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Achieving nonreciprocal unidirectional single-photon quantum transport using the photonic Aharonov–Bohm effect

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We show that nonreciprocal unidirectional single-photon quantum transport can be achieved with the photonic Aharonov–Bohm effect. The system consists of a 1D waveguide coupled to two three-level atoms of the V -type. The two atoms, in addition, are each driven by an external coherent field. We show that the phase of the external coherent field provides a gauge potential for the photon states. With a proper choice of the phase difference between the two coherent fields, the transport of a single photon can exhibit unity contrast in its transmissions for the two propagation directions. © 2015 Optical Society of America

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It has been recently noted that in the photonic structures undergoing dynamic modulation, the phase of the modulation provides a gauge potential for light [1]. As a result, photonic structures undergoing dynamic modulation exhibit a photonic Aharonov–Bohm (AB) effect. Such a photonic AB effect has been experimentally observed both in microwave and optical frequency ranges [2–4] and provides a nonmagnetic mechanism to achieve complete and linear optical isolation [1]. The gauge potential for light is also connected to the emerging area of topological photonics [5–10] since the gauge potential can be used to create topologically nontrivial properties for photons.

All previous works on the photonic Aharonov–Bohm effect have been carried out for classical electromagnetic waves. On the other hand, in recent years, there are also significant theoretical and experimental efforts aiming to achieve nonreciprocal transport for single photons in order to demonstrate important quantum-information-processing functionalities [11–16]. Therefore, achieving a photonic Aharonov–Bohm effect in order to realize nonreciprocal unidirectional transmission in single-photon quantum transport is also of interest.

In this Letter, we explore a system composed of a 1D waveguide coupled with two separated atomic-like systems, each

driven by an external coherent field (Fig. 1). We show that a single-photon effective gauge field can be created in this system, which can lead to a photonic Aharonov–Bohm effect and nonreciprocal unidirectional transport for single photons. Moreover, in the atom-waveguide system, the atom can induce strong photon–photon interaction at the two-photon level [17–22]. Our work points to a route of combining few-photon nonlinearity with a photonic gauge field.

In our system (Fig. 1), we assume that each atom consists of three levels of the V type, with a ground state $|g\rangle$ and two excited states $|e_1\rangle$ and $|e_2\rangle$ at frequencies ω_g , ω_{e_1} , and ω_{e_2} , respectively. Each atom is driven by an external coherent field, which couples between the two excited states. We assume that the frequency of the external coherent field at the two atoms is the same, but the phases can be different. The waveguide supports bidirectional propagation of photons. The two directions are labelled “L” and “R,” respectively. The waveguide photons couple to the atomic transitions between the ground state and the two excited states.

Before we discuss the case with two atoms, we first consider the interaction of a single waveguide photon with a single

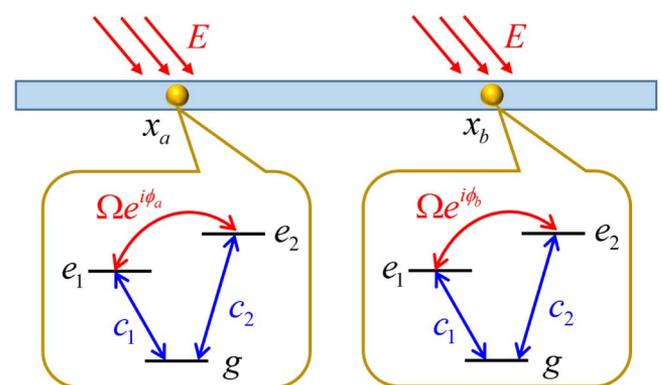


Fig. 1. Atom-waveguide system consists of two separated identical atoms. Both atoms have V -type energy levels and are located at x_a and x_b , respectively, inside the waveguide. The external fields that drive the two atoms have phases of ϕ_a and ϕ_b , respectively.

three-level atom under coherent drive. The system is described by the Hamiltonian [23]:

$$H = H_0 + H_1, \quad (1)$$

where

$$H_0 = \omega_g |g\rangle\langle g| + \sum_{m=1,2} \omega_{e_m} |e_m\rangle\langle e_m| + \sum_k \hbar \omega_k a_k^\dagger a_k, \quad (2)$$

$$H_1 = \sum_{m=1,2} \sum_k \hbar V (a_k^\dagger + a_k) (\sigma_{m,+} + \sigma_{m,-}) + 2\hbar \Omega \cos(\nu t + \phi) (|e_1\rangle\langle e_2| + |e_2\rangle\langle e_1|). \quad (3)$$

