Noncritical phase matching for guided-wave frequency conversion

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Inhomogeneities in waveguide dimensions are a serious problem for guided-wave frequency conversion devices. We discuss waveguide designs that make the phase matching "noncritical" with respect to small changes in dimensions. Application of noncritical phase matching results in larger fabrication tolerances, facilitating the practical realization of nonlinear devices with long interaction lengths. We experimentally demonstrate the existence of a noncritical thickness in a lithium niobate waveguide, and analyze the dimensional tolerances for second-harmonic generation in a polymer waveguide.

The confinement of electromagnetic radiation in waveguides greatly increases the efficiency of nonlinear optical interactions.[1] One of the main problems in waveguide frequency conversion is the need to maintain phasematching (PM) over the entire length of the device. The PM condition depends on the effective indices of the waveguide modes involved in the interaction, which in turn depend on the order of the mode, the wavelength used, and on the material and geometrical characteristics of the waveguide as manifested in the index profile of the waveguide structure. Thus, the inhomogeneities in the index profile along the length of the waveguide can limit the phase-matchable length for the interaction.

In this letter we address the problem of designing a waveguide for frequency conversion applications such that the PM is "noncritical" with respect to waveguide dimension, that is, the PM condition is independent, to first order, of small variations in the dimensions of the waveguide. Using second-harmonic generation (SHG) in a planar, step index waveguide as an example, we show that a value of waveguide thickness exists for which the PM is noncritical, and present expressions for the thickness tolerance in both the critical and noncritical cases. Two cases of practical importance, symmetric and strongly asymmetric waveguides, are examined, and the results are presented in normalized form. We experimentally demonstrate the existence of a noncritical thickness in proton exchanged waveguides in lithium niobate ($LiNbO_3$), and analyze the design of a polymer waveguide device. Use of noncritical PM increases the dimensional tolerances of waveguides, facilitating the fabrication of devices having long interaction lengths for high conversion efficiencies.

For SHG in a waveguide, a spatially uniform mismatch of the propagation constants, $\Delta\beta$, reduces the output power according to

$$P_{\text{out}} \propto \text{sinc}^2(\Delta\beta L/2), \tag{1}$$

where L is the length over which the interaction takes place, and $\Delta\beta$ is given by

$$\Delta\beta = \beta_{2\omega} - 2\beta_\omega = (4\pi/\lambda_\omega)(N_{2\omega} - N_\omega), \tag{2}$$

where β and N are the propagation constant and the effective index, respectively, evaluated at the frequencies indicated by the subscripts, λ_ω is the fundamental wavelength in free space, and where we have assumed that there is a single mode at the fundamental wavelength participating in the interaction. For a spatially varying $\Delta\beta$, the efficiency reduction is generally less than that given in Eq. (1), so a reasonably strict tolerance on the maximum value of $\Delta\beta$ is

$$|\Delta\beta L| \leqslant \pi. \tag{3}$$

The dependence of $\Delta\beta$ on the waveguide thickness, f, can be obtained from an expansion around the value f_0,

$$\Delta\beta(f) \simeq \Delta\beta(f_0) + \frac{\partial(\Delta\beta)}{\partial f}(\delta f) + \frac{\partial^2(\Delta\beta)}{\partial f^2}(\delta f)^2/2, \tag{4}$$

where $\delta f = f - f_0$. The interaction is phase matched when $\Delta\beta(f_0) = 0$. If $\partial\Delta\beta/\partial f \neq 0$, then we consider the PM to be "critical," and this first-order term determines, through Eq. (3), the allowable thickness variation δf. If, however, $\partial\Delta\beta/\partial f = 0$, the PM is "noncritical," and $\Delta\beta$ is a second-order function of δf.

Burns and co-workers proposed and demonstrated noncritical PM for SHG by tuning the cladding index of a planar waveguide, but the use of modal dispersion PM required the use of modes near cutoff.[2] Other methods of PM, including birefringent,[1] quasi-,[3-6] and balanced[7] PM, remove the constraints of modal dispersion PM, and enable the implementation of noncritically phase-matched waveguides for frequency conversion. Examples of theoretical dispersion curves showing noncritical operating points have appeared in Refs. 6 and 7.

To calculate tolerances for critically and noncritically phase-matched devices, and to establish design procedures for achieving noncritical PM, we must connect the derivatives appearing in Eq. (4) to the waveguide parameters. For specificity, we consider SHG in a planar waveguide, although the method is readily extended to other interactions. Assume a waveguide with substrate index n_s, cover index n_c, maximum index in the film n_f, and film thickness f. The normalized quantities describing the waveguide, following the notation of Kogelnik and Ramaswamy,[8] are given in Table I.

