \tag{1k}$$

$$\underline{P}_{i} \leqslant p_{itw} \leqslant \hat{p}_{it}: \mu^{(5)}_{itw}, \mu^{(6)}_{itw}, \forall\ itw, \tag{1l}$$

$$\left. \underline{D}_{i} \leqslant d_{itw} \leqslant \hat{d}_{it}: \mu^{(7)}_{itw}, \mu^{(8)}_{itw}, \forall\ itw \right\}. \tag{1m}$$

Uncertainties in the market are captured by probabilistic scenarios, indexed by w with probabilities ρ_w. The main source of uncertainty is the net demand D_{ntw}, which depends on the intermittent generation. Scenarios are constructed based on forecasts of the demand and intermittent generation. Other uncertainties, such as uncertainties in the bids of other generators, can also be captured through the use of scenarios. In the upper-level problem, the merchant storage owner maximizes the expected profit over the time horizon and the different scenarios. The resulting solution is the generation offer quantity $\hat{p}_{it}$, the demand bid quantity $\hat{d}_{it}$ and the price $\hat{c}_{it}$. Note that they are independent of the scenarios. Moreover, note that for $i \in \mathcal{I}^S$, symbols $\hat{p}_{it}$, $\hat{d}_{it}$ and $\hat{c}_{it}$ are upper-level variables but are considered to be parameters for $i \in \mathcal{I}^{NS}$, since the offers for the non-strategic players are fixed to their installed capacities ($\overline{P}_i$, $\overline{D}_i$) and their true cost parameters (c_i). For each scenario that the merchant storage owner takes into account, the ISO is assumed to solve a deterministic model in which the particular scenario is assumed to be the only one. The term $\sum_n C_{ni}\lambda^{(2)}_{ntw}$ in the upper-level objective function represents the nodal price that player i is exposed to. $\lambda^{(2)}_{ntw}$ is the nodal price and C_{ni} is a binary node-unit connection matrix. Constraint (1b) makes sure that in each time period, the storage units submit offers or bids to the market either as a generator or as a load [38]. The offer-bid values $\hat{p}_{it}$ and $\hat{d}_{it}$ must confine to the physical limits of each unit (1c). As noted earlier, the bids of the non-strategic generators in the market can however be assumed to be uncertain and this uncertainty can be captured by having several scenarios for their bids, indexed by w. This entails that the objective function of the lower-level problem becomes $\sum_{i \in \mathcal{I}^S, t} (p_{itw} - d_{itw})\hat{c}_{it} + \sum_{i \in \mathcal{I}^{NS}, t} p_{itw} e_{itw}$, where e_{itw} is the scenario dependent bid of the non-strategic generators. It is then straight forward to update the KKT conditions given in Section 2.2 accordingly. The simulation in Section 6.1 shows this modeling aspect. The energy balance of the storage units is captured by (1d) and the energy limits by (1e). Parameters ϵ^C_i and ϵ^D_i represent the charging and discharging efficiencies, respectively. Choosing a round-trip efficiency of less than one makes constraint (1b) redundant since there is no point in simultaneously charging and discharging. Constraint (1f) makes sure that the storage level at the end of the planning horizon should be no lower than at the beginning. Such a constraint may be very important when scheduling successive planning horizons. Upper case T denotes the last time period of the planning horizon. Thestorage owner is considered responsible for the energy limits so those constraints appear in the upper-level problem [38]. The lower-level ISO dispatch problem minimizes the as offered generation cost in the system while taking into account the power flow constraints (1h) and the energy balance on each bus (1i). There are also lower and upper limits on the real power flows f_{nmtw} (1j), the voltage angles θ_{ntw} (1k) and the dispatch values p_{itw} and d_{itw} (1l)–(1m). The lower-level dual variables are given after the colon. [4]

The formulation above assumes that the merchant storage owner submits *price-quantity pairs* to the market, that is offers or bids that contain both price and quantity. The model in (1) can also capture *self-scheduling*, e.g. offers or bids without a price component, if minor changes are made to the formulation. Specifically, the lower-level objective function becomes $\sum_{i,t} (p_{itw}\hat{c}_{it} - d_{itw}\hat{u}_{it})$ where for all storage units the submitted generation offer price $\hat{c}_{it}$ is zero, and the submitted demand bid price $\hat{u}_{it}$ is a sufficiently high constant [38].

The above model is in general very hard to solve because: (i) it is bilevel, (ii) it is non-linear and (iii) transmission congestion implies that small changes in offer behavior can create large changes in realized market outcomes.

2.2. The proposed stochastic disjunctive program

In order to compose a single-level stochastic optimization problem from the stochastic bilevel program given in (1), the KKT conditions are derived for the lower-level problem. The stationary conditions are given in (2).

$$\frac{\partial L}{\partial f_{nmtw}} = \lambda^{(1)}_{nmtw} - \lambda^{(2)}_{ntw}\mathbf{I}\left(n \neq m \right) - \mu^{(1)}_{nmtw} + \mu^{(2)}_{nmtw} = 0, \forall\ nmtw, \tag{2a}$$

$$\frac{\partial L}{\partial \theta_{nmtw}} = \sum_{m} \left[b_{nm}\lambda^{(1)}_{nmtw} - b_{mn}\lambda^{(1)}_{mntw} \right] - \mu^{(3)}_{nw} + \mu^{(4)}_{nw} = 0, \forall\ ntw, \tag{2b}$$

$$\frac{\partial L}{\partial p_{itw}} = \sum_{n} \left[C_{ni}\lambda^{(2)}_{ntw} \right] - \mu^{(5)}_{itw} + \mu^{(6)}_{itw} = \hat{c}_{it}, \forall\ itw, \tag{2c}$$

$$\frac{\partial L}{\partial d_{itw}} = \sum_{n} \left[C_{ni}\lambda^{(2)}_{ntw} \right] + \mu^{(7)}_{itw} - \mu^{(8)}_{itw} = \hat{c}_{it}, \forall\ itw. \tag{2d}$$

All complementary slackness conditions are collected in the strong duality condition given in (3). [5]

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[4] Note that storage degradation is not modelled directly in the model but such details could easily be incorporated. Storage degradation is important in the long-term, as mentioned in [39,40], since frequent charge–discharge cycling of batteries incurs extra operational costs and it may accelerate battery depreciation. In this paper, we have considered a short-term offer-bid model for battery storage where the degradation is not a key modelling aspect and it is therefore disregarded (as is also the case in references such as [35,38]).

