

## Bends and splitters in metal-dielectric-metal subwavelength plasmonic waveguides

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We investigate the performance of bends and splitters in metal-dielectric-metal subwavelength plasmonic waveguides. We show that bends and splitters with no additional loss over a very wide frequency range can be designed for metal-dielectric-metal waveguides with center layer thickness small compared to the wavelength. We also introduce the concept of characteristic impedance for such systems to account for their behavior. © 2005 American Institute of Physics.

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Light-guiding structures which allow subwavelength confinement of the optical mode are important for achieving compact integrated photonic devices.<sup>1-9</sup> The minimum confinement of a guided optical mode in dielectric waveguides is set by the diffraction limit and is of the order of  $\lambda_0/n$ , where  $\lambda_0$  is the wavelength in free space and  $n$  is the refractive index. As opposed to dielectric waveguides, plasmonic waveguides have shown the potential to guide subwavelength optical modes, the so-called surface plasmon polaritons, at metal-dielectric interfaces.

Several different plasmonic waveguiding structures have been proposed, such as metallic nanowires<sup>2-4</sup> and metallic nanoparticle arrays.<sup>5-7</sup> It is known that a metal-dielectric-metal (MDM) structure supports a subwavelength propagating mode at a wavelength range extending from dc to visible.<sup>10,11</sup> As an example, a gold-air-gold MDM waveguide with a center layer thickness  $d$  of 100 nm supports a mode at the optical communication wavelength of 1.55  $\mu\text{m}$  with a propagation length of  $\sim 10 \mu\text{m}$ .<sup>12</sup> Thus, such a waveguide could be potentially important in providing an interface between conventional optics and subwavelength electronic and optoelectronic devices.

In this letter, we investigate the performance of bends and power splitters in two-dimensional MDM plasmonic waveguides. Waveguide bends and splitters are basic structures for optical interconnects and therefore essential components of optical integrated circuits.<sup>8,13</sup> Here, of particular interest is the regime where the dimensions of bends and splitters are much smaller than the propagation length of the optical mode. In this regime, the relevant question is whether these bends and splitters will induce reflection or excess absorption loss on top of the propagation loss in the waveguides.

To answer this question, we calculate the transmission coefficient of bends and splitters and normalize it with respect to the transmission coefficient of a straight waveguide with the same length. We show that, even though the waveguides are lossy, bends and splitters with no *additional* loss can be designed over a wavelength range that extends from dc to near infrared, if  $d$  is small enough. This range includes the optical communication wavelength of 1.55  $\mu\text{m}$ . This remarkable effect is not observed in other light-guiding structures, such as high-index contrast or photonic-crystal

waveguides. We account for it with an effective characteristic impedance model based upon the real dispersion relation of the MDM waveguide structures.

We study the properties of silver-air-silver MDM waveguide bends and splitters using a two-dimensional finite-difference frequency-domain (FDFD) method.<sup>14,15</sup> This method allows us to directly use experimental data for the frequency-dependent dielectric constant of silver,<sup>16</sup> including both the real and imaginary parts, with no further approximation. Perfectly matched layer absorbing boundary conditions are used at all boundaries.<sup>17</sup> We use a spatial grid size of 2.5 nm in FDFD which we found to be sufficient for the convergence of numerical results.

To calculate the transmission coefficient of a 90° sharp MDM waveguide bend (inset of Fig. 1), we excite a dipole point source in the waveguide before the bend,<sup>18</sup> and measure the power flux of the transmitted optical mode after the bend. We perform a similar simulation in a straight waveguide and, by comparing the two cases, we extract the bending loss. In all cases,  $d$  is much smaller than the wavelength so that only the fundamental transverse magnetic (TM) waveguide mode (with magnetic field perpendicular to the direction of propagation) is excited. As an example, for  $d = 50$  nm, the optical mode is fully formed  $\sim 20$  nm away from the source, the mode travels  $\sim 200$  nm before the bend, and the bent wave is measured  $\sim 200$  nm after the bend. In

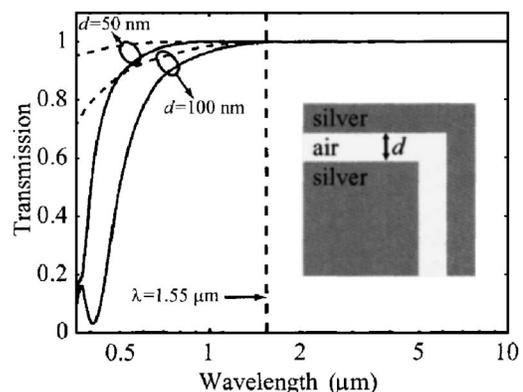


FIG. 1. Transmission spectra of a MDM waveguide bend (shown in the inset) calculated using FDFD. We also show with dashed line the transmission spectra of a PEC parallel-plate waveguide bend. Results are shown for  $d = 50, 100$  nm. The vertical dashed line marks the optical communication wavelength of 1.55  $\mu\text{m}$ .

