Competition in Interregional Taxation: The Case of Western Coal

Charles D. Kolstad

Los Alamos National Laboratory, University of California

Frank A. Wolak, Jr.

Harvard University

Markets for many products are dominated by a small group of states or countries with a natural advantage in the marketplace because of some initial endowment of resources, favorable climate, or location. The purpose of this paper is to explore how such markets involving a few political jurisdictions interact noncooperatively. We examine how such a market might be structured and operate and the extent of monopoly rent that can be extracted in the absence of collusion. We answer these questions for an empirically estimated model of western U.S. coal in which two states (Montana and Wyoming) dominate production. We demonstrate that in this market the amount of rent that can be extracted is greatly reduced through competition (relative to a cartel). Nevertheless, even with two producing states competing against each other, significant rents can be captured—significant enough to refute the contention that little rent can accrue without a cartel.

I. Introduction

Markets for many products are dominated by a small group of states or countries with a natural advantage in the marketplace because of

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some initial endowment of resources, favorable climate, or location. This is the case for many natural resources (e.g., uranium, copper, bauxite, coal, and oil) and some agricultural products (e.g., wheat and citrus). Many of the analyses of such markets focus on the viability of cartelization,[1] implying that should no cartel develop, opportunities for exercising market power are minimal. In a few cases (such as OPEC), producing regions have been successful in forming and operating a cartel. Yet the far more realistic market structure lies intermediate between the two extremes of perfect competition and what amounts to monopoly behavior by a cartel. It is common that a few producing states or countries act noncooperatively but individually still dominate a large enough part of the market to exercise some market power.

The purpose of this paper is to explore such markets involving a few noncooperative political jurisdictions. Two questions in particular are investigated. One is how such a market might be structured and operate. To this end, we propose tax-reaction-function Nash equilibria rather than the more conventional quantity-based Cournot and von Stackelberg equilibria. This approach is consistent with the use of severance taxes or export tariffs to exercise market power. The second question we investigate concerns the extent of monopoly rent that can be extracted in the absence of collusion. Where on the continuum between perfect competition and the perfect cartel do such noncooperative equilibria lie?

We answer these questions for an empirically estimated model of the western U.S. coal market where two states (Montana and Wyoming) dominate production. There has been a lively debate on the appropriate level of severance taxes in these two states. In fact, legislation to limit their taxing power has been introduced into the U.S. Congress. The results of our analysis, of course, contribute to this debate. At a more general level, this paper demonstrates a plausible model of tax competition among states or countries. We also demonstrate that in a major natural resource market (U.S. steam coal), the amount of rent that can be extracted is greatly reduced through competition (relative to a cartel). Nevertheless, even with two producing states competing against each other, significant rents can be captured—significant enough to refute the contention that little rent can accrue without a cartel.

In the next section, we develop further the idea of noncooperative tax equilibria focusing on variants of the Cournot and von Stackel-

[1] Pindyck (1978) measures the gains from cartelization in the copper, bauxite, and oil markets. Schmitz et al. (1981) conduct a similar analysis for wheat markets. Alt and Zimmerman (1980) examine a western U.S. coal cartel.

berg equilibria. We then apply this general analysis to the case of western coal and discuss the implications of the results.

II. A Model of Noncooperative Tax Equilibria

The conventional notion of oligopoly is that several producers dominate the market. There may be a large number of producers, but they are partitioned into groups that fall under a few political jurisdictions. It is these states or countries that exercise the market power. Without the political jurisdiction of the state or country, producers may be too numerous to exert any market power individually. This institutional distinction could be considered superficial, but it does bear on the instruments that are available to extract rent because the actual production decision is not made by the oligopolistic state or country. In most cases, tax or quota systems are used to control the market. Severance taxes or export tariffs result in any monopoly rent accruing directly to the taxing jurisdiction. Production quotas or export licenses restricting output achieve a similar level of activity but can result in rent accruing to producers.[2] Although quota systems do exist,[3] the more common instrument is the tax, perhaps because it is an unambiguous and direct way of raising revenue.

The prevalent approach to examining the behavior of oligopolists is through reaction-function equilibria (Friedman 1977).[4] In fact, most theories of oligopoly differ only in terms of the assumption about one agent's perception of the strategies of the other agents. The most common oligopoly model is due to Cournot and involves output quantity as the decision variable and the assumption that any agent perceives that its competitors will not respond to changes in that agent's output level. We examine reaction-function equilibria where the tax rate is the decision variable. On the surface, this may appear to be just as arbitrary as letting quantity be the decision variable. However, it is our view that if tax rates are the actual decision variables, then strategies for setting taxes will be based on how competitors set taxes, not on the indirectly determined output levels of competitors. As we show later, the choice of tax over quantity as the decision

[2] If production or export licenses are sold to producers, then rent can be captured by the political jurisdiction. In some cases, it can be advantageous for rent to accrue to producers through a quota system—e.g., when production costs are not well known to consumers and it is desirable to hide the amount of monopoly rent captured.

