Two-Settlement Locational Marginal Pricing for Distribution Networks

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***Abstract*—Advances in distributed energy resource technologies, including software embedded in energy consuming devices, can enhance the price-sensitivity of electric loads. We present a two-settlement (day-ahead and real-time) locational pricing mechanism that can take advantage of this demand flexibility to improve efficiency of operation and investment in distribution networks. We identify two key differences between locational pricing of the transmission network and locational pricing of the distribution network that our two-settlement pricing mechanism addresses. We then provide an illustrative example to demonstrate how our distribution network pricing mechanism provides incentives for improved efficiency of distribution network operation.**

***Index Terms*—distribution network pricing, locational marginal pricing, electricity market design, DLMP**

I. Introduction

Historically, the demand-side of electricity systems has been considered to be non-price sensitive in the short-run, i.e. electricity consumers are assumed to not have the ability to respond in their consumption to prices that change by the hour. This led to a paradigm under which electricity consumers pay a volumetric tariff and a system operator seeks to serve demand at minimum cost given an available fleet of generators. However, technological development in distributed energy resources (DERs) and increasing availability of software embedded in energy consuming devices has greatly reduced barriers to flexibility of demand that can be a valuable resource to balance the output of renewable generation.

Increased DER deployment and electrification of transportation and other energy services is leading to new challenges to distribution networks. Capital investment requirements for distribution networks is large and growing. The EIA reports that spending by major utilities on distribution systems was $57.4 billion in 2019, 64% higher than was spent in 2000 in real terms [1].

Enabling the widespread electrification of transportation and space heating necessary for a low carbon energy sector with acceptable reliability of electricity delivery will require large scale upgrades to current distribution networks. With flexible final demand, upgrades could only be undertaken where such upgrades cannot be avoided through more efficient use, reducing the need for distribution network upgrades. Locational pricing in distribution networks can achieve more efficient utilization of distribution network capacity by making use of the demand-side flexibility in controllable devices. An appropriate dynamic locational price signal can achieve consumption schedules and operation of DERs that avoid the need for distribution network expansions.

At the transmission level, multi-settlement wholesale electricity markets based on locational marginal prices (LMPs) common in North American restructured markets have been successful in achieving cost-efficient operation of generator fleets relative to previous market designs [2]. Locational pricing leads to a market design that respects deliverability constraints of the physical system and provides appropriate incentives to market participants for short-term operation and long-term investment decisions.

For the operation of transmission networks, a multi-settlement structure can incentivize generation unit owners and loads to indicate their true willingness to supply and consume in the day-ahead market.[1] Under a multi-settlement market design, a day-ahead (DA) market is cleared simultaneously for all 24 hours of the following day, while a real-time (RT) imbalance market is cleared each hour (or sub-hourly) during the operating day. Methods for pricing the transmission network and other operating constraints are well-developed at the transmission level, where a linear approximation of real power flow is used to ensure that the market clearing solution is physically feasible.

Locational pricing of electricity in distribution networks, or distribution locational marginal prices (DLMPs), has the potential to provide similar incentives for efficient operation of the distribution network. However, there exist important differences between transmission networks and distribution networks that require different solutions for pricing in these networks. First, distribution networks are generally radial (a single path exists from the transmission substation to a load node) or weakly meshed. In transmission networks where there exist many paths between two nodes, power flows according to the laws of physics rather than directly along

[1] Jha and Wolak [3] provide empirical evidence that a multi-settlement wholesale electricity market with virtual convergence bidders can provide strong incentives for this behavior.

the shortest path. Where transmission constraints would be violated under least-cost solution, generators can increase and decrease output to resolve constraints. Here, the constraints lead to diverse pricing across nodes, depending on shift factors from nodes to constraints. In radial networks, there exists only a single path for power to flow. Thus, a question arises about whether meaningful prices will emerge from a market solution. Second, in low-voltage distribution networks, the assumptions that allow for the linear approximation used for computing locational prices at the transmission level are not appropriate, in part due to higher R/X ratios in low voltage systems. In distribution networks, real power losses are greater as a proportion of power transmitted and voltage differences across nodes cannot be neglected.