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all cases, the waveguide lengths in the simulations were chosen large enough to ensure correct calculation of the additional loss of bends. To validate our method, we used it to calculate the transmission coefficient of perfect electric conductor (PEC) parallel-plate waveguide bends and splitters and found excellent agreement with analytical results<sup>19,20</sup> over the entire frequency range. In Fig. 1, we show the calculated bend transmission coefficient as a function of wavelength. We observe that at long wavelengths there is no bending loss. If the structure is small in comparison with the wavelength, the quasistatic approximation holds.<sup>21</sup> Under the quasistatic approximation, the bend is equivalent to a junction between two transmission lines with the same characteristic impedance, and there is, therefore, no bending loss. The limiting wavelength  $\lambda_c$  at which the transmission coefficient decreases below 99%, is  $1.27 \mu\text{m}$  ( $0.76 \mu\text{m}$ ) for  $d = 100 \text{ nm}$  ( $d = 50 \text{ nm}$ ). The operating wavelength range widens as  $d$  decreases, because for thinner structures the quasistatic approximation holds over a wider range of wavelengths.

In Fig. 1, we also show the calculated transmission coefficient of the bending structures when the PEC approximation is used for the metallic regions. In a PEC parallel-plate waveguide, the transmission coefficient of a  $90^\circ$  bend is only a function of  $d/\lambda_0$ , i.e.,  $T_{\text{PEC}} = T_{\text{PEC}}(d/\lambda_0)$ . If the device is small compared to the wavelength ( $d/\lambda_0 \ll 1$ ), there is no bending loss. The transmission coefficient decreases below 99% for  $d/\lambda_0 > 0.093$ . Thus, the limiting wavelength  $\lambda_c$  is  $1.08 \mu\text{m}$  ( $0.54 \mu\text{m}$ ) for  $d = 100 \text{ nm}$  ( $d = 50 \text{ nm}$ ). We observe that the transmission spectra of the PEC parallel-plate waveguide bend and of the MDM waveguide bend differ significantly. The limiting wavelength  $\lambda_c$  is lower in the PEC case.

In order to interpret the difference between the PEC and MDM transmission spectra, we calculated the guide wavelength  $\lambda_g$  of the fundamental TM mode in the MDM waveguide. The guide wavelength  $\lambda_g$ , defined as  $\lambda_g \equiv 2\pi/\beta_{\text{MDM}}$ , where  $\beta_{\text{MDM}}$  is the real part of the mode propagation constant,<sup>21</sup> is calculated using FDFD by exciting the fundamental mode in a straight MDM waveguide with a dipole source. To validate our method, we compared our results with results obtained by directly solving the dispersion relation of the MDM waveguide and found excellent agreement.<sup>11</sup> The calculated guide wavelength  $\lambda_g$  of the fundamental TM mode in the MDM waveguide is smaller than the free-space wavelength  $\lambda_0$ , which is the guide wavelength of the transverse electromagnetic (TEM) mode in the PEC waveguide. Since  $\lambda_g < \lambda_0$ , the PEC waveguide structure is “smaller” (in comparison to the optical mode wavelength) than the MDM waveguide structure, and this can explain the lower  $\lambda_c$  in the PEC case. We actually found that the transmission spectra of the MDM waveguide bend  $T_{\text{MDM}}$  is well approximated by the spectra of the PEC waveguide bend  $T_{\text{PEC}}$ , if the difference between  $\lambda_g$  and  $\lambda_0$  is taken into account, i.e.,  $T_{\text{MDM}} \approx T_{\text{PEC}}(d/\lambda_g)$ . This approximation typically holds for  $\lambda \geq \lambda_c$ , where the bending loss of the MDM waveguide is dominated by reflection. At shorter wavelengths, the bending loss is dominated by excess absorption and therefore this approximation no longer holds.

We also calculate the transmission spectra of MDM splitters. The calculation method using FDFD is similar to the one described above for the  $90^\circ$  bend. In Fig. 2, we show the calculated transmission coefficient as a function of wavelength for a MDM  $T$ -shaped splitter (inset of Fig. 2). The

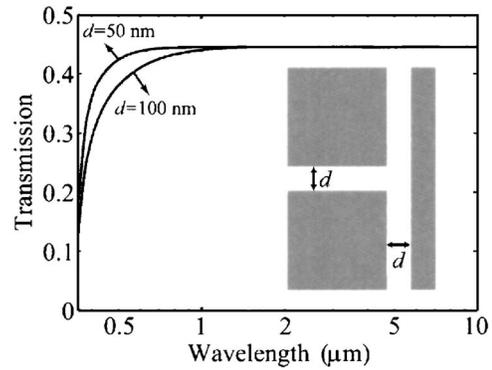


FIG. 2. Calculated transmission spectra of a MDM  $T$ -shaped splitter (shown in the inset). Results are shown for  $d = 50, 100 \text{ nm}$ .

frequency response of the MDM splitter is quite similar to the response of the MDM bend. At long wavelengths, the transmission is equal to 44.4%. Under the quasistatic approximation, which holds at long wavelengths, the splitter is equivalent to a junction of three transmission lines with the same characteristic impedance  $Z_0$ . The load connected to the input transmission line at the junction consists of the series combination of the two output transmission lines. Thus, the equivalent load impedance is  $Z_L = 2Z_0$  and the reflection coefficient is  $R = |(Z_L - Z_0)/(Z_L + Z_0)|^2 = 1/9$ . Because of the symmetry of the structure, the transmitted optical power is equally distributed between the two output waveguide branches, so that the transmission coefficient is  $T = 4/9$ . As in the MDM bend, the operating wavelength range widens as  $d$  decreases. At  $\lambda \approx \lambda_c$ , the splitter loss is dominated by reflection, while at shorter wavelengths it is dominated by excess absorption.