We begin with the first derivative term in Eq. (4). With Eq. (2) we have

$$\frac{\partial(\Delta\beta)}{\partial f} = \left(\frac{4\pi}{\lambda_\omega}\right)\left(\frac{\partial N_{2\omega}}{\partial f} - \frac{\partial N_\omega}{\partial f}\right). \tag{5}$$

TABLE I. Normalized waveguide quantities.

Normalized quantity	Symbol	Equation	
film thickness	V	$(2\pi/\lambda)f\sqrt{n_f^2-n_s^2}$	
effective index	b	$(N^2-n_s^2)/(n_f^2-n_s^2)$	
		$\approx(N-n_s)/(n_f-n_s)$	(for $n_f\approx n_s$)
asymmetry-parameter	a	$(n_s^2-n_c^2)/(n_f^2-n_s^2)$	(TE)
		$(n_f/n_c)^4(n_s^2-n_c^2)/(n_f^2-n_s^2)$	(TM)
mode size	W	$V+1/\sqrt{b}+1/\sqrt{b+a}$	

To find $\partial N/\partial f$ we use the normalized effective index b and normalized film thickness V given in Table I, and noting that $\partial V/\partial f = V/f$, we obtain

$$\frac{\partial N}{\partial f}=\left(\frac{n_f^2-n_s^2}{2Nf}\right)V\frac{\partial b}{\partial V}\approx\left(\frac{n_f-n_s}{f}\right)V\frac{\partial b}{\partial V}. \quad (6)$$

The approximate form in this and the following equations holds for weakly guiding waveguides ($n_f\approx n_s$). Defining a dispersion parameter r as the ratio of the differences of the film and substrate refractive indices at the fundamental and harmonic wavelengths

$$r\equiv\frac{n_{f,2\omega}^2-n_{s,2\omega}^2}{n_{f,\omega}^2-n_{s,\omega}^2}\approx\frac{n_{f,2\omega}-n_{s,2\omega}}{n_{f,\omega}-n_{s,\omega}}, \quad (7)$$

we can rewrite Eq. (5) with Eqs. (6) and (7) as

$$\frac{\partial(\Delta\beta)}{\partial f}=\frac{4\pi}{f\lambda_\omega}\frac{(n_{f,\omega}^2-n_{s,\omega}^2)}{2N_\omega}\left[\frac{N_\omega}{N_{2\omega}}r\left(V\frac{\partial b}{\partial V}\right)_{2\omega}-\left(V\frac{\partial b}{\partial V}\right)_\omega\right]$$
$$\approx\frac{4\pi}{f\lambda_\omega}(n_{f,\omega}-n_{s,\omega})\left[V_\omega\left(2r^{3/2}\frac{\partial b}{\partial V}\bigg|_{V=2\sqrt{r}V_\omega}-\frac{\partial b}{\partial V}\bigg|_{V=V_\omega}\right)\right], \quad (8)$$

where the additional assumption of weak dispersion ($n_{f,2\omega}+n_{s,2\omega}\approx n_{f,\omega}+n_{s,\omega}$) is necessary for the validity of the approximate form. The bracketed expression is a dimensionless quantity which describes the difference in dispersion of the harmonic and fundamental modes. We isolate this quantity by defining a sensitivity function g_c:

$$g_c\equiv\left(\frac{N_\omega}{N_{2\omega}}\right)r\left(V\frac{\partial b}{\partial V}\right)_{2\omega}-\left(V\frac{\partial b}{\partial V}\right)_\omega$$
$$\approx V_\omega\left[2r^{3/2}\left(\frac{\partial b}{\partial V}\right)\bigg|_{V=2\sqrt{r}V_\omega}-\left(\frac{\partial b}{\partial V}\right)\bigg|_{V=V_\omega}\right]. \quad (9)$$

The approximate form is a function only of r and V_ω, and thus can be written $g_c(r,V_\omega)$.

