Comment on "Bose-Einstein Condensation of Magnons in Cs_2CuCl_4"

In a recent paper [1], Radu *et al.* report experimental results they claim to support Bose-Einstein condensation (BEC) of magnons in Cs_2CuCl_4. It is true that an experimentally measured critical power law scaling exponent in agreement with the BEC universality class would support the realization of a BEC in magnetic systems that order as a canted antiferromagnet. It can be shown, however, that the claim of Radu *et al.* is overstated in this instance, because their determination of the critical exponent ϕ relies on a model-dependent theoretical approximation to the critical field H_{c1} for which the associated errors are neglected. We show that when these errors are included, the uncertainty in the obtained exponent is so large that the published experimental data [1] cannot be used to differentiate between contending universality classes.

A two-parameter fit to only a few data points delineating the critical ordering temperature (T_c) versus magnetic field (H) in the vicinity of the quantum critical point, to the power law

$$T_c \sim (H - H_{c1})^{1/\phi} \quad (1)$$

with both H_{c1} and the critical exponent ϕ varying has been shown to be unreliable [2,3]. An independent experimental determination of H_{c1} is therefore required to obtain an accurate estimate of ϕ. Given that neutron scattering measurements on Cs_2CuCl_4 presented in Ref. [4] have provided such a determination, yielding $H_{c1} = 8.44 \pm 0.01$ T, this would be an appropriate value to use in the fit to Eq. (1). Radu *et al.* instead use a value of $H_{c1} = 8.51$ T in their fit to Eq. (1), calculated using an approximate theoretical Hamiltonian, that is subsequently assumed to have *zero error* in their analysis. This assumption has two principal inaccuracies. The first is that the model Hamiltonian neglects higher order interactions, thereby introducing an unknown systematic error in H_{c1}. The second is that the exchange couplings used in its computation have significant experimental uncertainty, introducing a large error in H_{c1}. We obtain $H_{c1} = 8.51 \pm 0.12$ T on using the published errors in the exchange interactions [4].

Figure 1 shows fits of Eq. (1) to the experimentally measured phase boundary data points using both the experimental value of $H_{c1} = 8.44 \pm 0.01$ T of Coldea *et al.* [4] and the theoretical estimate of $H_{c1} = 8.51 \pm 0.12$ T, yielding $\phi = 2.8 \pm 0.4$ and $\phi = 1.5 \pm 0.9$ respectively, on considering the dominant contribution to the error: $\delta\phi = \frac{d\phi}{dH}|_{H_{c1}} \delta H_{c1}$. The single most important factor responsible for the very large error of ~60% in the case of the latter, as compared to the error of ~14% in the former fit, is the extreme sensitivity of the fit ϕ to the theoretical estimate of the critical field H_{c1}, as depicted graphically in the inset to Fig. 1.

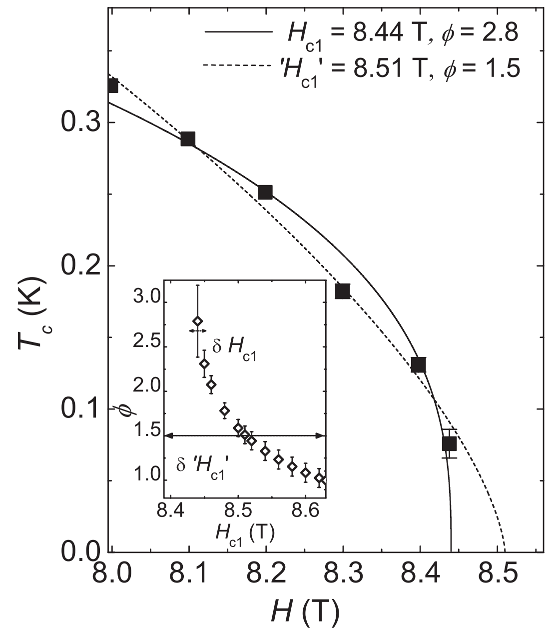

FIG. 1. Points on the ordering phase boundary from the experimental data in Ref. [1]. The solid line represents the best fit to ϕ using the experimentally measured value of $H_{c1} = 8.44$ T from Ref. [4]. The dashed line represents the best fit to ϕ using the theoretical estimate of $H_{c1} = 8.51$ T as per the analysis technique used in [1]. The inset shows the variation in the fit value of ϕ with the value of H_{c1}.

Given the substantial uncertainty in the value of ϕ that is obtained from a rigorous analysis, it is clear that the published experimental data [1] do not favor the $3d$ BEC universality class ($\phi = 1.5$) over other possibilities, including the $3d$ Ising universality class ($\phi = 2$).

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