Phase correction in double-pass quasi-phase-matched second-harmonic generation with a wedged crystal

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Compensation for dispersive elements is necessary for efficient multiple-pass and intracavity nonlinear frequency-conversion devices. We describe the use of a wedged quasi-phase-matched crystal to compensate for the phase shifts introduced by mirrors in such devices, taking advantage of the periodic variation in the relative phases of the interacting waves in a quasi-phase-matching grating. A representative double-pass second-harmonic generation experiment with a 5-cm-long periodically poled lithium niobate crystal showed the expected conversion efficiency enhancement.

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To increase the efficiency of nonlinear optical frequency-conversion devices it is common to use multiple-pass or intracavity configurations. One can obtain the greatest enhancements by resonating all the waves involved in the interaction, but such designs require precise control over the relative phase of the waves.[1] Several approaches to adjusting this phase shift, such as fine tilting of the birefringent crystal,[2] fine adjustment of the phase-matching temperature,[1] and relying on the dispersion of air over an appropriate path length,[2,3] have been proposed. Although these schemes solve the phase problem, the compromises in compactness, robustness, or efficiency that they entail prevented their widespread application.

Quasi-phase matching is widely used today in optical frequency-conversion devices.[4] Here we describe the use of a quasi-phase-matched (QPM) grating tilted with respect to the crystal boundary as a simple alternative solution to the problem of phase correction in multiple-pass and intracavity nonlinear devices. In a QPM device the nonlinear susceptibility changes sign every coherence length l_c, effectively compensating for the dispersion.[4] The relative phase of the interacting waves changes by π every l_c, so the appropriate choice of the length of the final domain permits control over the relative phase. In this Letter we analyze such a device and report the results of a representative double-pass second-harmonic-generation (SHG) experiment with a periodically poled lithium niobate (PPLN) crystal. We show that it is possible to compensate in a simple manner for the phase shift introduced by the folding mirror.

Let us consider the cw SHG process in the case of quasi-phase matching. The envelope of the second-harmonic (SH) field, E_2, evolves according to[4]

$$\frac{\mathrm{d}E_2(z)}{\mathrm{d}z} = i\gamma E_1{}^2 d(z)\exp(-i\Delta k z)\,, \tag{1}$$

where E_1 is the envelope of the fundamental field, $\gamma = \omega/n_2 c$, ω is the fundamental frequency, c is the speed of light, n_2 is the refractive index at the SH frequency, and $d(z)$ is the spatially varying nonlinear coefficient. The wave-vector mismatch is $\Delta k = k_2 - 2k_1 = \pi/l_c$, where k_1 and k_2 are the wave vectors at the fundamental and the SH, respectively, and the coherence length l_c is defined implicitly by this relation.

For the first-order QPM the nonlinear coefficient changes sign in adjacent domains, each of length l_c, leading to the nonlinear coefficient's being a square wave of period $2l_c$ and amplitude d_{eff}. We assume that the crystal boundaries do not necessarily coincide with any of the positions where $d(z)$ changes sign. Let N be the number of full-length domains in the grating and l_0 and l_{N+1} be the lengths of the zeroth and the $(N+1)$th domains, respectively; we do not limit their lengths, allowing them to be longer than l_c.

We start analyzing the double-pass SHG process by calculating the SH field after a single (forward) pass through the crystal of length L. The framework for this calculation is depicted in Fig. 1. In the undepleted-pump approximation the total fields propagating in the crystal are

$$\mathcal{E}_1{}^f(z) = E_1{}^f \exp(ik_1 z)\,, \tag{2}$$

$$\mathcal{E}_2{}^f(z) = E_2{}^f(z)\exp(ik_2 z)\,, \tag{3}$$

where $E_1{}^f$ and $E_2{}^f$ are the fundamental and the SH envelope fields, respectively, and $E_1{}^f$ is a real-valued constant such that $\mathcal{E}_1{}^f(z)$ is defined to have a zero phase at $z = 0$. The envelope of the SH field after a single pass, $E_2{}^f(Nl_c + l_{N+1})$, is obtained by integration of Eq. (1) between 0 and Nl_c under the realistic assumption that $N \gg 1$, so the SH fields generated in the zeroth and the $(N+1)$th domains are neglected:

$$E_2{}^f(Nl_c + l_{N+1}) \approx -\gamma g_0 d_Q L (E_1{}^f)^2, \tag{4}$$

where $d_Q = 2d_{\mathrm{eff}}/\pi$ is the effective QPM nonlinear

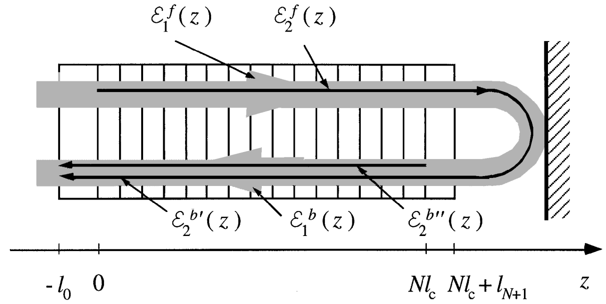

Fig. 1. Layout for analyzing the double-pass SHG process.

