Creating anyons from photons using a nonlinear resonator lattice subject to dynamic modulation

Luqi Yuan,[1] Meng Xiao,[1] Shanshan Xu,[2] and Shanhui Fan[1]
[1]*Department of Electrical Engineering, and Ginzton Laboratory, Stanford University, Stanford, California 94305, USA*
[2]*Department of Physics, Stanford University, Stanford, California 94305, USA*

(Received 13 July 2017; published 27 October 2017)

We study a one-dimensional photonic resonator lattice with Kerr nonlinearity under the dynamic modulation. With an appropriate choice of the modulation frequency and phase, we find that this system can be used to create anyons from photons. By coupling the resonators with external waveguides, the anyon characteristics can be explored by measuring the transport property of the photons in the external waveguides.

DOI: 10.1103/PhysRevA.96.043864

Quantum particles satisfy the commutation relation [1]

$$a_j a_k - e^{if(\theta)} a_k a_j = 0,$$
$$a_j a_k^\dagger - e^{-if(\theta)} a_k^\dagger a_j = \delta_{jk}, \tag{1}$$

where the a's are the annihilation operators. For elementary particles, $f(\theta)$ in Eq. (1) can only take the values of zero or π, corresponding to bosons or fermions, respectively. The possibility of creating anyons, described in Eq. (1) by a more complex function of θ, has long fascinated many physicists, both from a fundamental perspective [1–5] and also due to potential applications in quantum information processing [6,7]. Collective excitations that behave as anyons have been constructed from electrons in the fractional quantum Hall systems [8–10], or from atoms in one-dimensional optical lattices [11–13]. Moreover, there have been several proposals on using a two-dimensional cavity array to create a fractional quantum Hall effect for photons [14–16].

In this paper we propose to construct anyons from photons in a one-dimensional array of cavities. We consider a photonic resonator lattice [17–22] with Kerr nonlinearity and moreover that is subject to dynamic refractive index modulation [23–26]. We show that, with the proper choice of the temporal modulation profile, the Hamiltonian of the system can be mapped into a one-dimensional Hamiltonian for anyons. Moreover, by having the resonator lattice couple to external waveguides (see Fig. 1), the resulting open system naturally enables the use of photon transport experiment to measure anyon properties, including a standard beam splitter experiment that can be used to determine particle statistics. Our work opens an avenue of exploring fundamental physics using photonic structures. The use of a one-dimensional cavity array is potentially simpler to implement as compared to previous works that seek to demonstrate fractional quantum Hall effects in two-dimensional cavity arrays. Moreover, the capability for achieving anyon states may point to possibilities for achieving nontrivial many-body photon states that are potentially interesting for quantum information processing. Related to, but distinct from, our work, Ref. [27] shows that the dynamics of two anyons in a one-dimensional lattice can be simulated with one photon in a two-dimensional waveguide array. But Ref. [27] does not construct anyons out of photons.

Our work is inspired by the recent works in the synthesis of anyons in optical lattices [11–13]. Consider a Hamiltonian of interacting anyons in one dimension [11]

$$H_A = -\kappa \sum_j (a_j^\dagger a_{j-1} + \text{H.c.}) + \frac{U}{2}\sum_j a_j^\dagger a_j^\dagger a_j a_j, \tag{2}$$

where κ is the coupling constant between two nearest-neighbor lattice sites and U is the on-site interaction potential. $a_j^\dagger(a_j)$ is the creation (annihilation) operator for the anyon at the jth lattice site, which satisfies the commutation relations of Eq. (1) with $f(\theta) = \theta\,\text{sgn}(j-k)$, where $\theta \in [0,2\pi)$. The anyonic Hamiltonian in Eq. (2) can be mapped to the bosonic Hamiltonian

$$H_B = -\kappa \sum_j (b_j^\dagger b_{j-1} e^{i\theta b_j^\dagger b_j} + \text{H.c.}) + \frac{U}{2}\sum_j b_j^\dagger b_j^\dagger b_j b_j, \tag{3}$$

with the bosonic creation (annihilation) operator $b^\dagger(b)$, under the generalized Jordan-Wigner transformation [28,29]

$$a_j = b_j \exp\left(i\theta \sum_{m=j+1} b_m^\dagger b_m\right). \tag{4}$$

Therefore, to create anyon one needs a bosonic system with a particle-number-dependent hopping phase that breaks mirror and time-reversal symmetry.

Reference [13] showed that the Hamiltonin in Eq. (3) can be achieved by considering a time-dependent Hamiltonian

$$H(t) = -g\sum_j (b_j^\dagger b_{j-1} + \text{H.c.}) + \frac{V}{2}\sum_j b_j^\dagger b_j^\dagger b_j b_j$$
$$+ [\Delta\omega + F(t)]\sum_j j b_j^\dagger b_j \tag{5}$$

in the weak perturbation limit $g,|U| \ll \Delta\omega$. Here g is the coupling constant, $V = U + 2\Delta\omega$ is the interaction potential, $\Delta\omega$ is the potential tilt in the lattice, and $F(t) = -d\phi(t)/dt$, where $\phi(t) = p\,\cos(\Delta\omega t) + q\,\cos(2\Delta\omega t)$. In the limit where only one- or two-particle processes are significant, and under the high frequency approximation, the Hamiltonian of Eq. (5) maps to that of Eq. (3), provided that the parameters p and q are appropriately chosen. Briefly, in Eq. (5), the time-periodic force $F(t)$ is resonant with the tilted potential difference between nearest-neighbor lattice sites. And the presence of on-site interaction results in the particle-number-dependent phase in the coupling matrix elements after a gauge transformation is carried out [13].