[3] South Africa has a coal export license system that can be viewed as a mechanism for keeping coal prices above competitive levels.

[4] A number of authors have examined the pricing of natural resources through reaction-function equilibria (Salant 1976, 1982; Gilbert 1978; Ulph and Folie 1980; Chao and Peck 1982).

variable has a large effect on equilibrium levels of rent extraction in the western U.S. coal market.

The market we examine consists of several states or countries that produce a homogeneous good and are separated geographically from a single aggregate consumer. Each individual producer falls under a single political jurisdiction and will therefore be aggregated into one of the producing political jurisdictions. For the ith producing region, let q_i, t_i, τ_i, and $S_i(q_i)$ be, respectively, the output, tax rate, per unit transport cost to consumers, and supply (marginal cost) function for production of the good. Let the inverse demand function be denoted by $P(\Sigma_j q_j)$.

Consider first the classic Cournot analysis of a market equilibrium involving several producing entities with market power. In such a case, the state or country restricts output through the sale of export licenses or some other device in order to maximize the value of the licenses:

$$\max_{q_i} q_i\left[P\left(\sum_j q_j\right) - S_i(q_i) - \tau_i\right]. \tag{1}$$

The first-order conditions for this maximization are

$$P\left(\sum_j q_j\right) - S_i(q_i) - \tau_i + P'\left(\sum_j q_j\right) - S_i'(q_i) \begin{cases} \leq 0, & q_i = 0 \\ = 0, & q_i > 0 \end{cases}, \forall\, i, \tag{2}$$

making the standard Cournot assumption that $dq_j/dq_i = 0$ for $i \neq j$. Each equation (2) implicitly defines a reaction function for region i in terms of the other regions' outputs. These equations can be solved for a market equilibrium. The tax *rate* in each region that would support the above equilibrium is the consumer price less the marginal cost of production and transport divided by the marginal cost of production:

$$t_i^* = \frac{P\left(\sum_j q_j\right) - S_i(q_i) - \tau_i}{S_i(q_i)}. \tag{3}$$

We now turn to the case where the tax rate is the actual decision variable. The market equilibrium for a given set of taxes is determined by

$$(1 + t_i)S_i(q_i) + \tau_i = P\left(\sum_j q_j\right), \forall\, i. \tag{4}$$

Because there are many producers within the state or country, the market price net of tax will be marginal cost, $S_i(q_i)$. Thus, equation (4) states that the producer price (including tax) plus unit transport cost must equal the price to consumers.

Noncooperative Equilibria

For the purposes of tax equilibrium, we assume that each producing region i seeks to maximize tax revenues given the boundary equilibrium conditions (4):

$$\max_{t_i} t_i q_i S_i(q_i), \quad \forall\, i. \tag{5}$$

First-order conditions for state i are

$$\{t_i[S_i(q_i) + S_i'(q_i)q_i]\} \frac{dq_i}{dt_i} + S_i(q_i)q_i \begin{cases} \leq 0 & \text{for } t_i = 0. \\ = 0 & \text{for } t_i > 0. \end{cases} \tag{6}$$

Equation (6) is straightforward except for the dq_i/dt_i term, which indicates how production will change as a result of a change in the tax rate. A change in a particular t_i will bring about a movement to a new equilibrium and will involve a readjustment of all output levels and possibly a readjustment in the tax rates of other states or countries.[5] The evaluation of dq_i/dt_i thus involves taking the total derivative of the equilibrium conditions (4) with respect to each t_i.[6] The derivatives of the equilibrium conditions can be solved simultaneously with the equilibrium conditions to obtain expressions for q_i and dq_i/dt_i as functions of the vector of tax rates alone. Substituting these results into each producing region's first-order condition (eq. [6]) results in a set of tax reaction functions giving a state's or country's tax rate (t_i) as a function of all the other states' or countries' tax rates, $\mathbf{t}_{\langle i \rangle}$:

$$t_i = R_i(\mathbf{t}_{\langle i \rangle}). \tag{7}$$

This approach can be taken for each state or country yielding a tax reaction function such as equation (7) for each jurisdiction. This results in a set of equations in the same number of unknowns as tax rates, which can then be solved for a set of equilibrium tax rates.