Here, V is the strength that describes the coupling between a photon with a momentum k with the atom through the transition between the ground state and the m th excited state. For simplicity, we assume that the coupling strength is independent of both k and m . $\sigma_{m,+} = |e_m\rangle\langle g|$. Ω is the Rabi frequency of the external drive field. The drive field has a frequency chosen at $\nu = \omega_{e_2 e_1} = \omega_{e_2} - \omega_{e_1}$ and a phase of ϕ . Equation (1) can be transformed into a real-space Hamiltonian [24,25]:

$$H = \int dx \sum_{m=1,2} \left\{ c_{m,R}^\dagger(x) \left(-iv_g \frac{\partial}{\partial x} - \omega_{e_m g} \right) c_{m,R}(x) + c_{m,L}^\dagger(x) \left(iv_g \frac{\partial}{\partial x} - \omega_{e_m g} \right) c_{m,L}(x) + V \delta(x) \times [c_{m,R}^\dagger(x) \sigma_{m,-} + c_{m,R}(x) \sigma_{m,+} + c_{m,L}^\dagger(x) \sigma_{m,-} + c_{m,L}(x) \sigma_{m,+}] \right\} + \Omega e^{i\phi} |e_1\rangle\langle e_2| + \Omega e^{-i\phi} |e_2\rangle\langle e_1|, \quad (4)$$

where $\omega_{e_m g} = \omega_{e_m} - \omega_g$ is the atomic transition frequency between level $|e_m\rangle$ and level $|g\rangle$. In deriving Eq. (4), we set $\hbar = 1$ and use the rotating wave approximation. We assume that the $|\omega_{e_1 g} - \omega_{e_2 g}|$ is far larger than the linewidth of the atom. Therefore, a photon that is near resonant with the $g - e_m$ transition can only interact with this transition and can be labelled by m . In other words, we imagine two types of photons, each interacting exclusively with only one of the atomic transitions, and its type corresponding to the atomic transition that it interacts with. $c_{m,R(L)}^\dagger(x)$ is the creation operator for the right-going (left-going) photon of type m at x . We linearize the waveguide dispersion relation in the vicinity of the two transition frequencies. And, for simplicity, we assume that the group velocity v_g for the two types of photons is the same.

We consider a single photon of type 1, i.e., a single photon with frequency ω_1 near $\omega_{e_1 g}$, coming from the left. We define the detuning $\Delta \equiv \omega_1 - \omega_{e_1 g}$. Upon interacting with the atom, the photon may either remain at the same frequency or convert to a frequency $\omega_2 = \Delta + \omega_{e_2 g}$, i.e., convert to a single photon of type 2. Following the method in Ref. [25] that directly solves for the single-photon scattering eigenstates in real space, we can solve the time-independent Schrödinger equation $H|E_k\rangle = \Delta|E_k\rangle$ and obtain

$$t_1 = \frac{-i\Delta(V^2/v_g - i\Delta) + \Omega^2}{(V^2/v_g - i\Delta)^2 + \Omega^2} \equiv t, \quad (5)$$

$$r_1 = \frac{-(V^2/v_g)(V^2/v_g - i\Delta)}{(V^2/v_g - i\Delta)^2 + \Omega^2} \equiv r, \quad (6)$$

$$t_2 = r_2 = \frac{i(V^2/v_g)\Omega e^{-i\phi}}{(V^2/v_g - i\Delta)^2 + \Omega^2} \equiv s e^{-i\phi}. \quad (7)$$

Here, t_m , r_m are the transmission and reflection amplitudes, respectively, for the outgoing photons of the m th type. The results in Eqs. (5)–(7) satisfy $|t_1|^2 + |r_1|^2 + |t_2|^2 + |r_2|^2 = 1$, as required by probability conservation.