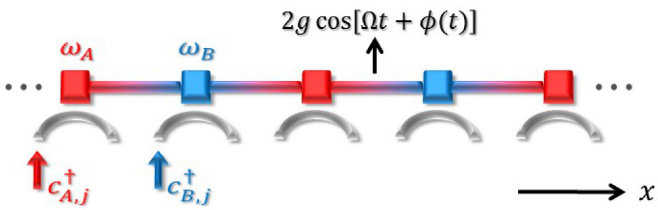

FIG. 1. Array of photonic resonators composed of alternating resonators with frequencies ω_A (red) and ω_B (blue). The coupling between nearest-neighbor resonators undergoes dynamic modulation. Each resonator is coupled with an external waveguide.

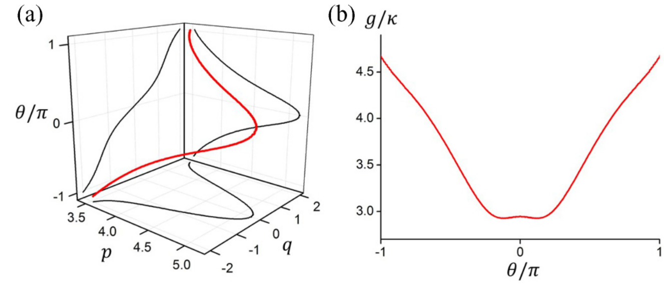

FIG. 2. (a) Trajectory that parameters p,q in the modulation phase $[\phi(t) = p\,\cos(\Delta\omega t) + q\,\cos(2\Delta\omega t)]$ obey, for the Hamiltonian (6) to map into Hamiltonian (3) in the two-photon limit, with θ in Eq. (3) varying from $-\pi$ to π. (b) The adjustment of the modulation strength g vs θ to achieve a constant κ in Eq. (3).

Building upon the previous works as outlined above, the main contributions of the present paper are the following. (1) We show that the one-dimensional Hamiltonian of Eq. (5), including its specific choice of parameters required for anyon synthesis, can be implemented in a photonic structure composed of resonators undergoing modulation. (2) Implementing Eq. (5) in a photonic resonator lattice also points to possibilities for probing the physics of anyons. In particular, coupling the resonator to an external waveguide (Fig. 1) enables one to directly conduct anyon interference experiments, and to probe anyon density distributions of both ground and excited states through a photon transport measurement.

We consider a photonic resonator lattice composed of two kinds of resonators (A and B) with frequencies ω_A and ω_B as shown in Fig. 1 [17]. The coupling between nearest-neighbor resonators undergoes dynamic modulation. Such a dynamic modulation of coupling can be implemented using refractive index modulation as discussed in Ref. [17]. Each resonator moreover has Kerr nonlinearity. Such a system is described by the Hamiltonian H_r:

$$\begin{aligned} H_r = {} & \sum_m \omega_A b^\dagger_{A,m} b_{A,m} + \sum_n \omega_B b^\dagger_{B,n} b_{B,n} \\ & - \sum_{\langle mn\rangle} 2g\ \cos[\Omega t + \phi(t)](b^\dagger_{A,m} b_{B,n} + \text{H.c.}) \\ & + \frac{V}{2}\sum_m b^\dagger_{A,m} b^\dagger_{A,m} b_{A,m} b_{A,m} + \frac{V}{2}\sum_n b^\dagger_{B,n} b^\dagger_{B,n} b_{B,n} b_{B,n}, \end{aligned} \tag{6}$$

where $b^\dagger(b)$ is the creation (annihilation) operator for the photon in the sublattice A and B. The third term in Eq. (6) describes the modulation. g, Ω, and $\phi(t)$ are the strength, the frequency, and the phase of the modulation, respectively. We assume a near-resonant modulation with $\Omega \approx \omega_A - \omega_B$. The last two terms describe the effect of Kerr nonlinearity with V characterizing the strength of the nonlinearity.

With the rotating-wave approximation and defining $\tilde{b}_m = b_{A,m}e^{i\omega_A t}$ ($\tilde{b}_n = b_{B,n}e^{i\omega_B t}$), we can transform Hamiltonian (6) to

$$\tilde{H}_r = -g\sum_j (\tilde{b}^\dagger_j \tilde{b}_{j-1} e^{i\Delta\omega t - \phi(t)} + \text{H.c.}) + \frac{V}{2}\sum_j \tilde{b}^\dagger_j \tilde{b}^\dagger_j \tilde{b}_j \tilde{b}_j, \tag{7}$$

where $\Delta\omega = (\omega_A - \omega_B) - \Omega$ is the detuning.

