Pulse compression during second-harmonic generation in aperiodic quasi-phase-matching gratings

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We propose a simple means for compressing optical pulses with second-harmonic generation. Aperiodic quasi-phase-matching gratings impart a frequency-dependent phase shift on the second-harmonic pulse relative to the fundamental pulse and can be engineered to correct for arbitrary phase distortions. The mechanism is discussed, and a detailed analysis of the compression of quadratic phase (linear frequency) chirped pulses is presented. © 1997 Optical Society of America

Optical pulse compression,[1] used in many ultrafast laser systems, has become increasingly important since the development of chirped pulse amplification.[2] Several compression schemes[3] utilizing dispersive linear systems and $\chi^{(3)}$ nonlinear effects are in common use. Here we present a fundamentally different method for compressing pulses by using frequency conversion in media with aperiodic quasi-phase-matching (QPM) gratings. In particular, we discuss the generation of a transform-limited second harmonic from chirped fundamental pulses.

Second-harmonic generation (SHG) is also widely used with ultrafast laser systems. Phase matching is usually required for efficient SHG but can be achieved only for certain ranges of wavelengths, and with limited spectral bandwidths, in most nonlinear materials. QPM[4] permits the use of a material chosen for qualities other than the possibility of phase matching and can permit doubling of a wide range of wavelengths at high conversion efficiencies. A theory for ultrashort-pulse SHG in homogeneous materials was given early in the development of nonlinear optics[5,6] and was recently extended to SHG in periodic QPM structures.[7] Efficient SHG of picosecond[8] and femtosecond[9] pulses in periodic QPM structures has also been demonstrated. Aperiodic QPM gratings have been studied with a view to increasing the wavelength acceptance bandwidth in continuous-wave SHG[10]; however, the important implications of the phase response of these aperiodic QPM structures have not yet been addressed.

The compression process described in this Letter is a result of the interplay of two phenomena: group-velocity mismatch (GVM) between the fundamental and the second-harmonic pulses, intrinsic to the nonlinear material, and spatial localization of second-harmonic conversion of particular frequency components, a property of aperiodic QPM gratings. GVM causes walk-off of the fundamental and second-harmonic pulses and therefore implies that the second-harmonic field generated at each spatial position in the nonlinear medium undergoes a particular time delay relative to the fundamental pulse, as observed at the output of the material. For chirped fundamental (input) pulses, frequency components correspond to temporal slices. These temporal slices are frequency doubled at positions in the material where the grating period quasi-phase matches the interaction. By choosing the location of each spatial frequency component of the grating, one determines the time delay each temporal slice of the fundamental pulse experiences relative to the second harmonic. If the chirp rate (aperiodicity) of the QPM grating exactly matches the chirp of the input pulse, then the generated output pulse has all its spectral components coincident in time, as shown in Fig. 1. It is therefore compressed.

The equations describing plane-wave undepleted-pump SHG of ultrashort pulses with pulse lengths short enough that GVM is a significant effect but for which intrapulse group-velocity dispersion (GVD) can be neglected can be expressed as[6,7]

$$\frac{\partial A_1(z,t)}{\partial z} + \frac{1}{\nu_{g_1}}\frac{\partial A_1(z,t)}{\partial t} = 0\,, \tag{1}$$

$$\frac{\partial A_2(z,t)}{\partial z} + \frac{1}{\nu_{g_2}}\frac{\partial A_2(z,t)}{\partial t} = \Gamma \overline{d}(z) {A_1}^2(z,t)\exp(i\Delta k_0 z)\,, \tag{2}$$

with $\Gamma = -i\pi d_{\text{eff}}/\lambda_1 n_2$. $A_m(z,t)$ is the slowly varying pulse envelope, λ_m is the vacuum center wavelength, n_m is the refractive index, and ν_{g_m} is the group velocity, with $m = 1$ for the fundamental and $m = 2$ for the second-harmonic pulse. The k vector mismatch, $\Delta k_0 = 4\pi(n_2 - n_1)/\lambda_1$, in Eq. (2) is defined for refractive indices evaluated at the center angular frequency of the fundamental pulse ω_1 and the second harmonic ω_2. The nonlinear coefficient $d(z)$ is allowed to vary in the direction of propagation, z, and is normalized as $\overline{d}(z) = d(z)/d_{\text{eff}}$, where d_{eff} is the maximum

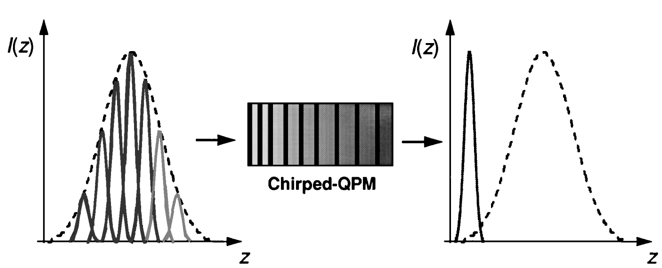

Fig. 1. Time-domain representation of pulse compression during SHG in aperiodic QPM gratings. Different shadings correspond to the optical frequency of each temporal slice of the fundamental pulse (dashed curve) and to the optical frequency for which SHG is quasi-phase matched in each spatial region of the aperiodic QPM grating.