We plot Eqs. (5)–(7) in Fig. 2. In the absence of the external drive field ($\Omega = 0$), the system has a resonance at $\Delta = 0$, where the single incident photon is completely reflected (i.e., $|r_1|^2 = 1$), which is consistent with Ref. [25]. In the presence of the external drive field (i.e., with $\Omega \neq 0$), the system supports two resonances. The difference in frequencies of the two resonances increases as Ω increases. This is the effect of Rabi splitting induced by the external drive field. In the limit of $\Omega \gg V^2/v_g$, the two resonances are located at $\Delta = \pm\Omega$, respectively. And, at the resonance, we have $|r_2| = |t_2| = |r_1| = |t_1| = 0.5$, indicating significant frequency conversion at these resonances.

The phase ϕ is a gauge degree of freedom. Following the argument in Ref. [1], we see that its value is related to a choice of the time origin and therefore can be arbitrarily set. As a result, the phase ϕ by itself has no measurable physical consequence. However, if one considers the system as shown in Fig. 1, where two atoms are both driven by the external coherent field, the difference in the phases of the coherent drive has a physical consequence as represented by the photonic Aharonov–Bohm effect [1].

To consider the system of two atoms in a waveguide as shown in Fig. 1, we first construct the scatter matrix $\mathbf{S}(\phi)$ for the system with one atom in the waveguide as solved above:

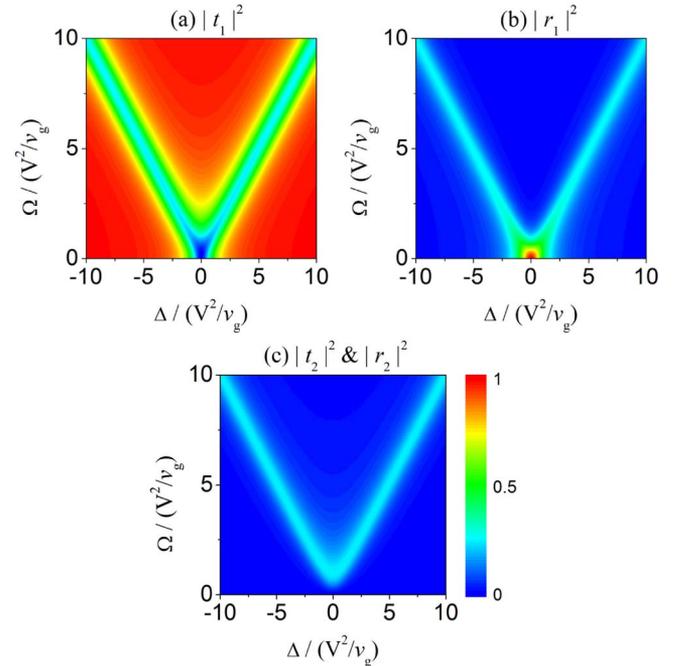


Fig. 2. Transmission and reflection spectra for the photon of type 1 in the system composed of one atom in a 1D waveguide driven by the external field. (a) $|t_1|^2$, (b) $|r_1|^2$, and (c) $|t_2|^2$ and $|r_2|^2$ are defined in Eqs. (5)–(7).

$$\begin{pmatrix} b_1 \\ a'_1 \\ b_2 \\ a'_2 \end{pmatrix} = \mathbf{S}(\phi) \begin{pmatrix} a_1 \\ b'_1 \\ a_2 \\ b'_2 \end{pmatrix}, \quad (8)$$

where $a_m(b'_m)$ and $b_m(a'_m)$ are the incoming and outgoing wave amplitudes of the m th type of photons at the left side (right side) of the atom. Using the results of Eqs. (5)–(7), and noticing that the system of Eq. (1) has a mirror symmetry, we have

$$\mathbf{S}(\phi) = \begin{pmatrix} r & t & se^{i\phi} & se^{i\phi} \\ t & r & se^{i\phi} & se^{i\phi} \\ se^{-i\phi} & se^{-i\phi} & r & t \\ se^{-i\phi} & se^{-i\phi} & t & r \end{pmatrix}. \quad (9)$$

Notice that when $\phi \neq 0$, $\mathbf{S}(\phi)$ is nonsymmetric, and hence, with the drive field, the system becomes nonreciprocal. From Eqs. (8) and (9), we can derive the transfer matrix $\mathbf{T}(\phi)$ defined as

$$\begin{pmatrix} a'_1 \\ b'_1 \\ a'_2 \\ b'_2 \end{pmatrix} = \mathbf{T}(\phi) \begin{pmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{pmatrix}. \quad (10)$$