Hamiltonians in Eqs. (5) and (7) are equivalent after a gauge transformation $\hat{U}(t) = \exp\{i[\Delta\omega t - \phi(t)]\sum_j j\tilde{b}^\dagger_j \tilde{b}_j\}$. Since, with proper choice of parameters, Ref. [13] showed that Eq. (5) can be mapped to the anyon Hamiltonian of Eq. (3), we have shown that a nonlinear photonic resonator lattice undergoing dynamic modulation in fact can be used to construct anyons from photons. In Eq. (6), the near-resonant modulation results in the coupling between resonators at different sites in the lattice. The small detuning between the modulation frequency and the frequency difference $\omega_A - \omega_B$ results in a tilt potential along the lattice [30,31]. The additional time-dependent modulation phase $\phi(t)$ provides the time periodic force. For the rest of the paper, we chose $\Delta\omega = V/2$. In the two-photon limit, $\tilde{H}_r$ in Eq. (7) then provides an anyon Hamiltonian of Eq. (3) that is noninteracting with $U = 0$. Here choices of parameters p,q as well as g follow the trajectories in Fig. 2. θ can be tuned from zero to 2π following the trajectory of p,q in Fig. 2(a). For each θ, g is chosen following Fig. 2(b) such that κ in Eq. (3) remains constant.

The photonic resonator lattice provides a unique platform for probing the physics of anyons. By coupling the nonlinear resonator as discussed above with the external waveguide (Fig. 1), it becomes possible to demonstrate anyon physics through photon transport measurement. The cavity-waveguide system is described by the Hamiltonian:

$$\begin{aligned} H = {} & \sum_m \int_k dk\ \omega_k c^\dagger_{k,A,m} c_{k,A,m} + \sum_n \int_k dk\ \omega_k c^\dagger_{k,B,n} c_{k,B,n} \\ & + \sqrt{\frac{\gamma}{2\pi}}\sum_m \int_k dk (c^\dagger_{k,A,m} + c_{k,A,m})(b^\dagger_{A,m} + b_{A,m}) \\ & + \sqrt{\frac{\gamma}{2\pi}}\sum_n \int_k dk (c^\dagger_{k,B,n} + c_{k,B,n})(b^\dagger_{B,n} + b_{B,n}) \\ & + H_r[b_{A,m}, b_{B,n}], \end{aligned} \tag{8}$$

where $c^\dagger_k(c_k)$ is the creation (annihilation) operator for the photon in the waveguide coupled with the $m(n)$th resonator of type $A(B)$. γ is the waveguide-cavity coupling strength. The Hamiltonian of the resonator lattice $H_r[b_{A,m}, b_{B,n}]$ is given in Eq. (6). By applying the rotating-wave approximation with

$\tilde{c}_{k,m} = c_{k,A,m}e^{i\omega_A t}$ ($\tilde{c}_{k,n} = c_{k,B,n}e^{i\omega_B t}$), we rewrite Eq. (8) to

$$\tilde{H} = \sum_j \int_k dk(\omega_k - \omega_{A(B)})\tilde{c}^\dagger_{k,j}\tilde{c}_{k,j} + \sqrt{\frac{\gamma}{2\pi}}\sum_j \int_k dk(\tilde{c}^\dagger_{k,j}\tilde{b}_j + \tilde{b}^\dagger_j\tilde{c}_{k,j}) + \tilde{H}_r[\tilde{b}_j], \quad (9)$$

where $\tilde{H}_r[\tilde{b}_j]$ is given in Eq. (7) [32,33].

The photon transport properties of Eq. (9) can be described by the input-output formalism, which is a set of operator equations in the Heisenberg picture [32]:

$$\frac{d\tilde{b}_j(t)}{dt} = i[\tilde{H}_r,\tilde{b}_j(t)] - \frac{\gamma}{2}\tilde{b}_j(t) + i\sqrt{\gamma}c_{\text{in},j}(t), \quad (10)$$

$$c_{\text{out},j}(t) = c_{\text{in},j}(t) - i\sqrt{\gamma}\tilde{b}_j(t). \quad (11)$$

Here $c_{\text{in},j}$ and $c_{\text{out},j}$ are the input and output operators for waveguide photons [34].