[5] Since the lower-level problem is a linear program, one knows that its duality gap is zero and therefore the strong duality condition is equivalent to the complementary slackness conditions. This can be seen by looking at the following LP primal and dual pair

$$\begin{array}{lllll} \text{maximize} & c^T x & & \text{minimize} & b^T y \\ s.\ t. & Ax \leqslant b & (P) & s.\ t. & A^T y \geqslant c \quad (D) \\ & x \geqslant 0 & & & y \geqslant 0. \end{array}$$

Let $s = b - Ax \geqslant 0$ denote the vector of slack variables and $t = A^T y - c \geqslant 0$ denote the vector of surplus variables. One can therefore write

$$\begin{aligned} c^T x &= (A^T y - t)^T x \\ &= y^T A x - t^T x \\ &= y^T (b - s) - t^T x \\ &= y^T b - y^T s - t^T x. \end{aligned}$$

At optimality (x^*, y^*, s^*, t^*), strong duality implies

$$c^T x^* = b^T y^*,$$

so

$$(y^*)^T s^* + (t^*)^T x^* = 0,$$

which gives the complementary slackness conditions

$$(y^*)^T s^* = 0,\ (x^*)^T t^* = 0,$$

since $y^*, x^*, s^*, t^* \geqslant 0$.

$$\begin{aligned}\sum\nolimits_{i,t}(p_{itw}-d_{itw})\hat{c}_{it} = & \sum\nolimits_{n,t}\lambda^{(2)}_{ntw}D_{ntw} + \sum\nolimits_{n,m,t}[\mu^{(2)}_{nmtw}\overline{F}_{nm} - \mu^{(1)}_{nmtw}\underline{F}_{nm}] \\ & + \sum\nolimits_{n,t}[\mu^{(4)}_{ntw}\overline{\Theta}_n - \mu^{(3)}_{ntw}\underline{\Theta}_n] + \sum\nolimits_{i,t}[\mu^{(6)}_{itw}\hat{P}_{it} - \mu^{(5)}_{itw}\underline{P}_i] \\ & + \sum\nolimits_{i,t}[\mu^{(8)}_{itw}\hat{d}_{it} - \mu^{(7)}_{itw}\underline{D}_i], \forall w.\end{aligned} \tag{3}$$

There are bilinear terms that appear in the objective function (1a) as well as the strong duality constraint (3). These terms are $\sum_n C_{ni}\lambda^{(2)}_{ntw}(p_{itw}-d_{itw})$, $\mu^{(6)}_{itw}\hat{p}_{it}$ and $\mu^{(8)}_{itw}\hat{d}_{it}$. Using (2c), (2d) and the complementary slackness (CS) conditions for constraints (1l) and (1m), the objective function can be rewritten as can be seen in (4).

$$\begin{aligned}&\sum_{i\in I^S,t,w}\rho_w\sum_n C_{ni}\lambda^{(2)}_{ntw}(p_{itw}-d_{itw}) \overset{(2c),(2d)}{=} \\ &\sum_{i\in I^S,t,w}\rho_w[(p_{itw}-d_{itw})\hat{c}_{it} + \mu^{(5)}_{itw}p_{itw} - \mu^{(6)}_{itw}p_{itw} + \mu^{(7)}_{itw}d_{itw} - \mu^{(8)}_{itw}d_{itw}] \overset{(CS)}{=} \\ &\sum_{i\in I^S,t,w}\rho_w[\underbrace{(p_{itw}-d_{itw})\hat{c}_{it}}_{\pi^C_{itw}} + \mu^{(5)}_{itw}\underline{P}_i - \underbrace{\mu^{(6)}_{itw}\hat{p}_{it}}_{\pi^P_{itw}} + \mu^{(7)}_{itw}\underline{D}_i - \underbrace{\mu^{(8)}_{itw}\hat{d}_{it}}_{\pi^D_{itw}}].\end{aligned} \tag{4}$$

The bilinear terms have therefore been reduced to $(p_{itw}-d_{itw})\hat{c}_{it}$, $\mu^{(6)}_{itw}\hat{p}_{it}$ and $\mu^{(8)}_{itw}\hat{d}_{it}$ where each term is a continuous variable multiplied by an offer-bid value. We assume discrete offer-bid values which can take values from a possible pool of ordered values $\hat{p}_{it}\in\{\hat{P}_1,\hat{P}_2,\ldots,\hat{P}_{np}\}$, $\hat{d}_{it}\in\{\hat{D}_1,\hat{D}_2,\ldots,\hat{D}_{nD}\}$ and $\hat{c}_{it}\in\{\hat{C}_1,\hat{C}_2,\ldots,\hat{C}_{nC}\}$. Therefore, one can rewrite the bilinear terms in the following disjunctive manner:

$$\begin{aligned}\mu^{(6)}_{itw}\hat{p}_{it} &= \bigvee_{l=1}^{np}\mu^{(6)}_{itw}\hat{P}_l, \\ \mu^{(8)}_{itw}\hat{d}_{it} &= \bigvee_{l=1}^{nD}\mu^{(8)}_{itw}\hat{D}_l, \\ (p_{itw}-d_{itw})\hat{c}_{it} &= \bigvee_{l=1}^{nC}(p_{itw}-d_{itw})\hat{C}_l,\end{aligned} \tag{5}$$

where the disjunction is represented by the disjunction (OR) operator $\bigvee$. One can then rewrite the whole stochastic bilevel program as the stochastic disjunctive program (6).

$$\begin{aligned}&\underset{\Omega}{\text{maximize}}\sum_{i\in I^S,t,w}\rho_w\left[\bigvee_{l=1}^{n_C}\left(p_{itw}-d_{itw}\right)\hat{C}_l + \right. \\ &\left.\mu^{(5)}_{itw}\underline{P}_i - \bigvee_{l=1}^{np}\mu^{(6)}_{itw}\hat{P}_l + \mu^{(7)}_{itw}\underline{D}_i - \bigvee_{l=1}^{nD}\mu^{(8)}_{itw}\hat{D}_l\right] \\ &\text{subject to:} \\ &\text{(1b)–(1f), (1h)–(1m), (2a)–(2d),} \\ &\text{(3) rewritten with (5),} \\ &\mu^{(1)}_{nmtw}, \mu^{(2)}_{nmtw}, \mu^{(3)}_{ntw}, \mu^{(4)}_{ntw} \leqslant 0, \\ &\mu^{(5)}_{itw}, \mu^{(6)}_{itw}, \mu^{(7)}_{itw}, \mu^{(8)}_{itw} \leqslant 0,\end{aligned} \tag{6}$$

where the set of decision variables is $\Omega = \{\hat{p}_{it}, \hat{d}_{it}, \hat{c}_{it}, p_{itw}, d_{itw}, s_{itw}, \theta_{ntw}, f_{nmtw}, \lambda^{(1)}_{nmtw}, \lambda^{(2)}_{ntw}, \mu^{(1)}_{nmtw}, \mu^{(2)}_{nmtw}, \mu^{(3)}_{ntw}, \mu^{(4)}_{ntw}, \mu^{(5)}_{itw}, \mu^{(6)}_{itw}, \mu^{(7)}_{itw}, \mu^{(8)}_{itw}\}$. Program (6) can be solved by the binary expansion approach [41]. Taking the disjunctive term $\bigvee_{l=1}^{np}\mu^{(6)}_{itw}\hat{P}_l$ as an example, one can introduce binary variables x^P_{itl} where $\sum_{l=1}^{np}x^P_{itl} = 1$ and write the disjunction as:

$$\begin{aligned}-M^P x^P_{itl} &\leqslant z^P_{itlw} \leqslant M^P x^P_{itl}, \forall\ itlw, \\ -M^P(1-x^P_{itl}) &\leqslant z^P_{itlw} - \mu^{(6)}_{itw}\hat{P}_l \leqslant M^P(1-x^P_{itl}), \forall\ itlw,\end{aligned}$$

where M^P is a sufficiently large constant, and z^P_{itlw} are continuous variables that are enforced to take the value of the bilinear term for a single index l. The disjunctive term can then be written as $\bigvee_{l=1}^{np}\mu^{(6)}_{itw}\hat{P}_l = \sum_{l=1}^{np}z^P_{itlw}$ and the offer value as $\hat{p}_{it} = \sum_{l=1}^{np}\hat{P}_l x^P_{itl}$. The same approach can be used to rewrite the other disjunctive terms. Additional constraints $\hat{p}_{it} \leqslant \overline{P}_i a_{it}$ and $\hat{d}_{it} \leqslant \overline{D}_i(1-a_{it})$ where $a_{it}\in\{0,1\}$ are introduced to ensure that in each period, each storage unit participates in the market either as a generator or as a load. This formulation allows the merchant storage problem to be written as a mixed-integer linear program (MILP) and the standard branch-and-bound algorithm can be used to solve it. A continuous linear relaxed optimization problem is then formed by allowing the binary variables to be continuous in the range from zero to one. Branches are created by finding a non-binary solution variable and setting it to zero at one node and to one at the other. The problem with the MILP formulation is however that it contains a large number of binary variables and the choice of M^P, M^D and M^C affects the performance of the solver. To tackle these shortcomings, following [41] we propose an alternative way to deal with the bilinear terms. The approach introduced in [41] was originally developed for the nonlinear and discrete *truss design problem* of finding the optimal placement and size of structural bars that can support a given load. The authors found that it can solve substantially larger problems than using mixed integer programming and the approach has been shown to be superior for several engineering design problems [42,43]. The quasi-relaxation technique that is the basis for the approach in [41] was later generalized in [44]. The approach is distinguished by the following elements:

Logic-based branching. The branch-and-bound procedure is a logic-based approach that dispenses with integer variables and branches directly on logical disjunctions as opposed to MILP approaches, which branch on integer variables. The two main advantages are that (i) it yields smaller subproblems at the nodes of the search tree, due to the absence of integer variables and (ii) it allows the use of a very effective branching rule that is not possible in an MILP context.

Nontraditional relaxations. The approach uses a relaxation other than the traditional continuous relaxation of an MILP. The MILP model has the advantage that the continuous relaxation is always readily available. However, the quality of the relaxation does not justify the overhead of including integer variables. The disjunctions used in the proposed approach do not have useful linear relaxations; nevertheless, the logic-based framework permits the use of a linear "quasi-relaxation" of the nonlinear problem at each node. It provides a lower bound on the optimum analogous to that of the continuous relaxation in an MILP.

In the mixed integer linear formulation, each of the offer-bid values is expressed as a convex combination of discrete values. Instead of such a formulation, we propose that only two discrete values are used for the offer-bid values; a lower bound and an upper bound. Taking $\hat{p}_{it}$ as an example, $\underline{p}_{it}$ and $\overline{p}_{it}$ represent the lower and upper bounds, respectively. A continuous variable y^P_{it} is introduced to span the range of $\hat{p}_{it}$:

$$\hat{p}_{it} = \underline{p}_{it}y^P_{it} + \overline{p}_{it}(1-y^P_{it}),\ 0 \leqslant y^P_{it} \leqslant 1. \tag{7}$$

This means that if $y^P_{it} = 0$, $\hat{p}_{it}$ will take the value of $\overline{p}_{it}$ and if $y^P_{it} = 1$, $\hat{p}_{it}$ will take the value of $\underline{p}_{it}$. If $0 < y^P_{it} < 1$, $\hat{p}_{it}$ will take a value that is between the two bounds. Fig. 1 illustrates of how the range of $\hat{p}_{it}$ is spanned by the variable y^P_{it} using the linear combination of the lower bound $\underline{p}_{it}$ and the upper bound $\overline{p}_{it}$. The disjunction is enforced by the constraint

$$\bigvee_{l=1}^{np}\left[\underline{p}_{it}y^P_{it} + \overline{p}_{it}\left(1-y^P_{it}\right) = \hat{P}_l\right]. \tag{8}$$

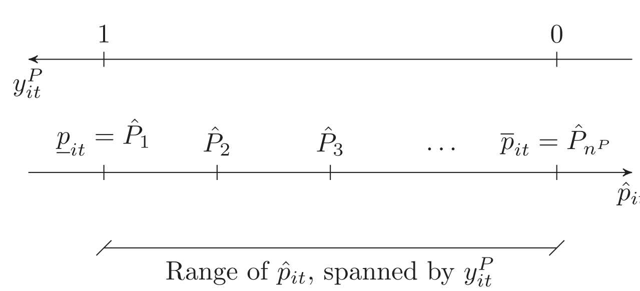

Fig. 1. An illustration of how the range of $\hat{p}_{it}$ is spanned by the variable y^P_{it} using the linear combination of the lower bound $\underline{p}_{it}$ and the upper bound $\overline{p}_{it}$.

The continuous variable $\mu_{itw}^{(6)}$ that appears in the disjunctive term is then represented by the sum of two values

$$\mu_{itw}^{(6)} = \mu_{itw}^{(6)-} + \mu_{itw}^{(6)+}, \tag{9}$$

and the bilinear term $\mu_{itw}^{(6)}\hat{p}_{it}$ can be written as

$$\pi_{itw}^{P} = (\mu_{itw}^{(6)-} + \mu_{itw}^{(6)+})[\underline{p}_{it}y_{it}^{P} + \overline{p}_{it}(1 - y_{it}^{P})]. \tag{10}$$

The reason for splitting $\mu_{itw}^{(6)}$ up in this manner will become clear when the quasi-relaxation is derived in (12). The reformulation is equivalent for $\mu_{itw}^{(8)}\hat{d}_{it}$ and $(p_{itw} - d_{itw})\hat{c}_{it}$ resulting in π_{itw}^{D} and π_{itw}^{C}, respectively. Subsequently, one can write the stochastic disjunctive program as given in (11).

$$\begin{aligned}
&\underset{\Omega}{\text{maximize}} \sum_{i \in \mathcal{I}^S, t, w} \rho_w [\pi_{itw}^{C} + \mu_{itw}^{(5)}\underline{P}_i - \pi_{itw}^{P} + \mu_{itw}^{(7)}\underline{D}_i - \pi_{itw}^{D}] \\
&\text{subject to:} \\
&\text{(1b)–(1f), (1h)–(1m), (2a)–(2d),} \\
&\text{(7)–(10)[Equiv. for } \mu_{itw}^{(8)}\hat{d}_{it} \text{ and} (p_{itw} - d_{itw})\hat{c}_{it}], \\
&\text{(3) rewritten with } \pi_{itw}^{P}, \pi_{itw}^{D}, \pi_{itw}^{C}, \\
&\mu_{nmtw}^{(1)}, \mu_{nmtw}^{(2)}, \mu_{ntw}^{(3)}, \mu_{ntw}^{(4)} \leqslant 0, \\
&\mu_{itw}^{(5)}, \mu_{itw}^{(6)}, \mu_{itw}^{(7)}, \mu_{itw}^{(8)} \leqslant 0,
\end{aligned} \tag{11}$$

where the set of decision variables is $\Omega = \{\hat{p}_{it}, \hat{d}_{it}, \hat{c}_{it}, p_{itw}, d_{itw}, s_{itw}, \theta_{ntw}, f_{nmtw}, \lambda_{nmtw}^{(1)}, \lambda_{ntw}^{(2)}, \mu_{nmtw}^{(1)}, \mu_{nmtw}^{(2)}, \mu_{ntw}^{(3)}, \mu_{ntw}^{(4)}, \mu_{itw}^{(5)}, \mu_{itw}^{(6)}, \mu_{itw}^{(7)}, \mu_{itw}^{(8)}\}$.