The difference between the various types of reaction-function equilibria depends on how one calculates the dq_i/dt_i in equation (6). Equation (4) can be solved to express q_i as a function of all tax rates. For a simple Cournot-type Nash equilibrium, each producing region assumes no change in the tax rates of the other states in response to

[5] Whether other states or countries adjust their tax rates depends on the ith state's conjecture about how his opponents react to changes in t_i.

[6] It is in taking the derivatives of these equilibrium conditions that assumptions about conjectures of actions and reactions enter. A traditional Cournot analysis (as discussed earlier) would hold that, in this tax-setting framework, as a particular tax rate changes, quantities produced by all other states or regions remain fixed. An alternate and possibly more plausible assumption is that all other tax rates remain fixed.

changes in its tax rate; therefore,

$$\frac{dq_i}{dt_i} = \frac{\partial q_i}{\partial t_i}. \tag{8}$$

If a producing region is a tax leader (in the spirit of von Stackelberg) and takes into account how other regions react to its tax rate, then

$$\frac{dq_i}{dt_i} = \sum_j \frac{\partial q_i}{\partial t_j} \frac{\partial t_j}{\partial t_i}, \tag{9}$$

where $\partial t_j/\partial t_i$ is determined by differentiating equation (7). In either case, we can determine an equilibrium tax rate.

Cooperative Tax Equilibria

In order to have a yardstick by which to measure the market power associated with noncooperative tax setting, it is useful to examine the results of collusive tax rate determination. The goal of a perfect cartel[7] is to maximize total cartel profits. If side payments are not permitted, then all that can be said about the cartel's behavior is that the cartel will operate on its contract curve, which can be determined by maximizing some weighted sum of member tax revenues:

$$\max_{t_i} \sum_i \alpha_i t_i S_i(q_i) q_i. \tag{10}$$

The first-order conditions for this maximization are

$$\sum_i \alpha_i \left\{ t_i [S_i(q_i) + S_i'(q_i) q_i] \frac{dq_i}{dt_j} \right\} + \alpha_j S_j(q_j) q_j \begin{cases} \leq 0, & t_j = 0 \\ = 0, & t_j > 0 \end{cases} \quad \forall j. \tag{11}$$

Expressions for dq_i/dt_j and q_i are obtained in the same fashion as the noncooperative case above (from equilibrium conditions [4] and their first derivatives). Given a set of weights ($\boldsymbol{\alpha}$), first-order conditions (11) can then be solved for a cooperative tax equilibrium. In contrast to the noncooperative case, the cartel solution is invariant as to whether taxes or quantities are the decision variables.

III. Montana-Wyoming Competitive Tax Setting

We are now in a position to apply the noncooperative model of tax-reaction-function equilibria developed in the previous section. We will hypothesize two plausible behavioral models for reaction-function equilibria and compare our results with the standard Cour-

[7] A perfect cartel is one where side payments among members are permitted.

not model and that of a perfect cartel. The situation is that Montana and Wyoming have vast coal resources and can produce desirable, low-sulfur coal very cheaply. This coal is by far the cheapest coal in the United States to produce and, even after significant transport costs are added, competes very favorably in distant midwestern and southern U.S. markets. The popularity of coal from these two states has caused a rapid increase in its production over the past few years (a 10-fold increase over the decade of the 1970s) and the imposition of significant severance taxes.[8]

To conduct this analysis, it is necessary to have estimates of the coal supply, demand, and transport functions discussed in the previous section. However, it is difficult, if not impossible, to estimate econometrically these functions based purely on historic data (Zimmerman 1977). Consequently, we have used a model of U.S. coal markets to generate pseudodata that we use to estimate the supply, demand, and transport relations. The model is an activity analysis, partial equilibrium model of the type widely used in examining coal market issues (Uri 1975; ICF 1976; Schlottman 1976; LeBlanc, Kalter, and Boisvert 1978; Alt and Zimmerman 1980; Bopp et al. 1981; Zimmerman 1981) and appears to give results consistent with other current coal market models (Wolak et al. 1981). The coal supply component is based on engineering estimates of marginal production costs, taking into account depletion effects. Details of the model may be found in the Appendix. This approach of generating pseudodata using a process model is not uncommon (Griffin 1977*a*, 1977*b*, 1980), although it can be faulted (Maddala and Roberts 1980). Nevertheless, the utility of the approach is clear in a case such as ours because it allows a far more sophisticated and enlightening analysis of the western coal market.

Although the coal market model cannot be solved explicitly for tax reaction function equilibria, it can simulate the market's response to a variety of tax rates. Our approach then is to exercise the coal market model for the year 1990[9] for a number of different combinations of severance taxes in Montana and Wyoming, ranging from 0 to 120

[8] Tax rates in both states are in terms of a percentage of the price, F.O.B. mine. The 1979 rate in Montana for surface coal is 30 percent; the rate in Wyoming, including county tax, is 16.4 percent (Blackstone 1979).