To probe anyon property one needs at least two particles. We therefore consider a normalized input state

$$|\Phi\rangle = \iint dt_1 dt_2 \chi(t_1 - t_2)\eta\left(\frac{t_1+t_2}{2}\right)c^\dagger_{\text{in},\alpha}(t_2)c^\dagger_{\text{in},\beta}(t_1)|0\rangle, \quad (j \leqslant m), \quad (12)$$

where the normalization condition $\langle\Phi|\Phi\rangle = 1$ requires that $\iint dt\, dt'\chi^2(t')|\eta(t)|^2 = 1$. In what follows, we assume that $\chi(t)$ has a very short temporal width to describe a scenario where we simultaneously inject two photons into α and β cavities through the waveguides. In the presence of such input state, we can compute the resulting two-photon probability amplitude inside the resonator lattice

$$v_{jm}(t) \equiv \langle 0|\tilde{b}_m(t)\tilde{b}_j(t)|\Phi\rangle \quad (j \leqslant m), \quad (13)$$

which from the input-output formalism satisfies (details in the Appendix)

$$\frac{dv_{jm}(t)}{dt} = i\langle 0|[\tilde{H}_r,\tilde{b}_m(t)\tilde{b}_j(t)]|\Phi\rangle - \gamma v_{jm}(t) - \gamma\eta(t)\delta_{j\alpha}\delta_{m\beta}. \quad (14)$$

The two-photon probability amplitude in (13) can be measured by coupling the resonator lattice to additional output waveguides and measuring the two-photon correlation function for the photons in the output waveguides.

Using Eq. (14), we then propose a set of photon transport experiments to probe the anyon statistics. When two anyons are confined in a potential well, the probability distribution of the eigenstates has characteristics that depend on the phase angle θ. Previous proposal on anyon in an optical lattice has focused on ground-state characteristics [11–13]. In our case, on the other hand, one can choose the frequency detuning ϵ of the input source to selectively excite either the ground state or the excited states. As an illustration, we choose $\eta(t) = e^{-i\epsilon t}[u(t) + u(T-t) - 1]/\sqrt{T}$ in Eq. (12), where $u(t)$ is the Heaviside step function and simulate Eq. (14) to obtain steady-state density distribution $N_j = \langle\Phi|\tilde{b}^\dagger_j(t)\tilde{b}_j(t)|\Phi\rangle = \sum_{m<j}|v_{mj}(t)|^2 + \sum_{m>j}|v_{jm}(t)|^2 + 2|v_{jj}(t)|^2$. In the simulation, we consider a lattice involving 21 resonators ($j = -10,\ldots,10$), and choose parameters $\gamma = 0.002\kappa$, $\Delta\omega = 100\pi\kappa$, and $T = 5/\gamma$. To probe the ground-state distribution, we use the input state in (12) with $\alpha = -4, \beta = 4$ and $\epsilon = -3.96\kappa, -3.91\kappa, -3.90\kappa$ for $\theta = 0,\pi/2,\pi$, respectively [see Fig. 3(a)]. When $\theta = 0$, the particle density distribution has the distribution with one peak in the zeroth resonator, which is consistent with the characteristic of bosonic particles. On the other hand, when $\theta = \pi$, the particle density distribution has two peaks near the ± 5 resonators, corresponding to the characteristic of two noninteracting fermionic particles. The case of $\theta = \pi/2$ has a distribution that is between the boson and the fermion cases. The simulation results by computing Eq. (14) match with the results obtained by a direct diagonalization of the anyon Hamiltonian of Eq. (2). We note that the simulation of Eq. (14) describes an open quantum system, whereas Eq. (2) describes a closed quantum system. Therefore, we show that with proper choice of system parameters, one can use the open system to probe the properties of a closed system. Similar agreement between the photonic resonator lattice system and the anyon Hamiltonian can be seen in excited-state properties as well, as can be seen in Fig. 3(b), where as an example we selectively excite the second excited state by setting $\alpha = -6, \beta = 6$ and $\epsilon = -3.84\kappa, -3.78\kappa, -3.74\kappa$ for $\theta = 0,\pi/2,\pi$, respectively, in Eq. (12). We note that in addition to the frequency detuning a different choice of the input state distribution is required in order to efficiently excite either the ground state or a particular excited state.

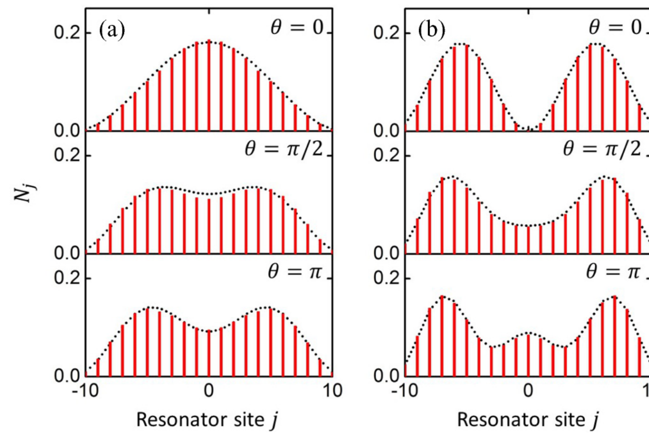

FIG. 3. (a) Ground-state and (b) the second excited-state particle density distributions N_j for $\theta = 0,\pi/2,\pi$, respectively, with an input photon pair. N_j is normalized to $\sum_j N_j = 2$. The density distributions are compared with the results obtained by a direct diagonalization of the anyon Hamiltonian of Eq. (2) (dotted curves).