The stochastic disjunctive program (11) can be solved directly by applying a specialized branch-and-bound solution approach that was proposed in [41]. The disjunctive formulation allows one to branch on the range of offer-bid values instead of binary variables. As an example, if an offer $\hat{p}_{it}$ can take values in the set {0, 10, 20, 30, 40, 50} MW, one can enforce the constraint $0 \leqslant \hat{p}_{it} \leqslant 20$ in one branch and the constraint $30 \leqslant \hat{p}_{it} \leqslant 50$ in the other. In order for such a branch-and-bound approach to work properly, one needs a relaxed optimization problem to give a upper bound on the objective value. Unfortunately, the straightforward continuous relaxation of (11) is both non-linear and non-convex and does therefore not provide a suitable way of obtaining an upper bound. It is however possible to obtain an upper bound by applying *quasi-relaxation.*

Definition 1. *For a given constrained maximization problem P, a problem Q is a quasi-relaxation of P if for every feasible solution of P with objective value equal to v, there is a feasible solution of Q having an objective function value greater than or equal to v. The optimal value of Q is an upper bound on the optimal value of P.*

One can derive a quasi-relaxation as follows. Looking at the stochastic disjunctive program given in (11), everything is linear except for (1b) and the constraints given in (8) and (10) as well as their equivalents for $\mu_{itw}^{(8)}\hat{d}_{it}$ and $(p_{itw} - d_{itw})\hat{c}_{it}$. First, we drop constraint (1b) which will be dealt with directly in the solution algorithm in Section 3. Then consider the following linear constraints in (12) which are obtained by dropping the disjunctive constraints in (8) and rewriting constraints (10). M^P represents a sufficiently large constant. The constraints for $\mu_{itw}^{(8)}\hat{d}_{it}$ and $(p_{itw} - d_{itw})\hat{c}_{it}$ are equivalent.

$$\pi_{itw}^{P} = \underline{p}_{it}\mu_{itw}^{(6)-} + \overline{p}_{it}\mu_{itw}^{(6)+}, \tag{12a}$$

$$-M^P y_{it}^{P} \leqslant \mu_{itw}^{(6)-} \leqslant 0, \tag{12b}$$

$$-M^P(1 - y_{it}^{P}) \leqslant \mu_{itw}^{(6)+} \leqslant 0, \tag{12c}$$

$$[\text{Equivalent for } \mu_{itw}^{(8)}\hat{d}_{it} \text{ and} (p_{itw} - d_{itw})\hat{c}_{it}]. \tag{12d}$$

Remark 1. *Let (11Q) denote the optimization problem that results from taking problem (11), dropping constraint (1b) and replacing constraints (8) and (10) as well as the equivalent constraints for the other bilinear terms* $\mu_{itw}^{(8)}\hat{d}_{it}$ *and* $(p_{itw} - d_{itw})\hat{c}_{it}$ *with constraints (12).*

Lemma 1. *Problem (11Q) is a linear program and a quasi-relaxation of problem (11). See Appendix for proof.*

Lemma 2. *If* $\hat{p}_{it}\hat{d}_{it} = 0, \forall\, (i \in \mathcal{I}^S)t$*, and each* $y_{it}^{P}, y_{it}^{D}, y_{it}^{C}$ *is binary in a solution of (11Q), that solution is feasible in (11). See Appendix for proof.*

3. Solution algorithm

Lemmas 1 and 2 can be used to derive a Specialized Branch-and-Bound (SBB) solution algorithm that solves the merchant storage problem (11). The SBB algorithm differs from the standard branch-and-bound algorithm (BB) used to solve the binary expansion MILP model in the following aspects:

1. The SBB algorithm branches on the lower and upper bounds of offer-bid values $(\underline{p}_{it}, \overline{p}_{it})$, $(\underline{d}_{it}, \overline{d}_{it})$, and $(\underline{c}_{it}, \overline{c}_{it})$ instead of branching on binary values in the BB algorithm.
2. The SBB algorithm uses the quasi-relaxation model in (11Q) to find valid upper bounds, while the BB algorithm uses a linear continuous relaxation by allowing the binary variables to be continuous in the range [0, 1].

The solution algorithm is shown in Algorithm 1. In the algorithm, variable $\hat{x}_{it}$ represents any of the offer-bid values $\hat{p}_{it}$, $\hat{d}_{it}$ or $\hat{c}_{it}$ in order to simplify the algorithm and it is accompanied by the corresponding spanning variable y_{it}^{X}. For clarity, sets $\underline{\mathcal{X}}$ and $\overline{\mathcal{X}}$ include all lower and upper bounds for all of the offer-bid values for all *i* and *t*. The algorithm can branch on any single offer-bid value.[6] In order to make sure that the merchant storage owner participates in the market either as a generator or a demand, the following feasibility cut is applied when a branch is created. Whenever the algorithm branches on a single offer quantity $\hat{p}_{it}$ and a branch is created where the lower bound $\underline{p}_{it}$ is greater than zero, the corresponding demand bid $\hat{d}_{it}$ is set to zero in that branch. Similarly, whenever the algorithm branches on a single bid quantity $\hat{d}_{it}$ and a branch is created where the lower bound $\underline{d}_{it}$ is greater that zero, the corresponding generation offer quantity $\hat{p}_{it}$ is set to zero in that branch. This enforces constraint (1b).

Algorithm 1. The SBB algorithm with linear quasi-relaxation.

Input : Linear quasi-relaxed problem (11Q) and discrete values $\{\hat{P}_1, \hat{P}_2, ..., \hat{P}_{n_P}\}$, $\{\hat{D}_1, \hat{D}_2, ..., \hat{D}_{n_D}\}$ and $\{\hat{C}_1, \hat{C}_2, ..., \hat{C}_{n_C}\}$.

Set lower and upper bounds $(\underline{p}_{it}, \overline{p}_{it}) = (\hat{P}_1, \hat{P}_{n_P})$, $(\underline{d}_{it}, \overline{d}_{it}) = (\hat{D}_1, \hat{D}_{n_D})$ and $(\underline{c}_{it}, \overline{c}_{it}) = (\hat{C}_1, \hat{C}_{n_C})$.

Set $LB = -\infty$.

Branch$(\underline{\mathcal{X}}, \overline{\mathcal{X}})$.

if $LB = -\infty$ **then**
 Problem is infeasible.
else
 *sol** is optimal for (11).

Function *Branch*$(\underline{\mathcal{X}}, \overline{\mathcal{X}})$
 if *(11Q) has a feasible solution sol with objective* $z > LB$ **then**
  if *some* $y_{it}^{X} \notin \{0, 1\}$ *and* $\underline{x}_{it} \neq \overline{x}_{it}$ **then**
   Let $\hat{X}_l$ be the largest value in the set $\{\hat{X}_1, \hat{X}_2, ..., \hat{X}_{n_X}\}$ that is smaller than $\underline{x}_{it}y_{it}^{X} + \overline{x}_{it}(1 - y_{it}^{X})$.
   Branch$(\underline{\mathcal{X}}', \overline{\mathcal{X}})$, where $\underline{\mathcal{X}}'$ is identical to $\underline{\mathcal{X}}$ apart from that $\underline{x}_{it} = \hat{X}_{l+1}$
   Branch$(\underline{\mathcal{X}}, \overline{\mathcal{X}}')$, where $\overline{\mathcal{X}}'$ is identical to $\overline{\mathcal{X}}$ apart from that $\overline{x}_{it} = \hat{X}_l$
  else
   Let $LB = z$ and $sol^* = sol$ with $\hat{x}_{it} = \underline{x}_{it}y_{it}^{X} + \overline{x}_{it}(1 - y_{it}^{X})$.

Output: Optimal solution *sol**.

[6] Since branching terminates when the *y*'s are binary, convergence is ensured just as is the case with the standard branch-and-bound algorithm.