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effective nonlinearity. $\overline{d}(z)$ is zero outside the nonlinear crystal (i.e., for $|z| > L/2$, where L is the crystal length). The boundary conditions $A_1(0, t) \equiv A_1(t)$ and $A_2(-L/2, t) \equiv 0$ apply, where $A_1(t)$ is the fundamental pulse envelope at the center of the nonlinear material. Substituting $\eta_1 \equiv t - z/\nu_{g_1}$ into Eq. (1), one confirms that $A_1(z, t) = A_1(\eta_1)$. A change of variables to $\eta_2 \equiv t - z/\nu_{g_2}$ and $\zeta \equiv z\delta$, where $\delta \equiv [\nu_{g_1}{}^{-1} - \nu_{g_2}{}^{-1}]$ is the GVM parameter, and integration of Eq. (2) yield

$$A_2(L/2, \eta_2) = \int_{-\infty}^{+\infty} D(\zeta')A_1{}^2(\eta_2 - \zeta')\mathrm{d}\zeta', \qquad (3)$$

where $D(\zeta) = (\Gamma/\delta)\overline{d}(\zeta/\delta)\exp(i\Delta k_0 \zeta/\delta)$. Equation (3) is a convolution integral, and therefore its Fourier transform can be written as

$$\widehat{A_2}(\Omega) = \hat{D}(\Omega) \cdot \widehat{A_1{}^2}(\Omega), \qquad (4)$$

where $\widehat{A_1{}^2}(\Omega)$ is the Fourier transform of the square of the fundamental pulse and $\Omega = \omega - \omega_m$ is the transform variable of η_2, with $\omega_m = \omega_1$ or $\omega_m = \omega_2$ for the fundamental or second-harmonic pulse, respectively. Returning to the position coordinate z,

$$\hat{D}(\Omega) = \Gamma \int_{-\infty}^{+\infty} \overline{d}(z')\exp[i(\Delta k_0 + \Omega\delta)z']\mathrm{d}z'. \qquad (5)$$

We recognize Eq. (5) as the usual continuous-wave SHG tuning curve implied by the spatial nonlinear coefficient distribution but with the effective phase mismatch given by $\Delta k = \Delta k_0 + \Omega\delta$, as would have been obtained with a Taylor expansion of Δk. Equations (4) and (5) indicate that the second harmonic can be interpreted as resulting from the input (fundamental) pulse squared and then acted on by a linear system with a transfer function related to the tuning curve for continuous-wave SHG. With periodic QPM gratings or homogeneous phase-matched materials, $\hat{D}(\Omega) = \Gamma L \sin(\Omega L\delta/2)/(\Omega L\delta/2)$, a real function that does not affect chirp but can cause spectral filtering, the frequency domain analog of temporal walk-off. With aperiodic QPM gratings, $\hat{D}(\Omega)$ has an engineerable phase response and can be used to cancel the phase structure of an arbitrarily chirped input pulse or to add any desired additional phase structure.

We begin analyzing the aperiodic QPM grating by assuming that it has a slowly varying spatial frequency that satisfies quasi-phase matching at the center of the nonlinear material for the center frequency of the pulse. Then

$$\overline{d}(z) = \exp[-i(\Delta k_0 z + D_{g_2}z^2 + D_{g_3}z^3 + \cdots)]\mathrm{rect}(z/L), \qquad (6)$$

where $\mathrm{rect}(x) = \{1 \text{ if } |x| \le 1/2,\ 0 \text{ if } |x| > 1/2\}$. Linearly chirped pulses can be compressed by use of only the first dispersive term. The local QPM period, Λ_{local}, is then given by $\Lambda_{\mathrm{local}}(z) = \Lambda_{\mathrm{QPM}}/(1 + \Lambda_{\mathrm{QPM}}D_{g_2}z/\pi)$, where $\Lambda_{\mathrm{QPM}} = 2\pi/\Delta k_0$. Higher-order terms can correct the higher-order phase of the input pulse. Substituting Eq. (6) into Eq. (5) and taking only $\Delta k_0, D_{g_2} \neq 0$, we find that