Arguably the most direct experiment for observing particle statistics is the two-particle scattering experiments at a 50:50 beam splitter [35–37]. Consider two quantum particles arriving simultaneously at the beam splitter from both sides [Fig. 4(a)]. Upon scattering at the beam splitter, for bosons the two particles appear at the same side of the beam splitter. For fermions, the two particles appear at the opposite sides. For anyons, depending on the phase angle θ, the outcome smoothly interpolates between the cases of bosons and fermions. While conceptually simple, there has not been a proposal for conducting such a two-particle scattering experiment for

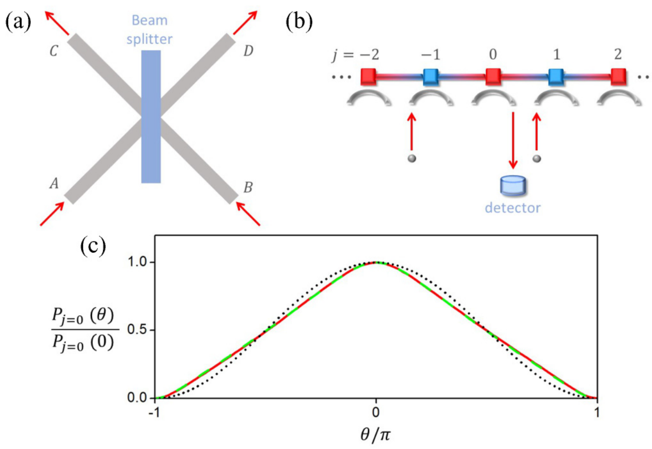

FIG. 4. (a) Beam splitter with input arms A and B as well as output arms C and D. (b) The cavity-waveguide system; two photons are incident into the first and (-1)st waveguides. The detection is performed at the zeroth waveguide. (c) Simulated $P_{j=0}$ vs θ from zero to 2π using $\tilde{H}_r$ in Eq. (14) (red solid curve), and the time-independent Hamiltonian H_B in Eq. (3) with $U = 0$ (green dashed curve). Black dotted curve corresponds to $\cos^2(\theta/2)$ as obtained from a simple approximation.

synthetic anyons, due in part to the difficulty of obtaining individual anyons in either electronic or atomic systems.

We show that the cavity-waveguide system can be used to perform the two-anyon interference experiment. For the input, we consider two photons injected into the first and (-1)st waveguides, respectively. These two waveguide channels correspond to the two input ports in the standard interferometer experiment. We consider the output photon at zeroth waveguide which emits from the zeroth photonic resonator. The zeroth waveguide channel represents one of the output ports. We define the two-photon correlation function $G_j^{(2)}(t,\tau) = \langle\Phi|c^\dagger_{\text{out},j}(t)c^\dagger_{\text{out},j}(t+\tau)c_{\text{out},j}(t+\tau)c_{\text{out},j}(t)|\Phi\rangle$. The total probability that two output photons at zeroth waveguide coincide in time is then

$$
\begin{aligned}
P_{j=0} &\equiv \int dt\, G^{(2)}_{j=0}(t,\tau=0) \\
&= \gamma^2 \int dt \langle\Phi|\tilde{b}_0^\dagger(t)\tilde{b}_0^\dagger(t)\tilde{b}_0(t)\tilde{b}_0(t)|\Phi\rangle \\
&= \gamma^2 \int dt |v_{00}(t)|^2. \qquad (15)
\end{aligned}
$$

We simulate the "beam splitter" experiment by solving Eq. (14). For the resonator lattice, we used the same parameters as in Fig. 3 except for $\gamma = 0.25\kappa$. For the input state, we use $\eta(t) = (2/\pi T^2)^{1/4}\exp(-t^2/T^2)$, where $T = 0.0425\kappa^{-1}$. We plot the simulated $P_{j=0}$ in Fig. 4(c). We compare such results to the case where we replace $\tilde{H}_r$ in Eq. (14) with the time-independent Hamiltonian H_B in Eq. (3) with $U = 0$. The two simulations show excellent agreement. Since H_B is rigorously equivalent to the anyon Hamitonian of Eq. (2), our simulation results prove that the cavity-waveguide Hamiltonian can indeed generate anyonic behavior in spite of all the approximates used to map Eq. (7) to Eq. (3).

The results in Fig. 4(c) can be qualitatively accounted for with a simple two-particle interference model. Our choice of the input state results in an excitation of the first and (-1)st resonators in the lattice:

$$
\begin{aligned}
|\Psi\rangle &= \tfrac{1}{2}(\tilde{b}^\dagger_{-1}\tilde{b}^\dagger_1|0\rangle + \tilde{b}^\dagger_1\tilde{b}^\dagger_{-1}|0\rangle) \\
&= \tfrac{1}{2}(e^{i\theta}a^\dagger_{-1}a^\dagger_1|0\rangle + a^\dagger_1 a^\dagger_{-1}|0\rangle), \qquad (16)
\end{aligned}
$$

where Eq. (4) is used to transform the bosonic to the anyonic operators. Upon time evolution, the state acquires a component in the zeroth resonator of approximately the form $\frac{1}{2}(e^{i\theta}+1)(a_0^\dagger)^2|0\rangle$. Thus $G^{(2)}_{j=0}(t,\tau=0) \approx \cos^2(\theta/2)$ at the zeroth waveguide as shown in Fig. 4(c). This simple model, where the phase factor θ appears explicitly due to anyon exchange, provides a qualitative explanation of the numerical results shown in Fig. 4(c). In particular, both the analytic model and the numerical results indicate a peak of $P_{j=0}$ at $\theta = 0$, corresponding to the boson case, and $P_{j=0} = 0$ at $\theta = \pi$, corresponding to the fermion case. Our results here indicate that one can indeed perform a two-anyon interference experiment using photons in the cavity-waveguide system.