A variety of methods for distribution network market design and computing DLMPs have been proposed in the literature, however, certain challenges remain in implementation [4]–[6]. Papavasiliou [7] provides a physical intuition of DLMPs and illustrates the differences between DLMPs and transmission level LMPs. Wang et al. [8] provide a broad overview of research on locational pricing in distribution networks, desirable properties of DLMPs, and the role that DLMPs can play in a future electricity system with active distribution networks.

In this paper, we illustrate two major issues in distribution network pricing. First, we suggest that two-settlement (DA and RT) pricing is critical for providing appropriate signals to consumers to achieve efficient consumption schedules. A two-settlement pricing mechanism should be designed such that it: (i) provides incentives for consumers to bid their true willingness to buy energy and expected consumption schedule in a day-ahead market, and (ii) provides appropriate incentives for consumers to adhere to the schedule cleared in the DA market, except for response to unexpected RT conditions.

We hypothesize that much more demand flexibility can be achieved by scheduling in a DA market. Many loads may have limited ability to respond in RT on very short time scales to reduce or shift demand, where planning one day in advance can achieve much greater flexibility. Moreover, with demand schedules created in the DA market, the system operator is able to plan commitment and dispatch for the operating day.

A second key point in this paper is that RT pricing in distribution networks is different from that in transmission networks. In contrast to the wholesale market where controllable generators submit offers to increase and decrease their output to the independent system operator (ISO) and respond to dispatch instructions issued by the ISO, the majority of loads in a distribution network are unlikely to be controllable. In addition, physical constraints may be “soft” rather than “hard” constraints with temporary overloading permissible in RT operation. We propose a general framework to compute RT prices that can provide appropriate incentives to consumers in the absence of RT bids and allows for soft network constraints.

The remainder of this paper proceeds as follows. In Section II we define the market clearing problem and propose a DLMP pricing mechanism for DA and RT markets. In Section III we provide a simple example to illustrate how the pricing mechanism provides appropriate incentives to consumers. Finally, in Section IV we provide concluding remarks and describe areas for future research.

II. The Market Clearing Problem

In the following subsections we describe two-settlement (DA and RT) price formation and market clearing for a distribution network. In subsection II-A we begin by specifying the general network constrained surplus maximization problem. In subsection II-B we introduce assumptions to allow for a linear relaxation of the general formulation and a decomposition of locational DLMPs for DA pricing. Finally, in subsection II-C we propose an RT pricing mechanism that provides appropriate incentives for market participants to adhere to the schedule cleared in the DA market.

A. A General Market Clearing Problem

The general surplus maximizing problem in a distribution network with $N+1$ nodes for a given settlement interval is given below in (1a)-(1h). Here we will assume a single phase representation of a balanced three phase system but the problem readily generalizes to the three phase unbalanced case.

$$\min_{\boldsymbol{P^s},\boldsymbol{P^d},\boldsymbol{Q}} \quad F(\boldsymbol{P^s},\boldsymbol{P^d},\boldsymbol{Q}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{i=0}^{N} P_i - L^p(\boldsymbol{P},\boldsymbol{Q};\mathcal{S}) = 0 \tag{1b}$$

$$\sum_{i=0}^{N} Q_i - L^q(\boldsymbol{P},\boldsymbol{Q};\mathcal{S}) = 0 \tag{1c}$$

$$-K_l \leq H_l(\boldsymbol{P},\boldsymbol{Q};\mathcal{S}) \leq K_l \quad \forall l \in \mathcal{U} \tag{1d}$$

$$\underline{V} \leq V_i(\boldsymbol{P},\boldsymbol{Q};\mathcal{S}) \leq \overline{V} \quad \forall i \in \mathcal{N} \tag{1e}$$

$$\underline{\boldsymbol{P}}^{\boldsymbol{s}} \leq \boldsymbol{P^s} \leq \overline{\boldsymbol{P}}^{\boldsymbol{s}} \tag{1f}$$

$$\underline{\boldsymbol{P}}^{\boldsymbol{d}} \leq \boldsymbol{P^d} \leq \overline{\boldsymbol{P}}^{\boldsymbol{d}} \tag{1g}$$

$$\underline{\boldsymbol{Q}} \leq \boldsymbol{Q} \leq \overline{\boldsymbol{Q}} \tag{1h}$$