[9] A static analysis has been chosen because other authors have argued its adequacy. Cremer and Weitzman (1976) show that when depletion effects are small, a static analysis of monopoly power gives essentially the same results as an intertemporal analysis. Because western coal is usually characterized by nearly flat marginal cost curves (depletion effects are small), the errors introduced by our static analysis should be small compared to the gains in simplicity. Pindyck (1981) suggests that myopic behavior may track the actual performance of natural resource markets better than Hotelling-type intertemporal optimizing behavior.

percent. Taxes in other states are set at their 1979 rates. These market equilibrium points can then be used to estimate aggregate supply and demand functions, which can be readily manipulated to determine equilibrium tax rates under alternate assumptions about the tax-setting process.

Let us rewrite the equilibrium conditions (4) explicitly in terms of supply and demand. We assume that supply and demand functions are linear.[10] Transport costs (per unit) are assumed to be a function of the mine-mouth price of the coal (including tax), principally to capture the effect that as coal price goes up, a producer's market area shrinks, thus reducing average transport costs.[11] Letting p_m and p_w be prices of coal (F.O.B.) in Montana and Wyoming, respectively, and p_d be the delivered price, equilibrium conditions (4) can be rewritten as

$$p_m = (1 + t_m)(\alpha_m + \beta_m q_m) + \epsilon_m, \tag{12a}$$

$$p_w = (1 + t_w)(\alpha_w + \beta_w q_w) + \epsilon_w, \tag{12b}$$

$$p_d = a_d + b_d(q_w + q_m) + \epsilon_d, \tag{12c}$$

$$\tau_i = c_i + d_i p_i + \eta_i \text{ for } i = m, w, \tag{12d}$$

and

$$p_d = p_i + \tau_i \text{ for } i = m, w, \tag{12e}$$

where the α, β, a, b, c, and d terms are coefficients, and disturbance terms (ϵ, η) of zero mean have been introduced. There are two exogenous variables (t_m, t_w) in these seven equations and seven endogenous variables (p_m, p_w, p_d, q_m, q_w, τ_m, τ_w). Because several endogenous variables occur in all equations, one would expect the error terms of these equations to be correlated, suggesting that some type of simultaneous equation estimation is appropriate. For estimating these equations, the pseudodata presented in figure 1 were generated. Table 1 presents the results of a two-stage least-squares estimate of the coefficients of (12).[12] The fit of the equations to the pseudodata is

[10] The assumption of linearity is principally for computational convenience although the data appear to fit a linear model reasonably well.

[11] That total coal transport costs for a given origin/destination appear to be directly proportional to quantity, after initial economies of scale have been realized, is reported by numerous authors (ICF 1976; White and Hynes 1979; Zimmerman 1979). Up to the point of very large shipment levels, congestion does not appear to affect shipment costs. In addition, western producers face a competitive fringe in the Midwest and East. As mine-mouth prices for western coal (including taxes) increase, distant marginal markets are lost, lowering the average shipment cost. This justifies making per unit transport costs a function of the price of coal at the point of origin.

[12] A two-stage least-squares estimator is thought to be more robust than a full-information technique when samples are small and there is the potential for misspecification, since any error is restricted to a single equation and is not allowed to propagate throughout the entire system (Johnston 1972).

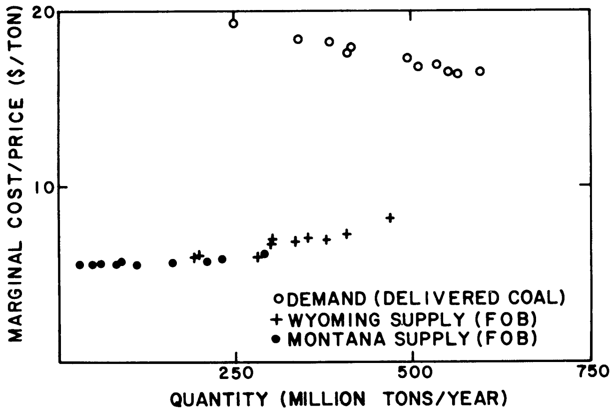

FIG. 1.—Activity analysis model results

reasonably good, although the very shallow slope of the supply curves, particularly for Montana, results in a large variation in quantity of coal produced for very small price changes.