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coefficient and g_0 is its sign in the zeroth domain. The corresponding total SH field for the single pass, $\epsilon_2{}^f(Nl_c + l_{N+1})$, is obtained by combination of Eq. (3) and relation (4):

$$\epsilon_2{}^f(Nl_c + l_{N+1}) = -\gamma g_0 d_Q L(E_1{}^f)^2 \times \exp[ik_2(Nl_c + l_{N+1})], \qquad (5)$$

which is the usual expression for ideal QPM SHG.[4] All the phase information is lost in the corresponding expression for the SH intensity, so the single-pass SH efficiency does not depend critically on l_{N+1}. However, this phase enters crucially in the double-pass efficiency.

After the forward pass through the crystal both fundamental and SH waves are reflected back by the mirror, which introduces the phase shifts ϕ_1 and ϕ_2, respectively. For an ideal mirror $\phi_1 = \phi_2 = \pi$; for a nonideal mirror the phase shifts can have any values between 0 and π. The backward-traveling fundamental wave at the entrance of the grating ($z = Nl_c$) is phase shifted by $\phi_1 + 2k_1 l_{N+1}$ compared with the forward wave at the same position:

$$\mathcal{E}_1{}^b(Nl_c) = \mathcal{E}_1{}^f(Nl_c)\exp(i\phi_1 + 2ik_1 l_{N+1}) = E_1{}^f \exp(ik_1 Nl_c + i\phi_1 + 2ik_1 l_{N+1}), \qquad (6)$$

and then it propagates freely according to

$$\mathcal{E}_1{}^b(z) = \mathcal{E}_1{}^b(Nl_c)\exp[-ik_1(z - Nl_c)]. \qquad (7)$$

$\mathcal{E}_1{}^b(Nl_c)$ can then be treated as the envelope of the fundamental field for the backward pass.

The output double-pass SH field, $\mathcal{E}_2{}^{\mathrm{DP}}$, has two contributions,

$$\mathcal{E}_2{}^{\mathrm{DP}} = \mathcal{E}_2{}^{b'} + \mathcal{E}_2{}^{b''}, \qquad (8)$$

where $\mathcal{E}_2{}^{b'}$ is the SH wave that was generated in the forward pass, reflected back from the mirror, and then propagated freely through the crystal in the backward pass, and $\mathcal{E}_2{}^{b''}$ is the wave generated in the backward pass. The free-traveling wave $\mathcal{E}_2{}^{b'}$ can be treated similarly to the backward-propagating fundamental wave, Eqs. (6) and (7), leading to the output SH field

$$\mathcal{E}_2{}^{b'}(-l_0) = \mathcal{E}_2{}^f(Nl_c + l_{N+1})\exp(i\phi_2 + ik_2 l_{N+1}) \times \exp[-ik_2(-l_0 - Nl_c)]. \qquad (9)$$

The $\mathcal{E}_2{}^{b''}$ wave is obtained analogously to the forward SH wave, relation (4) and Eq. (5), with the substitution of g_{N+1} for g_0 and the use of $\mathcal{E}_1{}^b(Nl_c)$ from Eq. (6) as the input fundamental envelope:

$$\mathcal{E}_2{}^{b''}(-l_0) = -\gamma g_{N+1} d_Q L[\mathcal{E}_1{}^b(Nl_c)]^2 \times \exp[-ik_2(-l_0 - Nl_c)]. \qquad (10);$$

Substituting Eq. (5) into Eq. (9) and Eq. (6) into Eq. (10) and summing the results as in Eq. (8), we obtain, omitting a common phase factor,

$$\mathcal{E}_2{}^{\mathrm{DP}} = -\gamma g_0 d_Q (E_1{}^f)^2[1 - \exp(i\Delta\phi + 2i\Delta k l_{N+1})], \qquad (11)$$

where $\Delta\phi = \phi_2 - 2\phi_1$ is the relative mirror phase shift and we have used $g_{N+1} = (-1)^{N+1}\ g_0$ and $\exp(i\Delta k N l_c) = \exp(i\pi N) = (-1)^N$. The two terms in Eq. (11) are phase shifted by $\delta\phi = \Delta\phi + 2\Delta k l_{N+1} = \Delta\phi + 2\pi l_{N+1}/l_c$, which is simply the relative phase shift between SH and fundamental waves accumulating during double passing of the last domain and reflecting from the mirror. The corresponding SH intensity, $I_2{}^{\mathrm{DP}}$, is