For critical PM $g_c\neq 0$, and we can use Eq. (3), (4), (8), and (9) to write an expression for the allowable fractional variation $(\delta f/f)_c$ in the thickness for a desired interaction length L

$$|(\delta f/f)_c|\leqslant\lambda_\omega/[4L(n_{f,\omega}-n_{s,\omega})|g_c|]. \quad (10)$$

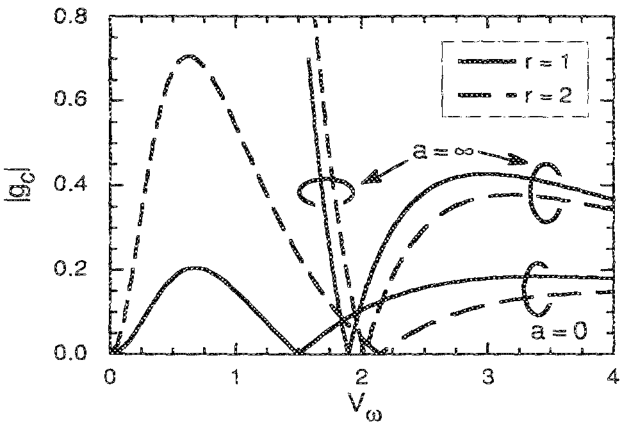

FIG. 1. Sensitivity $|g_c|$ vs normalized thickness V_ω, for dispersion parameter $r=1,2$, and for symmetric ($a=0$) and strongly asymmetric ($a=\infty$) step index profiles. For $|g_c|=0$ there is no first-order dependence of $\Delta\beta$ on thickness.

If $g_c=0$, the right-hand side of Eq. (10) diverges and the interaction is noncritically phase matched. The tolerance in this case, $(\delta f/f)_{nc}$, is obtained by an analysis similar to that leading to Eq. (10), but retaining the second-order term in Eq. (4). We find

$$|(\delta f/f)_{nc}|\leqslant\sqrt{\lambda_\omega/[2L(n_{f,\omega}-n_{s,\omega})|g_{nc}|]}, \quad (11)$$

where g_{nc} is the dimensionless sensitivity function

$$g_{nc}(r)=V_{nc}^2\left(4r^2\frac{\partial^2 b}{\partial V^2}\bigg|_{V=2\sqrt{r}V_{nc}}-\frac{\partial^2 b}{\partial V^2}\bigg|_{V=V_{nc}}\right), \quad (12)$$

where we give only the approximate form appropriate for weak guidance and weak dispersion. Note that for critical PM, the allowable variation in f scales as $1/L$, while for noncritical PM it scales as $1/\sqrt{L}$.

To this point the analysis is general, and applies to any waveguide index profile. The differences between various waveguides are manifested in the form of g_c, which depends on the index profile. As a specific example, we evaluate g_c for the asymmetric step index waveguide analyzed in Ref. 8, and assume weak guidance and weak dispersion so that the approximate forms of Eqs. (6)–(12) can be used. The analysis applies to either TE or TM modes if the appropriate form is taken for the asymmetry parameter a defined in Table I. With Eq. (16) of Ref. 8, $\partial b/\partial V=2(1-b)/W$, we can write Eq. (9) as

$$g_c(r,V_\omega)=2V_\omega\{2r^{3/2}[(1-b_{2\omega})/W_{2\omega}]-[(1-b_\omega)/W_\omega]\}. \quad (13)$$

where W is the normalized mode size given in Table I. In Fig. 1 $|g_c|$ is plotted as a function of V_ω for various values of r, and for the symmetric and strongly asymmetric cases, $a=0$ and $a=\infty$, respectively. For $V_\omega>4$, the asymptotic expression given for $b(V)$ in Eq. (19) of Ref. 8 can be used instead of solving the exact, transcendental relation, with a resulting error in g_c of less than 7% for the cases plotted here. In Fig. 2(a), the normalized "noncritical thickness"

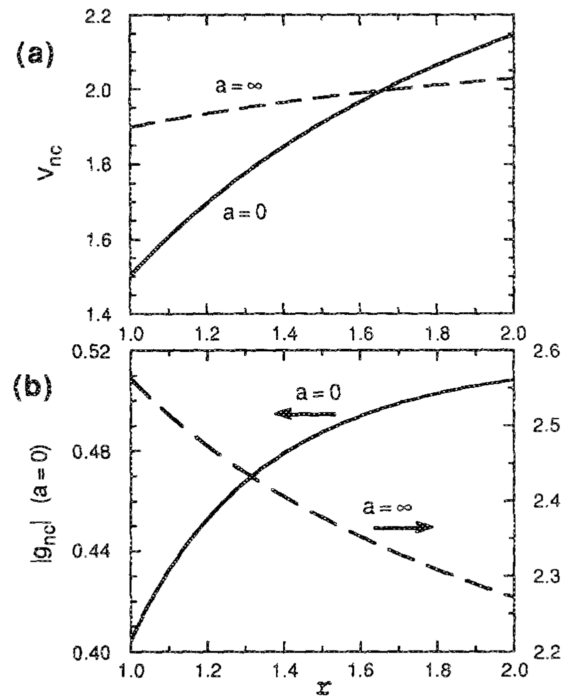

FIG. 2. (a) Normalized noncritical thickness V_{nc} and (b) sensitivity $|g_{nc}|$, vs dispersion parameter r for symmetric and strongly asymmetric step index profiles.