Here, (1a) is the maximization of consumer and producer surplus, where $F(\cdot)$ is defined as the negative of total surplus. The decision variables are the elements of $\boldsymbol{P^s} \in \mathbb{R}^{N+1}$, $\boldsymbol{P^d} \in \mathbb{R}^{N+1}$, and $\boldsymbol{Q} \in \mathbb{R}^{N+1}$ which are vectors of real power supplied, real power demand, and net supplied reactive power, respectively, at each bus $i : (0, 1, 2, ..., N)$ in the set of all buses $\mathcal{N}$. Here, $i = 0$ corresponds to the reference bus which we will define as the transmission substation. Let $\boldsymbol{P} \in \mathbb{R}^{N+1}$ denote net real power injections where $P_i = P_i^s - P_i^d$. Equations (1b) and (1c) correspond to real and reactive power balance, respectively. $L^p(\cdot)$ and $L^q(\cdot)$ represent real and reactive power system losses, and $\mathcal{S}$ represents the set of network parameters. Constraint (1d) is the branch flow limit. $H_l(\cdot)$ is apparent power flow (VA) on line $l \in \mathcal{U}$ where $\mathcal{U}$ is the set of all lines in the network. K_l is the maximum allowable flow (VA) on line l. Constraint (1e) is a

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nodal voltage magnitude constraint where $V_i(\cdot)$ is the voltage magnitude (p.u.) at bus i and $\underline{V}$ and $\overline{V}$ are the upper and lower voltage magnitude limits. Equations (1f), (1g), and (1h) are maximum and minimum constraints on the vectors $\boldsymbol{P^s}$, $\boldsymbol{P^d}$, and $\boldsymbol{Q}$.

B. Day Ahead Market Clearing for a Distribution Network

In this section, we impose three assumptions upon the the general formulation of the last section such that the market clearing problem can be solved and locational prices computed. Furthermore, let us suppose that we seek a solution in a neighborhood around an expected operating point. In this setting, let us assume (i) constant power factor for each nodal net real power injection,[2] (ii) bids and offers to buy and sell energy are provided in blockwise price and quantity pairs, and (iii) $L(\cdot)$, $H_l(\cdot)$, and $V_i(\cdot)$ can be approximated by linear functions of net real power injections at a constant power factor at each bus in the system, around a certain operating point. Previous work has demonstrated the use of linear sensitivities around a specified operating point for pricing in low voltage networks [9].

From assumption (i) reactive power is no longer a decision variable in the problem and we drop constraints (1c) and (1h) from the problem of the previous section. Assumption (ii) allows us to express $F(\cdot)$ as a linear function of blockwise quantity offers ($\boldsymbol{p^s} \in \mathbb{R}^M$) and bids ($\boldsymbol{p^d} \in \mathbb{R}^J$) with associated prices $\boldsymbol{c} \in \mathbb{R}^M$ and $\boldsymbol{b} \in \mathbb{R}^J$, respectively. From assumption (iii) we can express these functions with linear terms in $\boldsymbol{P}$:

$$\min_{\boldsymbol{p^s},\boldsymbol{p^d}} \quad F(\boldsymbol{p^s},\boldsymbol{p^d}) = \boldsymbol{c}^T\boldsymbol{p^s} - \boldsymbol{b}^T\boldsymbol{p^d} \tag{2a}$$

$$\text{s.t.} \quad \sum_{i=0}^{N} P_i - L_0 - \sum_{i=1}^{N} L_i P_i = 0 \quad \longleftrightarrow \quad \lambda \tag{2b}$$

$$-K_l \le H_{l0} + \sum_{i=1}^{N} H_{li} P_i \le K_l \quad \forall l \in U \quad \longleftrightarrow \quad \boldsymbol{\mu}^{min}, \boldsymbol{\mu}^{max} \tag{2c}$$

$$\underline{V} \le V_{j0} + \sum_{i=1}^{N} V_{ji} P_i \le \overline{V} \quad \forall j \in N \quad \longleftrightarrow \quad \boldsymbol{\gamma}^{min}, \boldsymbol{\gamma}^{max} \tag{2d}$$

$$0 \le \boldsymbol{p^s} \le \overline{\boldsymbol{p}}^{\boldsymbol{s}} \tag{2e}$$

$$0 \le \boldsymbol{p^d} \le \overline{\boldsymbol{p}}^{\boldsymbol{d}} \tag{2f}$$