$$I_2{}^{\mathrm{DP}} = \frac{4\omega^2}{n_2 n_1{}^2 c^3 \epsilon_0} d_Q{}^2 I_1{}^2 L^2[1 - \cos(\delta\phi)], \qquad (12)$$

which is a periodic function of l_{N+1} with a period of l_c. A change of magnitude l_c in l_{N+1} changes $\delta\phi$ by 2π, so tuning the output SH from a maximum at $\delta\phi = (2m - 1)\pi$ to a zero (at $\delta\phi = 2m\pi$) is possible independently of the magnitude of $\Delta\phi$. These cases correspond to perfect constructive and destructive interference, respectively, of the forward- and backward-generated SH waves. For $\delta\phi = (2m - 1)\pi$ the efficiency is the same as that of a crystal of length $2L$ operated at the same fundamental intensity.

This analysis can easily be extended to an N-pass device, and if the mirror phase shifts are compensated for we obtain an N^2-fold efficiency enhancement over the efficiency of a single-pass device. This scaling law, obtained for plane waves, holds true for Gaussian beams also, as long as the beam is reimaged to the same spot size in each pass. The trade-off between efficiency and bandwidth differs for single-pass and N-pass devices. For a single-pass device with length NL the efficiency for confocal focusing scales linearly with length and thus scales as NL. An N-pass device with crystal length L has the same acceptance bandwidth as the single-pass device with length NL, but the efficiency for the N-pass device scales as N^2L. This distinction is particularly important for high average power applications for which thermally induced perturbations can limit the maximum allowable harmonic power in a crystal of given length.

We can obtain the necessary control over the length of the final domain by polishing the end face of the crystal at a small angle θ to the domain boundaries and translating the crystal across the pump beam until the ideal value of l_{N+1} is reached. One possible difficulty with this method is that the wedge leads to a continuous linear phase shift across the generated SH beam, as follows from Eq. (11). If this phase shift is too large, it can reduce the output power and spoil the quality of the generated beam. To quantify these effects we calculated the SH output power P_2 and the M^2 value[5] as functions of the phase change $\Delta\Phi$ between points $\pm w_0$, with w_0 being the pump beam's $1/e$ electric-field radius. By definition, $\Delta\Phi$ is related to the wedge angle by

$$\Delta\Phi = 2\pi \frac{l_{N+1}(w_0) - l_{N+1}(-w_0)}{l_c} = 4\pi(\theta w_0/l_c). \qquad (13)$$

For $\Delta\Phi \le 1.5\pi$, both dependencies $P_2 = P_2(\Delta\Phi)$ and $M^2 = M^2(\Delta\Phi)$ can be approximated as parabolic functions:

$$P_2/P_2(\Delta\Phi = 0) = 1 - 7.0 \times 10^{-3}\Delta\Phi^2, \qquad (14)$$

$$M^2 = 1 + 7.2 \times 10^{-3}\Delta\Phi^2. \qquad (15)$$

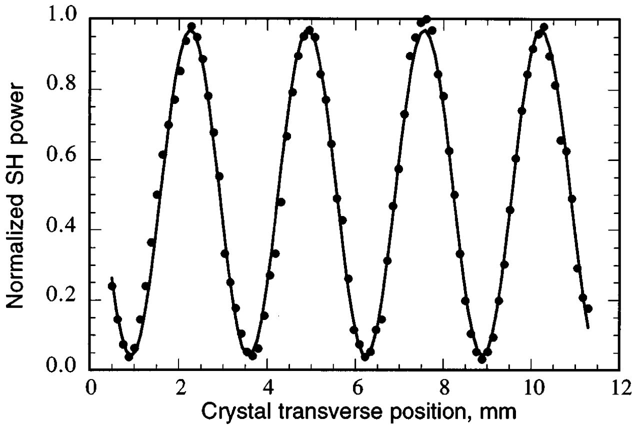

Fig. 2. Normalized output SH power as a function of the transverse position of the wedged PPLN crystal: circles, experimental data; solid curve, fit.