4. Illustrative example

4.1. Price-quantity bidding and offering

For the illustrative example, a 5-node system is used. The system is based on the PJM 5-bus system of the MATPOWER package [45]. The following changes have been made to the system in order to make it suitable for illustration:

1. A generator on bus 1 has been replaced by a wind farm on bus 3 with an installed capacity of 300 MW.
2. Two merchant storage units with an installed generation/demand capacity of 100 MW and a storage capacity of 400 MWh each have been connected to buses 3 and 4, respectively.
3. All transmission lines with unlimited capacity in the original system have a capacity of 400 MW.
4. The marginal cost of generator 3 has been increased from $40/MWh to $60/MWh.

The single-line diagram of the illustrative example is shown in Fig. 2. Generator data are given in Table 1.

The merchant storage owner optimizes its operation over a horizon of 4 time periods. The variable cost of the merchant storage units is considered to be negligible and their round-trip efficiency is considered to be 100%. The merchant storage owner submits price-quantity pairs to the market with a price of either 0 $/MWh or 50 $/MWh as well as quantity of either 0%, 50%, 75%, or 100% of the 100 MW installed capacity. The bus loads are increasing over the horizon and are assumed to be known deterministically. In order to add stochasticity to the system, there are 3 equiprobable scenarios possible for the wind farm connected to bus 3 (1: low, 2: medium, 3: high). In all scenarios the wind power production is decreasing over the horizon. The total load is considered to be the residual load after the wind power has been deducted from the bus loads.

Fig. 3 shows the charging and discharging (red) of the merchant storage units as well as their submitted offer-bid values to the market (black). For comparison, the figure also shows a *benchmark case* where the ISO controls the dispatch of the storage units completely (blue). Lastly, the figure shows the price deviation at the corresponding buses between the *benchmark case* and the *strategic case* (green).[7]

The results of the illustrative example show various types of strategic behavior from the merchant storage owner when compared to the benchmark case where the ISO completely controls the dispatch of the storage units. Some empirically relevant takeaways from the illustrative example are the following:

Demand withholding: The units withhold demand in periods 1 and 2 by bidding a fraction of their demand capacity. This results in a decrease in the price compared to the benchmark case on both buses in period 1 (for scenarios 2 and 3) which means that the storage units can charge at a lower price. This behavior also results in less stored energy in periods 3 and 4, which helps to drive up the price during those periods.

Generation withholding: In period 3, both units withhold their generation capacity. Furthermore in period 4, unit 6 withholds its generation capacity. This behavior results in a price increase compared to the benchmark case and therefore a higher expected profit for both units.

Portfolio effect: The abovementioned generation withholding of the units also shows a portfolio effect where the actions of one unit benefit the storage portfolio as a whole.

Increased profit: Table 2 shows the profit of the generating units over the horizon. The expected profit of the storage portfolio over the three scenarios and four periods is $5846.4. The last column shows how the expected profit of the generators compares with their expected profit from the benchmark case. The strategic actions of the two storage units result in an expected profit that is more than double the expected profit of the benchmark case ($2702.1). Moreover, the strategic actions of the storage units increase the expected profit of all of the other generators apart from generator 3 and the wind farm. This is because the storage owner manages to decrease the prices somewhat when there is high wind power production and increases the prices when there is lower wind production.

Under-use: While the expected profit is increased, the expected amount of energy sold by storage is around 25% less in the strategic case than in the benchmark case; storage is under-used compared to the welfare-maximising use. In the benchmark case, the ISO flattens out the prices to minimize the generation costs. When the storage portfolio is controlled strategically, the owner tries to maintain the price difference while finding a trade-off between sold energy and price.

4.2. Self-scheduling

In this case the storage owner submits self-schedule bids and offers, that is without a price component, to the market instead of price-quantity pairs. The SBB algorithm is run again for the illustrative example where the merchant storage owner is allowed to submit *self-schedule* offer-bid values of 0%, 20%, 40%, 60%, 80% or 100% of the installed capacity which for both units is 100 MW. For this case, the expected profit of the storage portfolio is $5815.0.

4.3. Self-scheduling with an increased number of scenarios

We simulate the same *self-schedule* case as before; however, the number of wind power scenarios has been increased to 9. The profit of the benchmark case is $2173.1 but is increased to $5672.9 for the strategic case.

5. Numerical simulation

This section shows numerical simulations of larger test systems along with a comparison of the generation cost for (i) the case without storage, (ii) the case with competitive storage as well as (iii) the case with strategic storage. Lastly, computational comparison demonstrates the superior performance of the proposed solution algorithm when benchmarked against current practices in the literature.

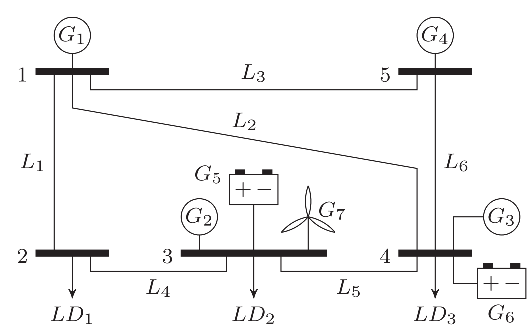

Fig. 2. The single-line diagram of the illustrative example.

[7] The results for the benchmark case show how the ISO dispatches the storage units differently depending on the particular scenario. The merchant storage owner however computes an expected profit-maximizing offer strategy that explicitly accounts for the fact that the storage owner does not know which scenario will actually occur at the time when the price and quantity bids are submitted. In other words, the storage owner has to bid to the market before the particular scenario materializes, with the aim of getting the highest weighted average profit from the three scenarios. This is evident in the strategic case where the dispatch is the same for all scenarios, corresponding to the best strategy on average, for all scenarios.

Table 1
Generator data for the illustrative example.

Unit #	Type	$\overline{P}_i / \overline{D}_i$ [MW]	Cost, c_i [$/MWh]	Bus
1	Dispatchable	40 / 0	15	1
2	Dispatchable	520 / 0	30	3
3	Dispatchable	200 / 0	60	4
4	Dispatchable	600 / 0	10	5
5	Storage	100 / 100	0	3
6	Storage	100 / 100	0	4
7	Wind	300 / 0	0	3

Table 2
Total profit by scenario as well as expected profit of the generating units over the horizon.

Unit #	Profit [$]				
	$w = 1$	$w = 2$	$w = 3$	E[Profit]	ΔE [Profit]
1	2933.7	2597.5	2597.5	2709.6	947.3
2	13932.9	13932.9	13932.9	13932.9	9288.6
3	0	0	0	0	0
4	0	3000.0	0	1000.0	1000.0
5	1339.7	2089.7	2339.7	1923.0	841.0
6	3008.6	4255.7	4505.7	3923.4	2303.3
7	5169.9	8539.7	15879.4	9863.0	− 451.5

5.1. IEEE 24-bus system with transmission constraints – self-scheduling

In this case, we run the SBB algorithm on the IEEE 24-bus, 32-unit system where the two 400 MW nuclear units in the system are assumed to be off-line. The simulation is carried out for 4 periods, 3 scenarios and there are two merchant storage units in the market (units 33 and 34); both of which have a storage capacity of 1000 MWh. They are connected to buses 13 and 15. There are 3 equiprobable scenarios possible for a 400 MW wind farm connected to bus 17 (1: high, 2: medium, 3: low). The units are allowed to submit self-schedule offer-bid values of 0%, 25%, 50%, 75% or 100% of their 200 MW installed capacity to the market. The results are represented in Fig. 4 where one can see how the units are able to strategically decrease the system price substantially during the first 2 periods as well as increase the price in the third period. These strategic actions increase their expected profit over the 3 scenarios and 4 periods from $655.9 in the benchmark case to $3856.0 in the strategic case.