The equilibrium conditions (12) can be solved for coal output for the two states as a function of tax rates:

$$q_m = -\frac{(4.1t_w + 11.6)t_m - 16.6t_w - 1.81}{(t_w + 2.80)t_m + 13.3t_w + 15.1} \ (10^6 \text{ tons/year}) \qquad (13a)$$

and

$$q_w = -\frac{(.722t_m + 9.6)t_w - 9.1t_m - 7.58}{(t_w + 2.80)t_m + 13.3t_w + 15.1} \ (10^6 \text{ tons/year}). \qquad (13b)$$

We will first examine two of many possible cartel solutions to illustrate the maximum market power obtainable by these two states. The first and simplest pricing model is to assume there is a single tax rate applied to both states. In this case, the cartel-revenue-maximizing solution corresponds to a tax rate of 87 percent.[13]

The other possible cartel objective is to maximize joint tax revenue—each state sets a tax rate such that cartel revenues are a maximum. Joint revenues are maximized when the Montana rate is 83 percent and the Wyoming rate is 96 percent. This generates only 2 percent more tax revenue than the uniform tax solution.

We are now in a position to examine specific models of noncooperative state tax-setting behavior. We first consider the case where both states react to the other state purely on the basis of the level of the

[13] This can be compared to the cartel solution of 67 percent obtained by Alt and Zimmerman (1980).

TABLE 1

REGRESSION RESULTS FOR WYOMING/MONTANA TAX EQUILIBRIUM CASE

	Equation	α, c	β, d	R^2	Standard Error of Regression	Number of Observations
Montana supply	(12a)	5.57 (136)	.00134 (4.5)	.99	.11	11
Wyoming supply	(12b)	4.69 (13.4)	.00664 (5.7)	.89	.43	11
Demand	(12c)	21.29 (86)	−.00835 (−15.9)	.97	.18	11
Montana transport	(12d)	13.18 (18.8)	−.493 (−6.2)	.81	.42	11
Wyoming transport	(12d)	10.41 (16.3)	−.287 (−11.1)	.93	.25	11

NOTE.—*t*-statistics (coefficient/standard error) are in parentheses. Units are dollars per ton (p) and millions of tons per year (q). Estimation is by two-stage least squares.

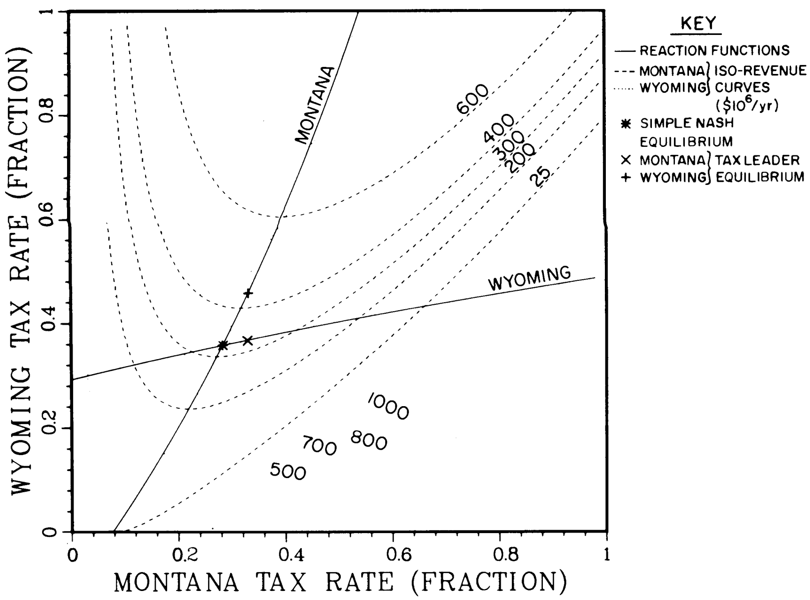

FIG. 2.—Tax equilibria and reaction functions

other's tax (i.e., $\partial t_i/\partial t_j = 0, i \neq j$). This corresponds to a simple Nash equilibrium (in the spirit of Cournot). For the case of Montana responding only to the *level* of Wyoming's tax, we can take the derivative of equation (13a) with respect to t_m and substitute this expression along with equations (13) into equation (6) to obtain a Montana tax reaction function giving Montana's optimal response to a given Wyoming tax rate. This can similarly be done for Wyoming to obtain its tax reaction function.

Unfortunately, the simultaneous solution of these reaction functions is not generally amenable to a closed-form solution. They can be solved numerically, however. Figure 2 is a plot of the two functions. Also shown in the figure are several isorevenue lines for the two states. As expected, the minima of the isorevenue lines trace out the reaction functions. Note that the simple Nash tax-rate equilibrium (indicated by an asterisk) is approximately 27 percent for Montana and 33 percent for Wyoming. These are considerably smaller numbers than were obtained previously for the case of collusion in regional tax setting. The effect of the competition between the two states is quite significant, dropping the optimal tax rates from 87 percent (for the case of the same tax rate for both cartel members) to near 30 percent. Revenue, however, declines more modestly to 62 percent of the maximum revenue cartel solution.