To implement the concepts presented above experimentally, we note that the experimental feasibility of the Hamiltonian in Eq. (6), without the nonlinear term, has been discussed in detail in Ref. [17]. For our purpose here, the frequency difference between two resonators of different types can be chosen as $\omega_A - \omega_B \sim 1$ GHz. The dynamic modulation is applied with the modulation detuning $\Delta\omega \sim 100$ MHz and the modulation strength $g \sim 10$ MHz. Such a modulation strength and speed is consistent with what is achievable in experiments based on silicon electro-optic modulators [25,38,39]. We also estimate the relevant experimental parameters for the nonlinear term. In Ref. [13], in order to obtain Eq. (3) from Eq. (5), one needs to assume $|V - 2\Delta\omega| \ll \Delta\omega$; therefore, the nonlinearity parameter $V \sim 2\Delta\omega$, which means that adding an extra photon in the resonator will shift the resonant frequency of the resonator by approximately 100 MHz. Such a strength of nonlinearity can be achieved by coupling a two-level quantum system with a resonator and reaching the strong coupling regime [40–42]. The nonlinear parameter in the range 10 MHz $\sim$ 10 GHz has been demonstrated in the recent atom-cavity experiments [43–47], which is sufficient for our experimental proposal.

In summary, we propose a mechanism to achieve a one-dimensional anyon from photons by using a nonlinear photonic resonator lattice under dynamic modulation. With this mechanism, both the ground state and the excited states of the anyon system can be selectively probed. Our system also enables the use of a two-photon interference experiment to directly probe the statistics of anyons. This platform can be useful for demonstrating a wide variety of other anyon physics effects [48–51], such as quantum works of two interacting anyons [52] and Bloch oscillation of anyons [53], that are potentially important for quantum information processing [54–58]. On the other hand, we also note that the present proposal represents a simulation of anyon physics in one-dimensional systems. Some of the topological properties associated with anyons in higher dimensions may not be preserved in such simulations [59].

ACKNOWLEDGMENT

This work is supported by U.S. Air Force Office of Scientific Research Grants No. FA9550-12-1-0488 and No. FA9550-17-1-0002.

The first two authors contributed equally to this work.

APPENDIX: EQUATIONS FOR TWO-PHOTON AMPLITUDES INSIDE THE CAVITY AS DERIVED FROM THE INPUT-OUTPUT FORMALISM

The photon transport properties of Eq. (9) can be described by the input-output formalism, which is a set of operator equations in the Heisenberg picture [32,34]:

$$\frac{d\tilde{b}_j(t)}{dt} = i[\tilde{H}_r,\tilde{b}_j(t)] - \frac{\gamma}{2}\tilde{b}_j(t) + i\sqrt{\gamma}c_{\text{in},j}(t), \tag{A1}$$

$$c_{\text{out},j}(t) = c_{\text{in},j}(t) - i\sqrt{\gamma}\tilde{b}_j(t), \tag{A2}$$

where

$$c_{\text{in},j}(t) \equiv \lim_{t_0\to-\infty}\int dk\tilde{c}_{k,j}(t_0)e^{-ik(t-t_0)}/\sqrt{2\pi}, \tag{A3}$$

$$c_{\text{out},j}(t) \equiv \lim_{t_1\to\infty}\int dk\,\tilde{c}_{k,j}(t_1)e^{-ik(t-t_1)}/\sqrt{2\pi}. \tag{A4}$$

We can rewrite Eq. (A1) as

$$\tilde{b}_j(t) = \tilde{b}_j(-\infty) + i\int_{-\infty}^{t} dt'\left\{[\tilde{H}_r,\tilde{b}_j(t')] + i\frac{\gamma}{2}\tilde{b}_j(t')\right\} + i\sqrt{\gamma}\int_{-\infty}^{t} dt' c_{\text{in},j}(t'). \tag{A5}$$