The DLMP is defined as the cost to serve an additional unit of load at node i. We can express a decomposition of the DLMP at node i as:

$$\text{DLMP}_i = \lambda_{\text{Sub}} + \sum_{l=1}^{U} H_{li}\mu_l - \lambda_{\text{Sub}} L_i - \sum_{j=1}^{N} V_{ji}\gamma_j \tag{3}$$

where $\mu_l = \mu_l^{max} - \mu_l^{min}$ and $\gamma_j = \gamma_j^{max} - \gamma_j^{min}$. Here, the first term ($\lambda_{\text{Sub}}$) represents the DA LMP at the transmission substation and serves as the energy component at the distribution network level. The next three terms are the congestion, loss, and voltage components, respectively. The decomposition in (3) illustrates that the price at location i depends on the marginal impact of consumption or production on binding constraints and losses.[3]

[2] This assumption implies that reactive power can be expressed as $Q_i \approx \alpha_i P_i$ for some constant α_i.

[3] For a detailed treatment of LMP decomposition see [10].

C. Locational Pricing in the Real-Time Market

We seek to define a pricing mechanism that provides appropriate signals to consumers. In RT consumers may consume more or less than the DA schedule, and the pricing mechanism should provide incentives for loads to adhere to the DA schedule unless they are responding to other conditions (for example, less renewable generation in RT than expected). Moreover, we seek to allow for distribution networks elements operated with "soft" constraints. For example, transformers may be loaded above a thermal limit for short amounts of time, but a price signal should indicate the high loading of the constraint providing a financial penalty to consumers that consume above their schedule and incentive to consumers that have RT flexibility to resolve the constraint. We seek to provide a price signal in this context that prices constraints based on RT loading, and DLMPs that reflect the marginal effect that a load has on highly loaded constraints. For example, at times loading may include planned loading beyond nameplate rating, long-term emergency loading, or short-term emergency loading. In a distribution network it may the case that we seek to have some positive constraint price when loading gets near a nameplate rating with a high price as that value is exceeded.

Consider the DLMP decomposition for RT:

$$\text{DLMP}_i^{rt} = \lambda_{\text{Sub}}^{rt} + \sum_{l=1}^{U} H_{li}\mu_l^{rt} - \lambda_{\text{Sub}}^{rt} L_i - \sum_{j=1}^{N} V_{ji}\gamma_j^{rt} \tag{4}$$

where the rt superscript indicates RT values. As in (3), $\lambda_{\text{Sub}}^{rt}$ indicates the LMP at the transmission substation. When RT net injections are sufficiently close to the neighborhood of the DA solution, the only unknown values in (4) are the constraint shadow prices, μ_l^{rt} and γ_j^{rt}.

Suppose that the pricing of a distribution network constraint can be determined by some function $g_l(\omega) = \mu_l^{rt}$ of the real-time loading of constraint l relative to the day-ahead solution. Let ω represent RT loading of a conductive element as a proportion of the thermal limit, note that DA loading equals one for a binding constraint. To provide appropriate incentives, we seek a function $g_l(\omega)$ with certain properties. First, if net injections and substation price remains unchanged from DA, pricing in RT should remain unchanged from DA. If loading changes such that a constraint is no longer highly loaded price of that constraint to should be reduced or go to zero in RT. Finally, if a constraint is more highly loaded than in the DA solution, price in RT should increase relative to DA. This function could take many forms (for example, based on degradation schedules of equipment overloading) and this is an

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area of future work. For the current example we will assume the simplest form that satisfies the above properties:

$$\mu_l^{rt} = g_l(\omega) = \begin{cases} 0 & \text{for } \omega < 1 - \eta \\ \mu_l^{da} & \text{for } 1 - \eta \leq \omega < 1 + \eta \\ \Theta_l & \omega \geq 1 + \eta \end{cases} \quad (5)$$

where Θ_l represents an administratively set thermal overload price for element l and η is a tolerance factor. The key question is whether $g_l(\omega)$ functions can be defined such that they provide the correct incentives to market participants under a broad range of conditions.[4]