5.2. IEEE 24-bus system without transmission constraints – self-scheduling

The SBB algorithm is run on the same IEEE 24-bus system without considering transmission constraints. The simulation is carried out for 12 periods and there is a single merchant storage unit in the market which has a storage capacity of 1000 MWh. The unit is allowed to submit self-schedule offer-bid values of either 0%, 25%, 50%, 75%, or 100% of its 300 MW installed capacity to the market. The unit is able to strategically apply generation withholding in order to increase the system price substantially and increase its expected profit from $7983.6 in the benchmark case to $40049.9 in the strategic case.

5.3. IEEE 118-bus without transmission constraints – price-quantity bidding

Finally, the SBB algorithm is run on the IEEE 118-bus system without considering transmission constraints for a horizon of 8 periods. A 2000 MWh storage unit submits price-quantity pairs with a price of either 0 $/MWh or 40 $/MWh as well as quantity of either 0%, 50% or 100% of its installed capacity of 600 MW. The strategic actions of the player increase its expected profit over the eight periods from zero in the benchmark case to $12000 in the strategic case.

5.4. Generation cost comparison

Table 3 shows a comparison of the expected generation cost for the benchmark case and the strategic case. The generation cost is decreased when storage is introduced for all simulations, both in the benchmark

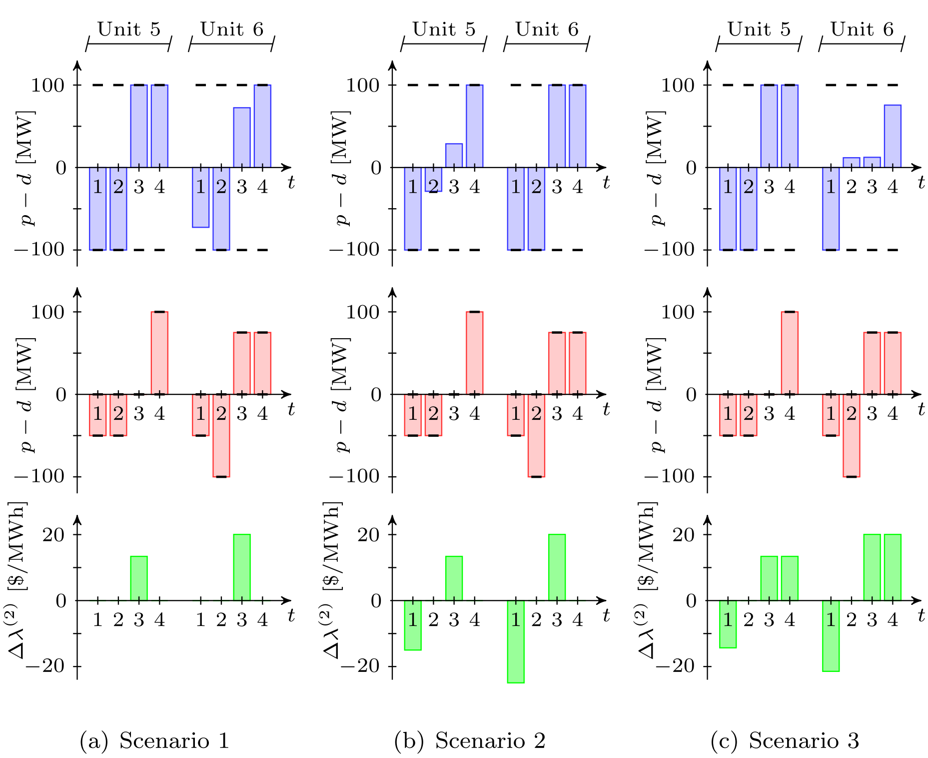

Fig. 3. Illustrative example results for the two storage units. Benchmark case (blue, top), strategic case (red, middle) and price deviation at the buses corresponding to the two units (green, bottom). Thick black lines represent offer-bid values and colored bars represent dispatch or price deviation.

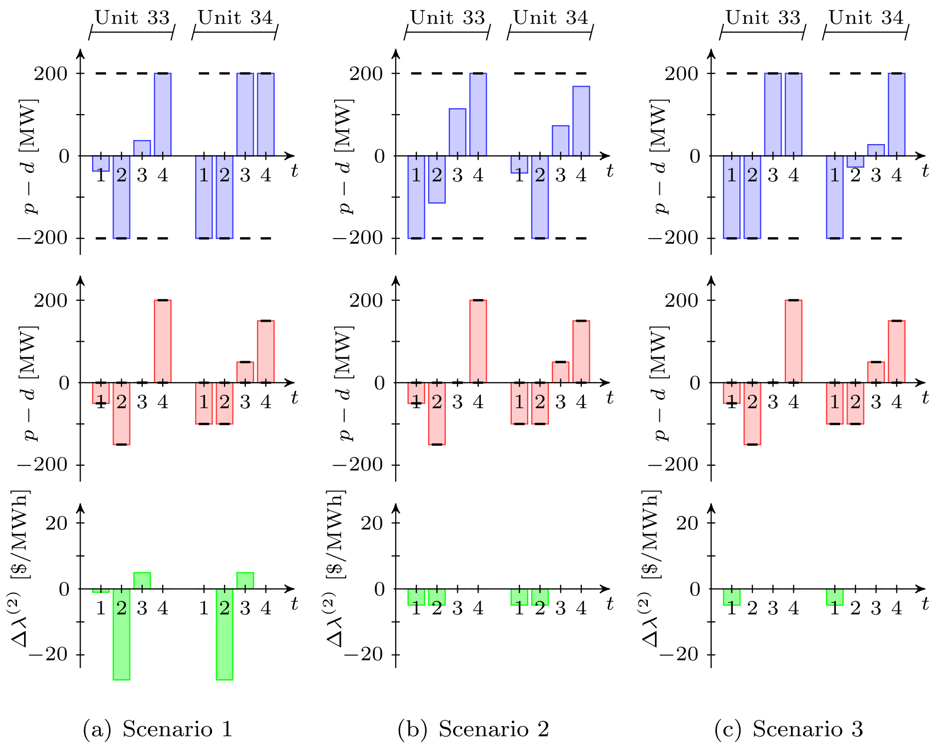

Fig. 4. IEEE 24-bus system results for the two storage units. Benchmark case (blue, top), strategic case (red, middle) and price deviation at the buses corresponding to the two units (green, bottom). Thick black lines represent offer-bid values and colored bars represent dispatch or price deviation.

Table 3
Comparison of expected generation cost for the simulations.

	Case Without Storage	Case With Competitive Storage (Benchmark)	Case With Strategic Storage (Strategic)
Illustrative example I	$80580	$72367	$74246
Illustrative example II	$80580	$72367	$74277
Illustrative example III	$81838	$73748	$75501
24-bus I	$125170	$118896	$119524
24-bus II	$691567	$641036	$645797
118-bus	$852852	$839325	$840852

case and in the strategic case. The generation cost is however lower in the benchmark case than in the strategic case.

5.5. Computational comparison

Table 4 shows the computational requirements of the Gurobi BB algorithm for solving the binary expansion MILP model and the SBB algorithm for solving the proposed disjunctive program (11). The SBB algorithm is implemented by the authors using a Python interface. In order to get a fair comparison, the Gurobi BB does not apply pre-solve algorithms or other heuristics. For the simulations performed, the proposed SBB algorithm is orders of magnitude more efficient than Gurobi BB in terms of nodes explored. For the two price-quantity bidding cases (the illustrative example I and the 118-bus), as well as the self-schedule 24-bus II case, SBB finds the optimal solution while Gurobi BB fails to find a proven optimal solution. Solving times for the two algorithms are moreover shown in Table 4. Note that Gurobi BB utilizes multithreading with 8 threads but SBB does not. Therefore the difference between the two approaches is even greater than the reported solving times would suggest. Fig. 5 shows a graphical comparison of the optimality gap convergence for the two algorithms (proposed SBB and Gurobi BB) for the 24-bus I case.