Using Eqs. (A1) and (A5), we obtain

$$\begin{aligned}\frac{d\tilde{b}_m(t)\tilde{b}_j(t)}{dt} &= i[\tilde{H}_r,\tilde{b}_m(t)\tilde{b}_j(t)] - \gamma\tilde{b}_m(t)\tilde{b}_j(t)\\ &\quad + i\sqrt{\gamma}\tilde{b}_m(-\infty)c_{\text{in},j}(t) - \sqrt{\gamma}\int_{-\infty}^{t} dt'\left\{[\tilde{H}_r,\tilde{b}_m(t')] + i\frac{\gamma}{2}\tilde{b}_m(t')\right\}c_{\text{in},j}(t) - \gamma\int_{-\infty}^{t} dt' c_{\text{in},m}(t')c_{\text{in},j}(t)\\ &\quad + i\sqrt{\gamma}c_{\text{in},m}(t)\tilde{b}_j(-\infty) - \sqrt{\gamma}\int_{-\infty}^{t} dt' c_{\text{in},m}(t)\left\{[\tilde{H}_r,\tilde{b}_j(t')] + i\frac{\gamma}{2}\tilde{b}_j(t')\right\} - \gamma\int_{-\infty}^{t} dt' c_{\text{in},m}(t)c_{\text{in},j}(t').\end{aligned} \tag{A6}$$

To probe the anyon property one needs at least two particles. We therefore consider a normalized input state

$$|\Phi\rangle = \iint dt_1 dt_2 \chi(t_1 - t_2)\eta\left(\frac{t_1+t_2}{2}\right)c^{\dagger}_{\text{in},\alpha}(t_2)c^{\dagger}_{\text{in},\beta}(t_1)|0\rangle \quad (j \leqslant m), \tag{A7}$$

where the normalization condition $\langle\Phi|\Phi\rangle = 1$ requires that $\iint dt\,dt'\chi^2(t')|\eta(t)|^2 = 1$. In what follows, we assume that $\chi(t)$ has a very short temporal width to describe a scenario where we simultaneously inject two photons into αth and βth cavities through the waveguides. Such an input state corresponds to a strongly correlated photon pair. Such an input state can be prepared by the four-wave-mixing process [60–63].

In the presence of such input state, we can compute the resulting two-photon probability amplitude inside the resonator lattice:

$$v_{jm}(t) \equiv \langle 0|\tilde{b}_m(t)\tilde{b}_j(t)|\Phi\rangle \quad (j \leqslant m). \tag{A8}$$

Sandwiching Eq. (A6) with $\langle 0|\ldots|\Phi\rangle$, we obtain the differential equation:

$$\frac{dv_{jm}(t)}{dt} = i\langle 0|[\tilde{H}_r,\tilde{b}_m(t)\tilde{b}_j(t)]|\Phi\rangle - \gamma v_{jm}(t) - \gamma\eta(t)\delta_{j\alpha}\delta_{m\beta}. \tag{A9}$$

Here we have used the property that $\chi(t)$ has a very short temporal width. Equation (A9) is Eq. (14) in the main text.

[1] J. M. Leinaas and J. Myrheim, Nuovo Cim. B **37**, 1 (1977).
[2] G. A. Goldin, R. Menikoff, and D. H. Sharp, J. Math. Phys. **22**, 1664 (1981).
[3] F. Wilczek, Phys. Rev. Lett. **48**, 1144 (1982).
[4] F. Wilczek, Phys. Rev. Lett. **49**, 957 (1982).
[5] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[6] A. Yu. Kitaev, Ann. Phys. (NY) **303**, 2 (2003).
[7] S. Das Sarma, M. Freedman, and C. Nayak, Phys. Rev. Lett. **94**, 166802 (2005).