For the two-atom system as shown in Fig. 1, when a photon is incident from the left, the transfer matrix is then

$$\mathbf{T}_L = \mathbf{T}(\phi_b)\mathbf{T}_f(x_b - x_a)\mathbf{T}(\phi_a), \quad (11)$$

where

$$\mathbf{T}_f(x_b - x_a) = \begin{pmatrix} e^{ik_1(x_b - x_a)} & 0 & 0 & 0 \\ 0 & e^{-ik_1(x_b - x_a)} & 0 & 0 \\ 0 & 0 & e^{ik_2(x_b - x_a)} & 0 \\ 0 & 0 & 0 & e^{-ik_2(x_b - x_a)} \end{pmatrix}. \quad (12)$$

When a photon of type 1 is incident from the right side, the transfer matrix is

$$\mathbf{T}_R = \mathbf{T}(\phi_a)\mathbf{T}_f(x_b - x_a)\mathbf{T}(\phi_b). \quad (13)$$

Notice that $\mathbf{T}_L \neq \mathbf{T}_R$ when $\phi_a \neq \phi_b$, since $\mathbf{T}(\phi_{a(b)})$ and $\mathbf{T}_f(x_b - x_a)$ do not commute. With this, we can compute the single photon transport for the system in Fig. 1.

Using the formalism as shown above, we now show that the system in Fig. 1 can support complete nonreciprocal unidirectional transmission, i.e., a single photon of type 1 incident from the left has unity coefficient of transmission to a photon at the same frequency, whereas the same photon incident from the right has zero transmission to a photon at the same frequency. We first argue qualitatively the theoretical condition for such complete nonreciprocal unidirectional transmission. Suppose we send a single photon of type 1 with $\Delta = 0$ from the left end and detect the transmission of the photon of type 1 at the right end. There are two pathways contributing to this process (Fig. 3). In the first pathway, the photon remains at k_1 after it interacts with the first atom. It acquires a phase shift $k_1(x_b - x_a)$ during its propagation to the second atom and then interacts with the second atom while keeping its frequency unchanged. In the second pathway, the photon state undergoes a frequency conversion to a wave vector k_2 after it interacts with the first atom. It then propagates to the second atom and

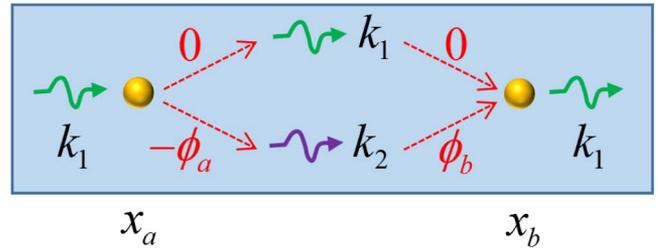


Fig. 3. Two pathways for a single photon of type 1 transmitted into a photon of the same type, through the system as shown in Fig. 1.

undergoes a frequency conversion back to a wave vector k_1 after it interacts with the second atom. Through this second pathway, the photon acquires a total phase shift of $-\phi_a + k_2(x_b - x_a) + \phi_b$ (Fig. 3). The phase difference between the two pathways is therefore $\phi_b - \phi_a - (k_1 - k_2)(x_b - x_a)$. In contrast, for a single photon of type 1 incident from the right end and transmitted to the single photon of the same type at the left end, there is a phase difference of $\phi_a - \phi_b - (k_1 - k_2)(x_b - x_a)$. Notice the change of the sign of the phases that arises from the external drive.

To achieve complete nonreciprocal unidirectional transport, the two pathways as discussed above should result in constructive interference when the photon is incident from the left and destructive interference when the photon is incident from the right. This can be accomplished with the choice of

$$(k_1 - k_2)(x_b - x_a) = \pi/2 + 2n\pi, \quad (14)$$

and

$$\phi_b - \phi_a = -\pi/2. \quad (15)$$

The argument presented above can be checked quantitatively. Assuming Eq. (14) is satisfied, using Eqs. (8)–(13), we obtain the coefficients of transmission (T) and reflection (R) into a photon of either type 1 or 2 when a single photon of type 1 is incident from the left:

$$T_1 = [\Omega^2 e^{-i(\phi_b - \phi_a)} + i2(V^2/v_g)^2]/M, \quad (16)$$

$$R_1 = -[(V^2/v_g)^2(1 + e^{-i2(\phi_b - \phi_a)})]/M, \quad (17)$$

$$T_2 = R_2 = [\Omega(V^2/v_g)e^{-i2\phi_b}(e^{i\phi_b} + ie^{i\phi_a})]/M, \quad (18)$$

where

$$M = \left(\frac{V^2}{v_g}\right)^2 (1 + e^{-i2(\phi_b - \phi_a)}) - i \left[\Omega^2 + 2 \left(\frac{V^2}{v_g}\right)^2 \right] e^{-i(\phi_b - \phi_a)}. \quad (19)$$

For a single photon incident from the right, its transport properties are also described by Eqs. (16)–(18), except we replace $\phi_b - \phi_a$ with $\phi_a - \phi_b$. From Eq. (16), we see that the complete unidirectional transport is indeed achieved with the choice of the phases of the drive field in Eq. (15), as well as a constraint on the strength of the drive field:

$$\Omega = \sqrt{2}V^2/v_g. \quad (20)$$

In Fig. 4, we plot the transmission and reflection coefficients for photon incident from either left or right as a function of

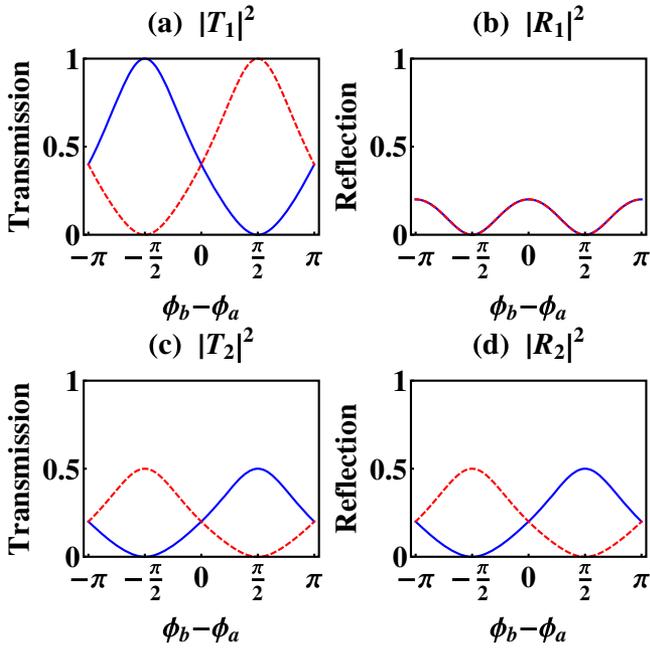


Fig. 4. Transmission [(a) $|T_1|^2$; (c) $|T_2|^2$] and reflection [(b) $|R_1|^2$; (d) $|R_2|^2$] spectra when the photon of type 1 is incident from the left end (blue solid curves) or from the right end (red dash curves) in the system as shown in Fig. 1.

$\phi_b - \phi_a$. The coefficients oscillate as a function of $\phi_b - \phi_a$. At $\phi_b - \phi_a = -\pi/2$, for photon incident from the left, there is no frequency conversion, and the system has 100% transmission. For a photon incident from the right, there is a complete frequency conversion, and the converted photons propagate with equal probability to the left and right. We therefore have shown complete nonreciprocal unidirectional transport by controlling the phase and the strength of the external drive field through the photonic Aharonov–Bohm effect.

We contrast our scheme here with previous work aiming to achieve nonreciprocal single-photon transport. In Refs. [11–14], the reciprocity was broken either through an external magnetic field or by exploiting the magnetic field of a nucleus magnetic moment that induces hyperfine level splitting. In comparison, the scheme here, which does not require such magnetic interaction, may allow a broader range of quantum emitters to be used for achieving nonreciprocal transport, which can be particularly useful in on-chip implementations, including implementations using the superconducting qubits [26–29] in the microwave frequency and various quantum emitters in the optical frequency. Related to our work here, there have been several previous works aiming to break reciprocity in few-photon transport without the use of magnetic effects. Reference [15] uses an opto-mechanical interaction. The use of a quantum emitter here, where the light–matter interaction is far stronger, should result in the reduction of the pump power. Reference [16] uses a nonlinear effect, which is not applicable at the single-photon level.