5.6. Computational comparison with a continuous MILP formulation

As shown in several publications, it is possible to linearize a bilevel electricity market model using the strong duality condition to linearize

Table 4
Comparison of the computational requirements of the different simulations and algorithms. The complexity of the MILP model is also reported in terms of the number of continuous variables, binary variables and constraints.

Simulation	Nodes explored		Solving time [min]		MILP complexity		
	SBB	Gurobi BB	SBB	Gurobi BB	cont.	bin.	constr.
Illustrative example I	1.31×10^6	*	162.8	*	1584	88	2395
Illustrative example II	59623	1.96×10^6	5.2	20.5†	1608	104	2555
Illustrative example III	57191	2.05×10^6	16.1	128.3†	4728	104	7505
24-bus I	47155	8.90×10^6	31.4	1650.0†	7400	88	8203
24-bus II	69103	*	12.7	*	3348	132	3781
118-bus	1.11×10^6	*	296.2	*	4064	72	4313

*No proven optimal solution found after 10 million nodes explored.
† Note that Gurobi BB utilizes multithreading with 8 threads but SBB does not.

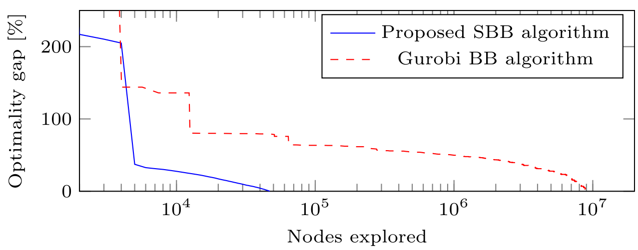

Fig. 5. Convergence of the optimality gap for the two solution algorithms for the 24-bus I case.

the objective function and use the Big-M technique and binary variables to linearize the complementary slackness conditions. This has for example been applied in works such as [35,38]. The resulting model is a continuous one and therefore the resulting bid/offer schedules are continuous. However, since the complementary slackness conditions have to be linearized using the Big-M technique, the model contains a great number of binary variables. This can lead to models which are impossible to solve, especially for electricity market models with many transmission constraints. This modelling limitation can be seen clearly in [38] where the author limits the number of binding transmission constraints in order to make the model tractable. Moreover in [35], the authors disregard the transmission constraints all together.

One of the main reasons for proposing the disjunctive formulation is to avoid this computational complexity tied to the Big-M linearization of the complementary slackness conditions. In the proposed disjunctive approach, transmission constraints in the lower level problem appear as regular linear constraints as opposed to complementary slackness conditions that involve binary variables. To illustrate this advantage of the proposed model, we have implemented the continuous MILP modelling approach on the IEEE 24-bus system with transmission constraints; the same system as in Section 5.1. This IEEE test system is however notorious for having very liberal transmission limits which are not likely to be binding. We therefore show what happens when the continuous model is solved for different scaling factors for the transmission limits. As an example, a scaling factor of 0.7 represents that the maximum flow of a transmission line is 70 % of the maximum flow in the original system. By doing so, one can see how the computational complexities of the continuous model are affected by the increasing number of binding complementary slackness conditions. Cases with 3 and 6 scenarios for the wind production are studied and the results can be seen in Table 5. Note that the continuous MILP model is solved by standard Gurobi BB, which utilizes multithreading with 8 threads as well as pre-solve algorithms and other heuristics but SBB does not. Therefore the difference between the two approaches is even greater than the reported solving times would suggest.

The results show that the computational complexity of the continuous MILP model grows very rapidly as the transmission limits are made more stringent, especially for the 6-scenario case where the number of Big-M complementary slackness conditions is increased even more. Gurobi BB is even unable to solve the two cases with the most stringent transmission limits. This is not the case with the proposed disjunctive method, where the computational complexity is more or less constant, no matter the level of transmission congestion in the system. For highly congested power systems, which are often prone to having market power issues, the proposed method is therefore a superior choice as it scales much better than its continuous counterpart.

6. Further analysis

In this section, further technical constraints are analyzed for the same system as in the illustrative example. The merchant storage owner is assumed to submit self-scheduling bids to the market as in illustrative example 2.

6.1. Uncertain bids by other generators

One can assume that in addition to the net demand being uncertain, there are also uncertainties in the bids of the other generators in the system. Let us assume that the value of the bid of the most expensive generator is uncertain but can be represented by three scenarios. Its most likely value is 60 $/MWh with a probability of 0.6 but other scenarios give values of 55 $/MWh and 50 $/MWh, each with a probability of 0.2. In this case the expected profit of the merchant storage owner is lowered to $ 5174.5 compared to $ 5815.0 in the original case, because the storage owner now faces the prospect of cheaper supply from its competitor.

6.2. Initial/final state-of-charge requirement

When scheduling successive planning horizons, it may be important to put a requirement on the storage level of the storage units at the end of the planning horizon. Such a requirement is captured by constraint (1f) in the merchant storage problem. It makes sure that the storage level at the end of the planning horizon should be no lower than that at the beginning. The storage capacity of each unit is 400 MWh. Table 6 shows how the expected profit of the merchant storage owner is affected when the initial storage level is varied and is accompanied by the constraint that the final storage level must be no lower than the initial one. The results are shown graphically in Fig. 6 and also include the benchmark case where both units are fully controlled by the ISO. When the initial storage level is relatively low (0–200 MWh), it has no impact on the expected profit ($ 2702.1 and $ 5815.0 for the benchmark and strategic cases, respectively). This is mainly because the units are still able to charge during the early periods when there is substantial wind power available and the net demand is low. However, when the initial storage level is increased further, the expected profit reduces, both for the benchmark and the strategic case. It even becomes zero when both storage units are fully charged at the beginning of the planning horizon and it is required that they are fully charged at the end of the planning horizon.

6.3. Arbitrage by other players

The proposed model in this paper is able to capture the behavior of other non-strategic storage units that have the opportunity to arbitrage between different periods. In this example it is assumed that the strategic merchant storage owner only has half of its previous capacity, that is a single storage unit (unit 6) with an installed capacity of 100 MW connected at bus 4. Furthermore there is also a non-strategic storage unit (unit 5) present at bus 3 which surrenders its control to the ISO. The ISO is therefore free to operate that storage unit in a way which best benefits the system in terms of minimizing the cost. When such a

Table 5
Computational comparison between the proposed disjunctive approach and a continuous MILP formulation.

	Transmission scaling factor	Nodes explored: SBB	Nodes explored: Continuous MILP	Solving time [min]: SBB	Solving time [min]: Continuous MILP
3 scenarios	0.8	45665	5236	24.6	0.2
	0.7	46795	22410	24.5	0.4
	0.6	69431	40863	37.1	1.0
	0.5	15333	4.15×10^6	9.9	92.1
6 scenarios	0.8	84209	95464	102.5	4.9
	0.7	80131	2.74×10^6	107.3	126.2
	0.6	89965	*	121.5	*
	0.5	32387	*	44.7	*

*No proven optimal solution found after 10 million nodes explored.

Table 6
The expected profit of the strategic storage owner as a function of the initial/final storage level.