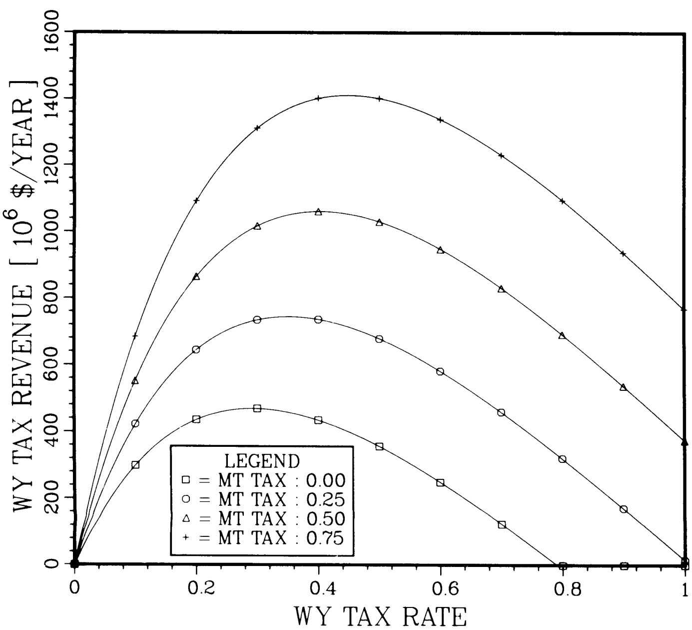

FIG. 3.—Wyoming severance tax revenue

Another possible tax-setting model is that one of the states is a leader in the spirit of von Stackelberg. To the extent that Montana often seems to be concerned about Wyoming's response to Montana's tax policies, Montana is a candidate as a leader. Wyoming, of course, could be a leader, being more guarded about its strategies. The "tax leader" would set his tax to maximize revenue, assuming the other state stays on his simple Nash reaction function. The cross and the plus in figure 2 indicate the equilibria that result when each of the states is a tax leader.[14] In this case, both the leader and the follower have slightly higher tax rates than for the simple Nash equilibrium. However, the leader can raise his tax rate more than the follower. In any event, the interesting thing to note is that the tax-leader equilibria are not significantly different from the simple Nash equilibrium. This is due to the very shallow slopes of the two reaction functions.

Figure 3 illustrates the background for these reaction functions. Shown in the figure, for various levels of tax in Montana, is the tax

[14] As is conventional in evaluating von Stackelberg price-leader equilibria, the point at which the follower's reaction function is tangent to the leader's isorevenue curve gives a revenue maximum for the leader.

revenue that accrues to Wyoming as it varies its tax rate. One thing appears clear from these two figures: that the optimum tax rate for each state is fairly insensitive to the tax rate of the other state. For each state, the optimum tax rate is less than 50 percent for tax rates in the other state of up to 75 percent. This suggests not only that our results are fairly robust, but that the tax equilibrium should be fairly "stable," not subject to rapid fluctuation as states react to each other's actions.

It is interesting to compare these results with the more conventional quantity-based Cournot equilibrium discussed in equations (1)–(3). One would expect that such a Cournot equilibrium would yield higher equilibrium tax rates because a state would assume that no customers would be driven to its competitor as a result of a change in its output level. This is, in fact, confirmed. The Cournot tax equilibrium is 57 percent for Montana and 67 percent for Wyoming. The amount of revenue accruing to the two states is 50 percent more than the simple tax-based Nash equilibrium. That simple Nash equilibrium is much closer to actual tax rates in the two states. This suggests the importance of selecting the more relevant decision variable in analyzing competition among a small number of producers.

IV. Conclusions

This paper has served three purposes. First, we have introduced an alternate methodology for analyzing competition among a few producing regions that dominate the market for a single good. Within this framework, we have shown that there are substantial losses in market power as a result of interregional competition, but despite these losses, producing regions can still individually exert a very noticeable amount of influence on the market as well as capture sizable amounts of rent. Finally, for examining the tax-setting process, we have illustrated the importance of working in tax space versus the more traditional quantity space.

Appendix

This Appendix provides a brief overview of the partial equilibrium activity analysis model of the U.S. coal market used to generate the pseudodata presented in this paper. The model is more fully documented in Wolak et al. (1981).