III. An Illustrative Example

In this section we construct a simple 24-hour example to illustrate how the proposed pricing mechanism provides appropriate incentives to market participants. For this simulation we use a modified version of the IEEE 13 Test Network [11]. We implement the simplifications defined in [12] and in addition remove the tap changing transformer while capacitor banks, line geometry, and loads were modified to represent a balanced three-phase system. Expected load shapes were based on four-week hourly averages of aggregated and anonymized loads from utility territory in California. We construct blockwise price sensitive DA bids for each load approximating an assumed linear demand function. To estimate linear sensitivity factors, we draw loads with randomized perturbations drawn from an assumed distribution around the expected operating point (5000 draws each hour) and solve power flow using GridLab-D software [13]. We then use ordinary least squares to estimate the parameters of the linear approximations of $L(\cdot)$, $H(\cdot)$, and $V(\cdot)$.

We consider a simple hypothetical case where the RT and DA LMPs are equal in every hour at the transmission substation to illustrate the effect of a binding constraint in the distribution network. In this example, DA price is high in the late afternoon when solar generation comes off-line. Loads then clear quantities in the market that are lower in during the high price periods relative to consumption at an average price. The DA market is a financially binding forward market, and we seek illustrate how the proposed pricing mechanism can provide incentive to loads to adhere to this schedule in the physical RT market.

Suppose there is a binding distribution network constraint in the network in the DA solution during hours 10 through 12 and Node 675 has a non-zero shift factor to the congested element. Then consider three cases: i) the load consumes exactly as much at RT as it cleared in DA, ii) the load consumes more in RT than cleared in DA, and iii) the load consumes less in RT than cleared in DA. These cases and associated RT prices outcomes are pictured in Figure 1.[5] Meanwhile, assume all other loads adhere precisely to their DA schedules.

[4]Price functions are defined similarly for voltage constraints with prices defined for upper and lower voltage limits.

[5]For these examples we assume values $\Theta = 250$ and $\eta = 0.05$, and the binding constraint is line segment 632-671.

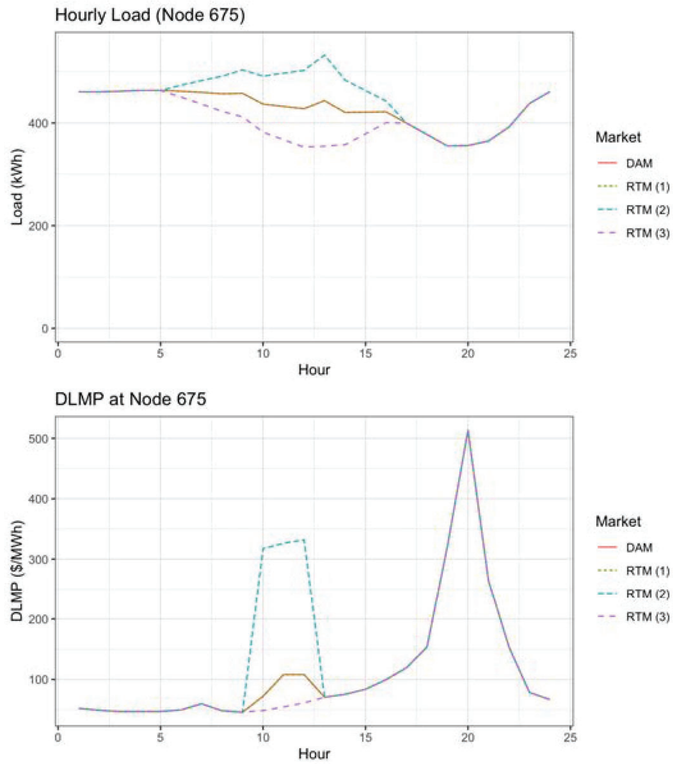

Fig. 1. Example 1. Hourly nodal load (top) and DLMP (bottom). (1): DA consumption = RT consumption, (2) DA consumption > RT consumption, and (3) DA consumption < RT consumption.