S_i^0 [MWh]	Expected profit [$]	
	Benchmark	Strategic
0 – 200	2702.1	5815.0
220	3594.2	5815.0
240	4040.2	5815.0
260	3594.2	5815.0
280	4486.3	5547.1
300	3817.2	4878.0
320	3741.5	4140.0
340	3289.2	3439.1
360	2725.4	2725.4
380	1534.6	1534.6
400	0.0	0.0

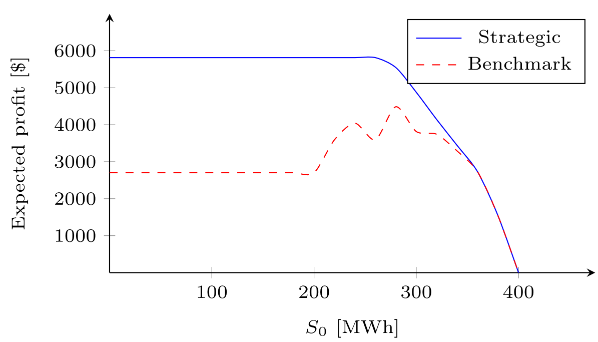

Fig. 6. The expected profit of the strategic storage owner as a function of the initial/final storage level S_i^0.

player is present in the market, the strategic merchant storage owner includes this in the lower-level problem, just like any other generator. This will add three constraints to the lower-level problem, identical to the ones given in (1d)–(1f). The KKT conditions are then updated accordingly. Table 7 shows how the expected profit of the merchant storage owner is affected by the presence of another arbitraging unit in the market that is completely controlled by the ISO. The results are shown graphically in Fig. 7 and also include the benchmark case where unit 6 is also fully controlled by the ISO. As can be seen, the influence on the expected profit is substantial, both for the benchmark and the strategic case. When the installed capacity of the non-strategic storage unit increases, the expected profit of the strategic unit deteriorates. However, the expected profit in the strategic case is higher than in the benchmark case.

7. Conclusion

This paper proposes a stochastic disjunctive programming model for finding the optimal offer-bid strategy of a merchant storage portfolio. The interaction between the merchant storage owner and the ISO is modeled as a stochastic bilevel optimization model and then reformulated as a stochastic disjunctive program. Employing the disjunctive nature of the optimization model, a specialized branch-and-bound algorithm is proposed. The proposed SBB solution algorithm branches on the ranges of discrete variables (rather than binary variables in the standard BB algorithm). To find a relaxed solution of the proposed stochastic disjunctive program, first the concept of quasi-relaxation is defined. Then a linear quasi-relaxation is derived for the case of the merchant storage model. Both the proposed disjunctive programming model and the SBB solution algorithm are benchmarked against current practices in literature for modeling and solving these

Table 7
The expected profit of the strategic storage unit (unit 6) as a function of the installed capacity of the non-strategic storage unit (unit 5).

$\overline{P}_5$, $\overline{D}_5$ [MW]	Unit 6 expected profit [$]	
	Benchmark	Strategic
0	4623.8	4672.8
20	4341.0	4341.0
40	3505.7	4100.2
60	2837.2	3639.4
80	2168.6	3238.3
100	1620.0	2537.1
120	951.4	2136.0
140	0.0	1568.6
160	0.0	1268.6
180	0.0	867.4
200	0.0	433.7
220	0.0	433.7
240	0.0	324.0

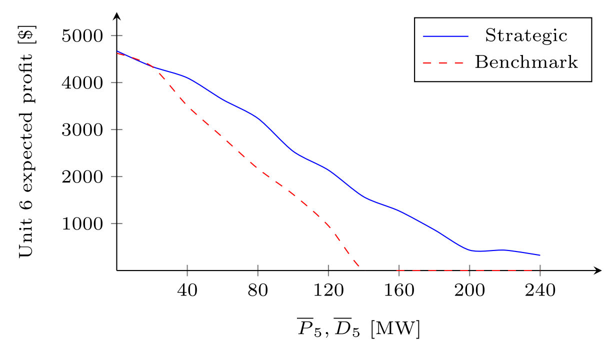

Fig. 7. The expected profit of the strategic storage unit (unit 6) as a function of the installed capacity of the non-strategic storage unit (unit 5).

types of problems (binary expansion MILP model and Gurobi BB). The numerical results confirm the performance of the modeling approach and its solution algorithm for dealing with the optimal offer-bid strategy of an energy storage portfolio. Simulations of test systems reveal the various abilities of the merchant storage owner to exercise unilateral market power. Those include *demand withholding, generation withholding* and *under-use* which result in an increased congestion in both space and time when compared to the welfare-maximizing use of storage. Further technical analysis was carried out to shed a light on how factors such as uncertain bids by other players, final state-of-charge requirements and arbitrage by other storage players affect the profit of the merchant storage owner. The results showed that all these factors have a tendency to limit the expected profit. Although the modeling approach and the SBB algorithm are derived in the context of a merchant storage offer-bid model, the same modeling approach and solution algorithm might be applicable for other stochastic bilevel optimization problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to thank the Swedish TSO, Svenska Kraftnät, for their financial support of the project.

Appendix A. Proofs

Proof of Lemma 1. The same approach as in [41] is used for proof of this lemma. Consider any feasible solution ($\hat{p}_{it}$, $\hat{d}_{it}$, $\hat{c}_{it}$, $\mu^{(6)}_{itw}$, $\mu^{(8)}_{itw}$, p_{itw}, d_{itw}) to (11). It suffices to construct a feasible solution ($\hat{p}_{it}$, $\hat{d}_{it}$, $\hat{c}_{it}$, $\mu^{(6)\prime}_{itw}$, $\mu^{(8)\prime}_{itw}$, $p_{itw}{}'$, $d_{itw}{}'$) of (11Q) since the latter has the same objective value in (11Q) as the former does in (11). Let

$$\mu^{(6)-\prime}_{itw} = y^{P}_{it}(\mu^{(6)-}_{itw} + \mu^{(6)+}_{itw}) \tag{A.1a}$$

$$\mu^{(6)+\prime}_{itw} = (1 - y^{P}_{it})(\mu^{(6)-}_{itw} + \mu^{(6)+}_{itw}) \tag{A.1b}$$

$$\mu^{(8)-\prime}_{itw} = y^{D}_{it}(\mu^{(8)-}_{itw} + \mu^{(8)+}_{itw}) \tag{A.1c}$$

$$\mu^{(8)+\prime}_{itw} = (1 - y^{D}_{it})(\mu^{(8)-}_{itw} + \mu^{(8)+}_{itw}) \tag{A.1d}$$

$$p^{-\prime}_{itw} = y^{C}_{it}(p^{-}_{itw} + p^{+}_{itw}) \tag{A.1e}$$

$$p^{+\prime}_{itw} = (1 - y^{C}_{it})(p^{-}_{itw} + p^{+}_{itw}) \tag{A.1f}$$

$$d^{-\prime}_{itw} = y^{C}_{it}(d^{-}_{itw} + d^{+}_{itw}) \tag{A.1g}$$

$$d^{+\prime}_{itw} = (1 - y^{C}_{it})(d^{-}_{itw} + d^{+}_{itw}), \tag{A.1h}$$

which is clearly feasible in (11Q). Furthermore, substituting (A.1a)–(A.1h) into (11Q) results in the same constraints as in (11), apart from the disjunctive constraints. All other constraints in (11Q) are identical to their counterparts in (11). □

Proof of Lemma 2. The same approach as in [41] is used for proof of this lemma. When a variable y_{it} is either 0 or 1, the offer-bid value is either at the lower bound or the upper bound of that branch, both of which are valid discrete values and therefore fulfill the disjunctive constraints. □

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