[8] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[9] A. Stern, Ann. Phys. (NY) **323**, 204 (2008).
[10] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).
[11] T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, Nat. Commun. **2**, 361 (2011).
[12] S. Greschner and L. Santos, Phys. Rev. Lett. **115**, 053002 (2015).
[13] C. Sträter, S. C. L. Srivastava, and A. Eckardt, Phys. Rev. Lett. **117**, 205303 (2016).
[14] J. Cho, D. G. Angelakis, and S. Bose, Phys. Rev. Lett. **101**, 246809 (2008).
[15] R. O. Umucalilar and I. Carusotto, Phys. Rev. Lett. **108**, 206809 (2012).
[16] M. Hafezi, M. D. Lukin, and J. M. Taylor, New J. Phys. **15**, 063001 (2013).
[17] K. Fang, Z. Yu, and S. Fan, Nat. Photon. **6**, 782 (2012).
[18] K. Fang and S. Fan, Phys. Rev. Lett. **111**, 203901 (2013).
[19] K. Fang, Z. Yu, and S. Fan, Opt. Express **21**, 18216 (2013).
[20] Q. Lin and S. Fan, Phys. Rev. X **4**, 031031 (2014).
[21] L. Yuan and S. Fan, Phys. Rev. Lett. **114**, 243901 (2015).
[22] L. Yuan and S. Fan, Phys. Rev. A **92**, 053822 (2015).
[23] K. Fang, Z. Yu, and S. Fan, Phys. Rev. Lett. **108**, 153901 (2012).
[24] K. Fang, Z. Yu, and S. Fan, Phys. Rev. B **87**, 060301(R) (2013).
[25] L. D. Tzuang, K. Fang, P. Nussenzveig, S. Fan, and M. Lipson, Nat. Photon. **8**, 701 (2014).
[26] E. Li, B. J. Eggleton, K. Fang, and S. Fan, Nat. Commun. **5**, 3225 (2014).
[27] S. Longhi and G. Della Valle, Opt. Lett. **37**, 2160 (2012).
[28] A. Kundu, Phys. Rev. Lett. **83**, 1275 (1999).
[29] M. T. Batchelor, X.-W. Guan, and N. Oelkers, Phys. Rev. Lett. **96**, 210402 (2006).
[30] L. Yuan and S. Fan, Optica **3**, 1014 (2016).
[31] S. Longhi, Opt. Lett. **30**, 786 (2005).
[32] S. Fan, Ş. E. Kocabaş, and J.-T. Shen, Phys. Rev. A **82**, 063821 (2010).
[33] L. Yuan, S. Xu, and S. Fan, Opt. Lett. **40**, 5140 (2015).
[34] C. W. Gardiner and M. J. Collett, Phys. Rev. A **31**, 3761 (1985).
[35] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[36] R. C. Liu, B. Odom, Y. Yamamoto, and S. Tarucha, Nature (London) **391**, 263 (1998).
[37] R. Loudon, Phys. Rev. A **58**, 4904 (1998).
[38] L. D. Tzuang, M. Soltani, Y. H. D. Lee, and M. Lipson, Opt. Lett. **39**, 1799 (2014).
[39] G. T. Reed, G. Z. Mashanovich, F. Y. Gardes, M. Nedeljkovic, Y. Hu, D. J. Thomson, K. Li, P. R. Wilson, S. Chen, and S. S. Hsu, Nanophotonics **3**, 229 (2014).
[40] D. E. Chang, V. Vuletić, and M. D. Lukin, Nat. Photon. **8**, 685 (2014).
[41] A. Imamoğlu, H. Schmidt, G. Woods, and M. Deutsch, Phys. Rev. Lett. **79**, 1467 (1997).
[42] I. Carusotto, D. Gerace, H. E. Tureci, S. De Liberato, C. Ciuti, and A. Imamoğlu, Phys. Rev. Lett. **103**, 033601 (2009).
[43] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Nature (London) **436**, 87 (2005).
[44] I. Fushman, D. Englund, A. Faraon, N. Stoltz, P. Petroff, and J. Vučković, Science **320**, 769 (2008).
[45] A. Kubanek, A. Ourjoumtsev, I. Schuster, M. Koch, P. W. H. Pinkse, K. Murr, and G. Rempe, Phys. Rev. Lett. **101**, 203602 (2008).
[46] M. Koch, C. Sames, M. Balbach, H. Chibani, A. Kubanek, K. Murr, T. Wilk, and G. Rempe, Phys. Rev. Lett. **107**, 023601 (2011).
[47] T. Volz, A. Reinhard, M. Winger, A. Badolato, K. J. Hennessy, E. L. Hu, and A. Imamoğlu, Nat. Photon. **6**, 605 (2012).
[48] Y. Hao, Y. Zhang, and S. Chen, Phys. Rev. A **78**, 023631 (2008).
[49] Y. Hao and S. Chen, Phys. Rev. A **86**, 043631 (2012).
[50] T. M. Wright, M. Rigol, M. J. Davis, and K. V. Kheruntsyan, Phys. Rev. Lett. **113**, 050601 (2014).
[51] G. Tang, S. Eggert, and A. Pelster, New J. Phys. **17**, 123016 (2015).
[52] L. Wang, L. Wang, and Y. Zhang, Phys. Rev. A **90**, 063618 (2014).
[53] S. Longhi and G. Della Valle, Phys. Rev. B **85**, 165144 (2012).
[54] G. Castagnoli and M. Rasetti, Int. J. Theor. Phys. **32**, 2335 (1993).
[55] C. Mochon, Phys. Rev. A **69**, 032306 (2004).
[56] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008).
[57] J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. A. Fisher, Nat. Phys. **7**, 412 (2011).
[58] T. Đurić, K. Biedroń, and J. Zakrzewski, Phys. Rev. B **95**, 085102 (2017).
[59] C.-E. Bardyn and A. İmamoğlu, Phys. Rev. Lett. **109**, 253606 (2012).
[60] S. Ramelow, L. Ratschbacher, A. Fedrizzi, N. K. Langford, and A. Zeilinger, Phys. Rev. Lett. **103**, 253601 (2009).
[61] X. Li, L. Yang, X. Ma, L. Cui, Z. Y. Ou, and D. Yu, Phys. Rev. A **79**, 033817 (2009).
[62] J. W. Silverstone, D. Bonneau, K. Ohira, N. Suzuki, H. Yoshida, N. Iizuka, M. Ezaki, C. M. Natarajan, M. G. Tanner, R. H. Hadfield, V. Zwiller, G. D. Marshall, J. G. Rarity, J. L. O'Brien, and M. G. Thompson, Nat. Photon. **8**, 104 (2013).
[63] H. Kim, H. J. Lee, S. M. Lee, and H. S. Moon, Opt. Lett. **40**, 3061 (2015).