$$\hat{D}(\Omega) = \Gamma \int_{-\infty}^{+\infty} \mathrm{rect}(z'/L)\exp[-i(D_{g_2}z'^2 + \Omega z'\delta)]\mathrm{d}z', \qquad (7)$$

which is the well-known Fresnel integral. When the bandwidth of $\hat{D}(\Omega)$ exceeds that of the pulse spectrum, $\hat{D}(\Omega)$ can be accurately approximated over the spectrum of the pulse by the result for an infinitely long crystal,

$$\hat{D}(\Omega) = \Gamma\sqrt{\pi/D_{g_2}}\,\exp(i\Omega^2\delta^2/4D_{g_2}), \qquad (8)$$

where we have neglected a constant phase factor. From the form of Eq. (8), we find that for any input pulse the second harmonic experiences an effective GVD of $\delta^2/2D_{g_2}$ relative to the fundamental.

As an example, we discuss the effect of this transfer function on a chirped Gaussian input pulse. We create the input pulse by dispersing a transform-limited pulse with $1/e$ power half-width τ_0 and real amplitude A_1 in a linear delay line with a GVD of D_p, resulting in a temporal envelope of

$$A_1(t) = A_1 \frac{\tau_0}{\sqrt{\tau_0{}^2 - iD_p}} \exp[-t^2/2(\tau_0{}^2 - iD_p)] \qquad (9)$$

and thus a pulse length of $\tau = [\tau_0{}^2 + (D_p/\tau_0)^2]^{1/2}$. The spectral envelope of the square of the pulse is

$$\widehat{A^1}_2(\Omega) = \sqrt{\pi}\,A_1{}^2\,\frac{\tau_0}{\tau}\sqrt{\tau_0{}^2 - iD_p} \times \exp\left[-\frac{1}{4}(\tau_0{}^2 - iD_p)\Omega^2\right]\cdot \qquad (10)$$

Assuming that $L > L_{\min}$, where $L_{\min} \cong 2|\delta/\tau_0 D_{g_2}|$, so that the simplified Eq. (8) applies, substitution of Eqs. (8) and (10) into Eq. (4) and inverse Fourier transformation yield

$$A_2(t) = \Gamma\sqrt{\pi/D_{g_2}}\,A_1{}^2\,\frac{\tau_0}{\tau}\,\frac{\sqrt{\tau_0{}^2 - iD_p}}{\sqrt{\tau_0{}^2 - i2D_{\mathrm{SH}}}} \times \exp\left\{-t^2/2\left[\left(\tau_0/\sqrt{2}\right)^2 - iD_{\mathrm{SH}}\right]\right\}, \qquad (11)$$

where $D_{\mathrm{SH}} = D_p/2 + \delta^2/2D_{g_2}$. The length of this second-harmonic pulse is $\tau_{\mathrm{SH}} = [(\tau_0/\sqrt{2})^2 + (\sqrt{2}D_{\mathrm{SH}}/\tau_0)^2]^{1/2}$. If $D_{g_2} = -\delta^2/D_p \equiv D_{g_2,\mathrm{opt}}$ the chirp on the input pulse is compensated, resulting in a second-harmonic pulse, to within a constant phase factor, of

$$A_2(t) = \Gamma\left(\sqrt{\pi D_p}/\delta\right)A_1{}^2\sqrt{\tau_0/\tau}\,\exp(-t^2/\tau_0{}^2). \qquad (12)$$

Note that the minimum second-harmonic pulse length is $\tau_0/\sqrt{2}$, as is obtained with SHG of unchirped Gaussian pulses in homogeneous materials. Figure 2(a) shows fundamental and second-harmonic pulse lengths versus fundamental pulse chirp for two values of D_{g_2}.

The energy conversion efficiency for a plane-wave input is the ratio of the time integrals of the square