V_{nc} for which $g_c = 0$ is plotted as a function of r and a. To evaluate g_{nc} we require the second derivative $\partial^2 b/\partial V^2$, which is given by

$$\frac{\partial^2 b}{\partial V^2}=\left(\frac{2(1-b)}{W^2}\right)\left[\left(\frac{1-b}{W}\right)[b^{-3/2}+(b+a)^{-3/2}]-3\right]. \tag{14}$$

In Fig. 2(b) we plot the sensitivity parameter $|g_{nc}|$, obtained with Eqs. (12) and (14), as a function of r and a.

To empirically demonstrate the existence of a noncritical PM point, we determined the mismatch $\Delta\beta$ between the propagation constants of two, harmonically related wavelengths in a planar waveguide as a function of guide thickness. Substrates of Z-cut $LiNbO_3$ were exchanged in pure benzoic acid at 200 °C, producing TM waveguides with nearly step index profiles, having thicknesses in the range 0.33–0.57 μm.[9,10] The effective indices of the lowest order modes at 458 and 916 nm were determined for each sample by prism coupling. The differences $\Delta\beta$ are shown in Fig. 3, together with theoretical curves calculated using extraordinary indices at 458 and 916 nm of 2.275 and 2.165,[11] and Δn_e values at 458 and 916 nm of 0.1775 and 0.115.[12] While the experimental data are displaced from the theoretical curves, it is clear that a turning point in $\Delta\beta$ is observed close to the predicted "noncritical" thickness.

To illustrate the increase in tolerances obtained with a noncritical design, we analyze a planar, periodically poled, polymer waveguide device recently applied to the quasi-phase-matched SHG of 0.67 μm radiation[13] The waveguide is symmetric with $f = 1.3$ μm, $n_{f,2\omega} = 1.606$, $n_{s,2\omega} = 1.488$, $n_{f,\omega} = 1.576$, and $n_{s,\omega} = 1.482$.[14] Thus, $r = 1.26$

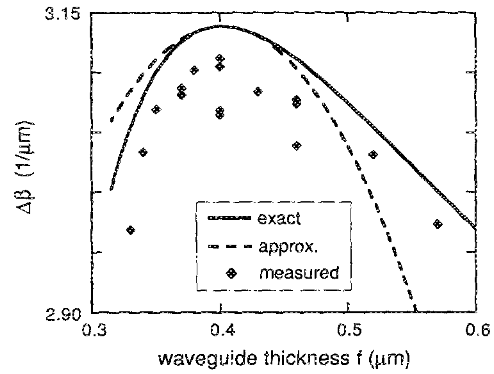

FIG. 3. Calculated and measured values of $\Delta\beta$ for proton-exchanged $LiNbO_3$ waveguides at 458 and 916 nm. The exact curve is obtained by solving the eigenvalue relations in Ref. 8, and the approximate curve results from Eq. (4).

and $V_\omega = 3.27$, so the device is critically phase matched, with $|g_c| = 0.17$ and $|(\delta f/f)_c| = 2\%$ mm. For noncritical PM, $V_{nc} = 1.75$ ($f = 0.7$ μm), $|g_{nc}| = 0.46$, and $|(\delta f/f)_{nc}| = 12\%\ \sqrt{\text{mm}}$. For an interaction length of 10 mm, the allowable thickness variations $|\delta f|$ are then 2.7 nm for the actual, critically phase-matched device, and 27 nm for the proposed, noncritically phase-matched structure.

We have described noncritical waveguide designs with no first-order dependence of phase matching on the waveguide dimensions, which should significantly increase fabrication tolerances for nonlinear guided wave devices. Results are given as a function of waveguide parameters for symmetric and strongly asymmetric step profile waveguides, but noncritical solutions can be found for other common waveguide profiles, and will be discussed in a future publication.

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