The model is a classic, static, spatial equilibrium model as described by Takayama and Judge (1971). The United States is divided into a number of coal supply regions where the marginal cost of producing a variety of coals is characterized by upward-sloping supply curves, which are approximated by step functions.[15] The coal supply curves are generated by the U.S. Department of Energy (DOE) by using a process analysis model developed for the DOE and based on geologic information and engineering estimates of mining costs. A rail and barge transport network connects these coal supply regions to a set of demand regions. Demand for coal for electric power generation is endogenous within the model and is driven by inelastic demands for electricity. It is the electric power part of the model that involves an activity analysis—the model chooses from a variety of generating technologies and fuels to satisfy electricity demand at least cost. Coal demand from sectors other than electric power is exogenously specified. Thus there are price-sensitive derived demands for coal from specific supply regions even though exogenously specified demands are inelastic. Of course, much of the price sensitivity of demand is due to the locational detail of the model which permits a wide variety of pairings of producers and consumers of coal based on relative coal prices. A market equilibrium is computed by finding the least-cost way of satisfying the exogenous demands. Thus, coal production and prices at the regional level are determined endogenously. The model has been used over the past few years for a number of analyses of western coal issues and appears to give results consistent with other current coal market models (see Wolak et al. 1981).

Model Logic

For convenience in understanding the following model description, table A1 presents a summary of indices, endogenous variables, and inputs used in the model. For clarity of exposition, some details of the model have been omitted here relative to the complete documentation in Wolak et al. (1981). The model is solved as a linear program using the MINOS software developed by Murtaugh and Saunders (1981). The linear program has approximately 4,000 constraints and 25,000 variables and is solved on a CRAY I computer.

Objective Function

The objective function is a straightforward expression of costs. Because exogenous demand is inelastic, the object is to minimize total costs. Minimize

$$\underbrace{\sum_i\sum_k\sum_p C_{ikp}cs_{ikp}}_{\text{coal mining}} + \underbrace{\sum_i\sum_k\sum_{k'} W_{ikk'}cw_{ikk'}}_{\text{coal washing}} + \underbrace{\sum_i\sum_j\sum_k T_{ijk}cct_{ijk}}_{\text{coal transport}}$$
$$+ \underbrace{\sum_j\sum_{j'} R_{jj'}ct_{jj'}}_{\text{electricity transport}} + \underbrace{\sum_j\sum_{l=1}^{19}\sum_m E_{jlm}ce_{jlm}}_{\text{generation costs}} . \quad \text{(A1)}$$

[15] In fact, the coal supply curves are functions of production rates rather than cumulative production. It is assumed that in order to produce at a certain rate, enough reserves must be set aside to correspond to the life of mining capital. Because this analysis is restricted to the medium term (~ 10 years), this distinction should not cause problems.

TABLE A1

INDICES, VARIABLES, AND FUNCTIONS

Indices:

$i = 1, \ldots, 30.$	Coal supply regions.
$j = 1, \ldots, 57.$	Coal demand regions.
$k = 1, \ldots, 40.$	Coal types.
$l = 1, \ldots, 25.$	Energy production sectors ($l = 1, \ldots, 7$: coal-fired electricity; $l = 8, \ldots, 19$: non-coal-fired electricity; $l = 20, \ldots, 25$: other coal demand).
$p = 1, \ldots, 40.$	Steps of coal supply curve.
$m = 1, \ldots, 4.$	Steps of load duration curve.

Endogenous variables (all ≥ 0):*

C_{ikp}	Amount of k-type coal mined from the pth step of the coal supply curve in coal supply region i (10^6 tons).
$W_{ikk'}$	Amount of k-type coal washed to k'-type coal in coal supply region i (10^6 tons of k coal).
D_{jlk}	Amount of k-type coal consumed by sector l in region j (10^{12} Btu).
T_{ijk}	Amount of k-type coal transported from region i to region j (10^6 tons).
$R_{jj'}$	Amount of electricity exported from region j to region j' (10^9 kWh).
E_{jl}	Electrical energy produced by sector l in region j (10^9 kWh).
CC_{jl}	Capacity of sector l in region j used to produce electricity (10^6 kW).