From Eqs. (14) and (15) we calculate that at $\Delta\Phi = 0.8\pi$ the double-pass SH power is reduced by $\approx 5\%$ below the ideal value and the M^2 value is 1.05. If this phase shift is accepted as a maximum tolerable phase shift across the beam, $\Delta\Phi_{\max}$, from Eq. (13) we obtain the restriction on the allowable wedge angle:

$$\theta < \theta_{\max} = \frac{\Delta\Phi_{\max}}{4\pi}\frac{l_c}{w_0} \approx \frac{0.2 l_c}{w_0}\,. \tag{16}$$

From relation (16) we than have an important design condition on the minimum crystal aperture, $W_{\min}$, which we need so we can compensate for an arbitrary mirror phase shift and can compensate and not exceed the maximum wedge angle:

$$W_{\min} = \frac{l_c}{\theta_{\max}} \approx 5 w_0\,. \tag{17}$$

A phase-corrected double-pass SHG experiment was set up as follows: The pump source was an Er:fiber laser that produced 3-mW cw output power at 1.54 μm. The laser beam was focused to a spot size $w_0 = 100\ \mu$m in a wedged PPLN crystal. The crystal was mounted upon a translation stage so it could be moved across the pump beam. A gold-coated mirror with a radius of curvature of ≈ 8 cm was used to reimage both the fundamental and the SH waves back through the crystal to the same spot size as in the forward pass. After the second pass, the beam splitter reflected the SH onto a silicon detector.

The 0.5-mm-thick PPLN chip with a 18.75-μm QPM period was fabricated according to the procedure described in Ref. 6. The crystal was 50 mm long by 12 mm wide and was polished so the total wedge with respect to the QPM grating lines was 42 μm over the whole width of 12 mm. The resulting wedge angle, $\theta = 3.5$ mrad, was smaller than $\theta_{\max} = 19$ mrad, as calculated with relation (16).

We measured the output SH power after a double pass through the chip as a function of the transverse position of the PPLN crystal, shown in Fig. 2. The experimental data were fitted to a functional form of Eq. (12), with $\Delta\phi$ treated as a parameter. From the fit we deduce the period of SH power variations as 2.65 mm, in excellent agreement with the value of 2.7 mm expected for the given wedge angle.

When the device was pumped with 2.27 mW of power we measured 89-nW single-pass SH power (external conversion efficiency $\eta_{SP} = 1.7\%/\mathrm{W}$) and 232 nW for the double-pass configuration, when the phase shift that was due to the mirror was completely compensated for ($\eta_{DP} = 4.5\%/\mathrm{W}$). With the Fresnel reflections ($\approx 14\%$ per surface) accounted for and with focusing looser than confocal, these figures corresponded to 90% and 89% of the theoretical values, respectively. For better conversion efficiency in a practical device, the crystal end faces must be antireflection coated and the pump beam must be confocally focused for both passes.[7] In this case for a 5-cm device we expect the single-pass conversion efficiency to be $\eta_{SP} = 4.0\%/\mathrm{W}$ and the double-pass conversion efficiency to be four times greater, $\eta_{DP} = 4\eta_{SP} = 16.0\%/\mathrm{W}$.

We have demonstrated the use of a wedged QPM crystal to compensate for the phase shifts introduced by mirrors in multiple-pass or intracavity frequency-conversion devices. A representative double-pass SHG experiment with a 5-cm-long PPLN crystal was in excellent agreement with theory. Clearly this approach should also work for other nonlinear mixing processes such as optical parametric amplification and generation. In the case of intracavity SHG, pump-resonant standing-wave optical parametric oscillation, or multiple-pass devices one needs to compensate for phase shifts that are due to two mirrors. One way to accomplish this is to put a slow wedge (say, l_c, over the whole width of the crystal) on one face and a fast wedge (several l_c) on the other face of the crystal. With an appropriate combination of wedge angles and crystal width a transverse position where both phase shifts are well compensated for can be found. Another possibility is to make the two wedges orthogonal to each other so the phase shifts can be adjusted independently of each other by orthogonal translations of the crystal.

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References

1. R. Paschotta, P. Kurz, R. Henking, S. Schiller, and J. Mlynek, Opt. Lett. **19,** 1325 (1994).
2. C. Zimmermann, R. Kallenbach, T. W. Hänsch, and J. Sandberg, Opt. Commun. **71,** 229 (1989).
3. G. C. Bhar, U. Chatterjee, and P. Datta, Appl. Phys. B **51,** 317 (1990).
4. M. M. Fejer, G. A. Magel, D. H. Jundt, and R. L. Byer, IEEE J. Quantum Electron. **28,** 2641 (1992).
5. M. W. Sasnett and T. F. Johnson, Proc. SPIE **1414,** 21 (1991).
6. L. E. Myers, R. C. Eckardt, M. M. Fejer, R. L. Byer, W. R. Bosenberg, and J. W. Pierce, J. Opt. Soc. Am. B **12,** 2102 (1995).
7. G. D. Boyd and D. A. Kleinman, J. Appl. Phys. **39,** 3597 (1968).