A consumer in the network seeks to maximize consumer surplus (CS) given by the following expression:

$$CS = q_{RT}(MB - \pi_{RT}) + q_{DA}(\pi_{RT} - \pi_{DA}) \quad (6)$$

where q_{RT} is the energy consumed at RT, and q_{DA} is the amount of energy cleared in the DA market. π_{DA} and π_{RT} are the prices faced by the consumer in DA and RT, respectively. In (6) the first term is surplus realized for the real-time use of energy, while the second term is a difference payment based on the outcome in the DA market. Rearranging terms yields:

$$CS = (q_{RT} - q_{DA})(MB - \pi_{RT}) + q_{DA}(MB - \pi_{DA}). \quad (7)$$

In case (i), we see in (7) that $q_{RT} = q_{DA}$ and the first term is zero leaving the DA surplus. In case (ii) the load consumes more in RT than was cleared in DA. An important property of the function $g_l(\cdot)$ is that $\mu_l^{rt} \geq \mu_l^{da}$ for a level of cleared demand which leads to $\pi_{RT} \geq \pi_{DA}^*$ where π_{DA}^* is the price that the load would face in the counterfactual where the higher consumption level was cleared in the DA market (i.e. the consumer bid higher for that quantity). When this holds, observe in (7) that $MB - \pi_{RT} \leq MB - \pi_{DA}^*$. Therefore the consumer is better off by clearing expected consumption in the DA market. When increasing consumption in hours where there is a binding constraint, the consumer pays a high price for energy, providing incentive to bid actual willingness to pay in the DA market and adhere to the cleared schedule.

In case (iii) the consumer consumes less in RT reducing the loading of the binding constraint in RT. In this case, the difference payment in (6) leads to negative surplus because of a higher price in the DA market. Again the consumer is incentivized to bid a true willingness to pay and to adhere to the schedule in RT.

In the previous example, the DA and RT prices were equal in hours during which the constraint was not binding. In actual market operation, prices will be different in the DA and RT ISO market clearing solutions, and locational pricing in the distribution network will reflect both the wholesale market conditions and effects of distribution network constraints. Through participation in the DA market, the consumer is able to purchase energy without exposure to potentially volatile RT prices. Notice that in (7) when a consumer adheres precisely to the DA schedule, the consumer's maximization problem does not depend on the RT price. Where the consumer consumes more or less in RT than cleared in the DA market the consumer will settle at the RT price for the difference. For example, if there are high prices in RT in the consumer is penalized for the over-consumption by paying the high RT price. This provides correct incentives for participation in the DA market and to provide incentives for RT flexibility when it is valuable. For instance, when less renewable generation is available in RT than was expected in the DA solution.

IV. Conclusion

In this paper we illustrate two important points about pricing in distribution networks. First, two-settlement (DA and RT) locational pricing in distribution networks will be critical to appropriately align incentives of market participants for efficient operation of price-sensitive loads. Second, pricing in distribution networks in the RT imbalance market is different than transmission level markets due to the the fact loads are generally not dispatchable in RT and that constraints may be "soft" allowing for temporary overloading. Moreover we provide a general framework to implement pricing in this context.

Additional research will be required to explore how the proposed method can be implemented in real-world systems. Data-driven approaches can be explored to estimate the linear approximations of the functions $L(\cdot)$, $H_l(\cdot)$, and $V_i(\cdot)$ in more complex networks and associated accuracy can be assessed. In addition, what constraints should be priced and functional forms for constraint pricing functions is an area that should be explored to define functions that can provide appropriate price signals under a broad range of system conditions.

In future work we plan to describe the market structure, with description of how the proposed market integrates with existing ISO markets including timing of respective market settlements. Competitive retailers could be an entity responsible for providing bids to the wholesale market for the customers they represent and providing desired hedging mechanisms to distribution network consumers. In addition, with 24-hour solving of the DA market intertemporal bidding can be implemented that creates efficient schedules for loads that can be shifted across hours (i.e. electric vehicle chargers).

Importantly, efficient consumption schedules can be achieved and decisions decentralized through an appropriate price signal. Acting in their own interest to maximize surplus, consumers behave efficiently with respect to both wholesale market conditions and distribution network constraints. Moreover, consumers are incentivized to signal their intended schedules in a DA market and then adhere to the cleared schedule in RT. If such a price signal is designed, we hypothesize that there are significant benefits that can be accrued with its implementation. In future research it will be important to quantify the benefits such a pricing mechanism in real-world settings. Locational pricing in distribution networks will be key component of electricity markets to achieve the flexibility necessary to operate high-renewable electricity systems. The structure described here provides an outline of the incentives that are important to consider in achieving efficient operation of distribution network markets.

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