Exogenous variables (inputs):*

α_{ik}	Heat content of type k coal produced in coal region i (10^6 Btu/ton).
β_{jl}	Fraction of incoming sulfur emitted from burning coal in sector l in region j.
BC_{jl}	Bound on capacity in sector l in region j (10^9 kWh).
ce_{jlm}	Variable cost of operating type l plants in region j in mode m excluding coal (mills/kWh).
$ct_{jj'}$	Cost of electricity transmission from region j to j' (mills/kWh).
cct_{ij}	Cost of coal transport from area i to region j ($/ton).
CM_{ikp}	Maximum coal available from pth step of k-coal supply curve in region i (10^6 tons).
cs_{ikp}	Cost of mining k coal from the pth step of the coal supply curve in region i ($/ton).
$cw_{ikk'}$	Cost of washing k-type coal to type k' coal in area i ($/ton of k coal).
$\epsilon_{jj'}$	Efficiency of electricity transmission from regions j to j' (fraction).
$\eta_{ikk'}$	Weight efficiency for washing k-type coal to k'-type coal in region i (fraction).
h_{jlm}	Heat rate of sector l facilities in region j, mode m (10^3 Btu/kWh).
λ_{jlm}	Availability factor for type l plants in region j, mode m electricity (fraction).
μ_{jm}	Fraction of year that mode m electricity is demanded in region j (fraction).
ρ_k	Sulfur content of type k coal (lb/10^6 Btu).
ps_{jl}	Point-source SO_2 emissions limitation for category l in region j (lb/10^6 Btu).
RM_j	Reserve margin in region j (fraction).
TDE_{jm}	Total demand for mode m electricity in region j (10^9 kWh).
TDN_{jl}	Total demand for nonutility coal by sector l, region j (10^{12} Btu).
θ_j	Average power demand $\div$ peak power demand in region j.

* On an annual basis, as appropriate.

Coal-Mass-Balance Constraints

Two constraints are obvious. Shipments of coal out of a supply region should be no more than coal production in that region, taking account of coal-washing (beneficiation) activities; that is,

$$\sum_j T_{ijk} \leqslant \sum_p C_{ikp} + \sum_{k'} \eta_{ik'k} W_{ik'k} - \sum_{k'} W_{ikk'}; \tag{A2}$$

and the tonnage of coal arriving at a demand region should be compatible with the heat content of that coal:

$$\sum_l D_{jlk} \leqslant \sum_i \alpha_{ik} T_{ijk}. \tag{A3}$$

Because coal supply curves are represented by multiple step functions, a bound must be placed on coal use from each step:

$$C_{ikp} \leqslant CM_{ikp}. \tag{A4}$$

Energy Demand Constraints

Because exogenous demand is inelastic, demand in quantity terms must be satisfied. Three types of demand must be satisfied. Demand for coal for other than electricity generation must be satisfied (for $l = 20, \ldots, 25$) in each demand region:

$$TDN_{jl} \leqslant \sum_k D_{jlk}. \tag{A5}$$

For electricity demand, we must satisfy both an energy demand constraint and a reserve margin constraint. Energy demand is categorized by mode (e.g., peak or baseload power):

$$\sum_{l=1}^{19} E_{jlm} + \sum_{j'} (R_{j'jm} E_{j'j} - R_{jj'm}) \geqslant TDE_{jm}. \tag{A6}$$

Reserve margins must also be maintained:

$$\sum_{l=1}^{19} CC_{jl} \geqslant (1 + RM_j) \sum_m \frac{TDE_{jm}}{8.76\theta_j}. \tag{A7}$$

Production-Possibility Constraints

Several constraints are necessary to assure that the electricity-generation sector operates in a physically or legally feasible manner. One constraint is that electricity generated not exceed utilized generating capacity (for $l = 1, \ldots, 19$):

$$\sum_m \frac{E_{jlm}}{8.76\lambda_{jlm}\mu_{jm}} \leqslant CC_{jl} \leqslant BC_{jl}. \tag{A8}$$

Another constraint is that the energy of coal used must be sufficient for electric energy generation in each coal demand region:

$$\sum_k D_{jlk} \geqslant \sum_m h_{jlm} E_{jlm}. \tag{A9}$$

An air pollution regulation constraint requires that point-source SO_2 emission standards be satisfied (for $l = 1, \ldots, 7$):

$$\sum_k (2\rho_k \beta_{jl} - ps_{jl}) D_{jlk} \leq 0. \tag{A10}$$

Data Sources

A wide variety of data sources have been used for the model. These are documented fully in Wolak et al. (1981). Costs of generating electricity and equipment-operating characteristics are adapted from DOE and Electric Power Research Institute studies. Inventories of electricity-generating equipment are taken from the annual reports of the regional electricity reliability councils. Expectations developed by the DOE of nuclear capacity are used as bounds on new nuclear capacity. Transport costs and SO_2 emission limits are those used in the original DOE National Coal Model. Electricity demand is taken from an econometric model of state demand developed at Oak Ridge National Laboratory. Nonutility coal demand is extrapolated from current use. Electric power load-duration curves are as forecast by utilities. Coal supply curves are those developed by DOE for their 1979 report to Congress. Coal-washing data are from the U.S. Bureau of Mines.

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