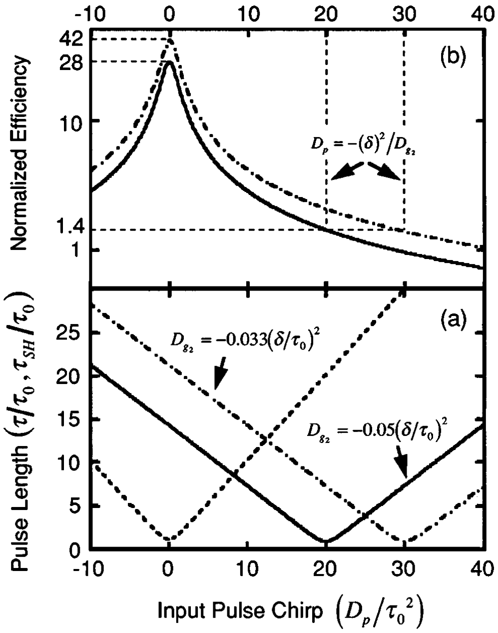

Fig. 2. Normalized input (dashed curve) and output (a) pulse lengths and (b) conversion efficiencies plotted against the chirp of the input pulse (expressed in terms of normalized delay line GVD) for SHG in a chirped QPM grating for $D_{g2} = -0.05(\delta/\tau_0)^2$ (solid curve) and for $D_{g2} = -0.033(\delta/\tau_0)^2$ (dashed–dotted curve). Pulse lengths are normalized to the minimum fundamental pulse length, τ_0. Efficiencies are normalized to that of the SHG of unchirped pulses in homogeneous materials of optimum length, $\eta_{0_{\text{PW}}}$.

magnitudes of $A_2(t)$ and $A_1(t)$, which can be written with Eq. (11) as

$$\eta_{\text{PW}} = \frac{\pi}{\sqrt{2}}\,\Gamma^2({A_1}^2\tau_0)\,\frac{1}{\tau}\,\frac{1}{|D_{g_2}|} = 1.4\,\eta_{0_{\text{PW}}}\,\frac{\delta^2}{\tau_0}\,\frac{1}{\tau}\,\frac{1}{|D_{g_2}|}\,, \tag{13a}$$

where

$$\eta_{0_{\text{PW}}} = 1.6\Gamma^2({A_1}^2\tau_0)\tau_0/\delta \tag{13b}$$

is the efficiency for SHG of unchirped pulses in a homogeneous (or periodic QPM) material of the maximum length allowed by GVM, $L_{\max} = \tau_0/\delta$, as defined in Ref. 9. The scaling of the efficiency depends on which experimental parameters are held constant. If the chirp of the input pulse, τ/τ_0, is varied while the chirp of the QPM grating, D_{g_2}, is fixed, η_{PW} scales with the peak intensity of the stretched fundamental pulse, as shown in Fig. 2(b). However, if $D_{g_2} = D_{g_2,\text{opt}}$ and $\tau/\tau_0 \gg 1$, then $\eta_{\text{PW}} = 1.4\eta_{0_{\text{PW}}}$, which no longer depends on the amount of stretching, τ/τ_0.

Most applications for pulse compression require high efficiency and therefore confocal focusing of the fundamental beam in the nonlinear material. In the near-field limit, the confocal efficiency, η_{conf}, is related to the plane-wave efficiency when the effective area of the beam is taken to be $L\lambda/2n_1$. Thus for fixed pulse energy and τ_0, ${A_1}^2$ in Eqs. (13) is inversely proportional to the crystal length. Because all other factors dependent on focusing are identical for chirped and unchirped QPM, it is convenient to express η_{conf} normalized to $\eta_{0_{\text{conf}}}$, the efficiency for SHG of unchirped pulses in a homogeneous (or periodic QPM) material of length $L_{\max}$. With definitions given above,

$$\eta_{\text{conf}}/\eta_{0_{\text{conf}}} = (\eta_{\text{PW}}/\eta_{0_{\text{PW}}})\,(L_{\max}/L_{\min}) = 0.7\tau_0/\tau\,. \tag{14}$$

Because many laser systems that generate chirped pulses are peak power limited and therefore produce pulse energies roughly in proportion to the pulse length, this τ_0/τ scaling is not a significant limitation. In particular, because SHG efficiencies of $\sim$100%/nJ are available with unchirped QPM in the near infrared in materials such as periodically poled lithium niobate[9] and multinanojoule pulse energies are readily available from several ultrafast laser systems, SHG in aperiodic QPM gratings promises to be an efficient means for implementing ultrashort pulse compression.

This high efficiency implies that the undepleted-pump approximation made in Eqs. (1) and (2) can easily be violated, resulting in complications such as efficiency saturation and cascading. The neglect of intrapulse GVD is also an approximation; intrapulse GVD may cause a shift in $D_{g_2,\text{opt}}$ but is unlikely to cause pulse distortions that cannot be corrected with optimized aperiod grating structures.

In conclusion, we have shown that aperiodic quasi-phase-matching gratings can be used for simultaneous second-harmonic generation and pulse compression. Arbitrary high-order phase correction can be implemented in a simple, compact, and efficient device.

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