In summary, we have shown that nonreciprocal single-photon transport can be accomplished by creating a gauge potential for photons through coherently driving two quantum emitters. In addition to achieving nonreciprocal transport, the use of the photonic gauge field provides a power mechanism to control single-photon transport. The combination of quantum emitters with a photonic gauge field may also lead to a route to novel quantum many-body photonic states, such as the fractional quantum Hall effect for light [30].

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REFERENCES

1. K. Fang, Z. Yu, and S. Fan, *Phys. Rev. Lett.* **108**, 153901 (2012).
2. L. D. Tzuang, K. Fang, P. Nussenzveig, S. Fan, and M. Lipson, *Nat. Photonics* **8**, 701 (2014).
3. E. Li, B. J. Eggleton, K. Fang, and S. Fan, *Nat. Commun.* **5**, 3225 (2014).
4. K. Fang, Z. Yu, and S. Fan, *Phys. Rev. B* **87**, 060301 (2013).
5. M. Hafezi, E. A. Demler, M. D. Lukin, and J. M. Taylor, *Nat. Phys.* **7**, 907 (2011).
6. K. Fang, Z. Yu, and S. Fan, *Nat. Photonics* **6**, 782 (2012).
7. A. B. Khanikaev, S. H. Mousavi, W.-K. Tse, M. Kargarian, A. H. MacDonald, and G. Shvets, *Nat. Mater.* **12**, 233 (2012).
8. M. C. Rechtsman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, *Nature* **496**, 196 (2013).
9. L. Lu, J. D. Joannopoulos, and M. Soljačić, *Nat. Photonics* **8**, 821 (2014).
10. L. Yuan and S. Fan, arXiv:1504.07299.
11. Y. Shen, M. Bradford, and J.-T. Shen, *Phys. Rev. Lett.* **107**, 173902 (2011).
12. E. J. Lenferink, G. Wei, and N. P. Stern, *Opt. Express* **22**, 16099 (2014).
13. K. Xia, G. Lu, G. Lin, Y. Chen, Y. Niu, S. Gong, and J. Twamley, *Phys. Rev. A* **90**, 043802 (2014).
14. I. Söllner, S. Mahmoodian, S. L. Hansen, L. Midolo, A. Javadi, G. Kiršanskė, T. Pregnolato, H. El-Ella, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, *Nat. Nanotechnol.* **10**, 775 (2015).
15. M. Hafezi and P. Rabl, *Opt. Express* **20**, 7672 (2012).
16. D. Roy, *Phys. Rev. B* **81**, 155117 (2010).
17. J.-T. Shen and S. Fan, *Phys. Rev. Lett.* **98**, 153003 (2007).
18. L. Zhou, J. Lu, and C. P. Sun, *Phys. Rev. A* **76**, 012313 (2007).
19. D. Roy, *Phys. Rev. Lett.* **106**, 053601 (2011).
20. H. Zheng, D. J. Gauthier, and H. U. Baranger, *Phys. Rev. A* **85**, 043832 (2012).
21. L. Zhou, L.-P. Yang, Y. Li, and C. P. Sun, *Phys. Rev. Lett.* **111**, 103604 (2013).
22. X.-Y. Yu, J.-H. Li, and X.-B. Li, *J. Opt. Soc. Am. B* **30**, 649 (2013).
23. M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University, 1997).
24. J. T. Shen and S. Fan, *Opt. Lett.* **30**, 2001 (2005).
25. J.-T. Shen and S. Fan, *Phys. Rev. Lett.* **95**, 213001 (2005).
26. J. Koch, A. A. Houck, K. Le Hur, and S. M. Girvin, *Phys. Rev. A* **82**, 043811 (2010).
27. A. Kamal, J. Clarke, and M. H. Devoret, *Nat. Phys.* **7**, 311 (2011).
28. D. Underwood, W. E. Shanks, J. Koch, and A. A. Houck, *Phys. Rev. A* **86**, 023837 (2012).
29. A. A. Houck, H. E. Tureci, and J. Koch, *Nat. Phys.* **8**, 292 (2012).
30. R. O. Umucaklar and I. Carusotto, *Phys. Rev. Lett.* **108**, 206809 (2012).