Based on the above discussion, in order to improve the transmission coefficient of the MDM splitter, we can adjust the characteristic impedance of the input waveguide  $Z_{\text{in}}$  so that  $Z_{\text{in}} \approx Z_L = 2Z_0$ . The input impedance  $Z_{\text{in}}$  can be adjusted by varying the thickness  $d_{\text{in}}$  of the input waveguide. In Fig. 3, we show the calculated reflection coefficient  $R$  of the MDM  $T$ -shaped splitter at  $\lambda_0 = 1.55 \mu\text{m}$  as a function of  $d_{\text{in}}/d_{\text{out}}$ , where  $d_{\text{out}} = 50 \text{ nm}$  is the thickness of the two output waveguide branches (inset of Fig. 3). We note that at  $\lambda_0 = 1.55 \mu\text{m}$ , the propagation length of the fundamental MDM mode is much larger than the splitter dimensions so that the

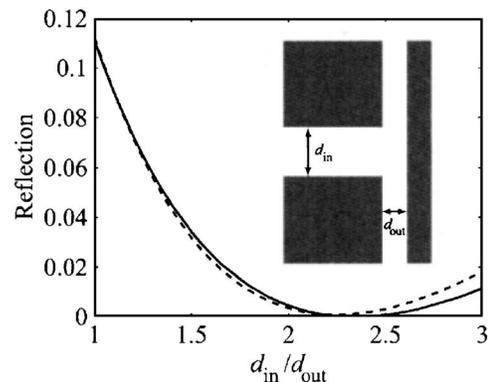


FIG. 3. Reflection coefficient  $R$  of a MDM  $T$ -shaped splitter (shown in the inset) as a function of  $d_{\text{in}}/d_{\text{out}}$  at  $\lambda_0 = 1.55 \mu\text{m}$  calculated using FDFD. We also show with dashed line the reflection coefficient  $R$  calculated based on the characteristic impedance  $Z_{\text{MDM}}$  and transmission-line theory. Results are shown for  $d_{\text{out}} = 50 \text{ nm}$ .

contribution of excess absorption to the splitter loss is negligible. We observe that the reflection coefficient is below 1% for  $1.8 < d_{\text{in}}/d_{\text{out}} < 2.8$  and is minimized for  $d_{\text{in}}/d_{\text{out}} \approx 2.25$ . We also found that the limiting wavelength  $\lambda_c$  of the optimized splitter is almost the same as the limiting wavelength  $\lambda_c$  of the symmetric splitter of Fig. 2.

The characteristic impedance of the fundamental TEM mode in a PEC parallel-plate waveguide is uniquely defined as the ratio of voltage  $V$  to surface current density  $I$  and is equal to<sup>21</sup>

$$Z_{\text{TEM}} \equiv \frac{V}{I} = \frac{E_x d}{H_y} = \frac{\beta_{\text{TEM}}}{\omega \epsilon_0} d = \sqrt{\frac{\mu_0}{\epsilon_0}} d,$$

where  $E_x, H_y$  are the transverse components of the electric and magnetic field, respectively, and we assumed a unit-length waveguide in the  $y$  direction. For non-TEM modes, such as the fundamental MDM mode, voltage and current are not uniquely defined. However, metals like silver satisfy the condition  $|\epsilon_{\text{metal}}| \gg \epsilon_{\text{diel}}$  at the optical communication wavelength of  $1.55 \mu\text{m}$ .<sup>16</sup> Thus,  $|E_{x \text{ metal}}| \ll |E_{x \text{ diel}}|$  so that the integral of the electric field in the transverse direction can be approximated by  $E_{x \text{ diel}} d$ , and we may therefore define the characteristic impedance of the fundamental MDM mode as

$$Z_{\text{MDM}}(d) \equiv \frac{E_{x \text{ diel}} d}{H_{y \text{ diel}}} = \frac{\beta_{\text{MDM}}(d)}{\omega \epsilon_0} d,$$

where  $\beta_{\text{MDM}}(d) = 2\pi/\lambda_g(d)$ , and the guide wavelength  $\lambda_g$  is calculated as mentioned above. In Fig. 3, we show the reflection coefficient of the MDM  $T$ -shaped splitter calculated based on  $Z_{\text{MDM}}$  as

$$\bar{R} = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|^2 = \left| \frac{2Z_{\text{MDM}}(d_{\text{out}}) - Z_{\text{MDM}}(d_{\text{in}})}{2Z_{\text{MDM}}(d_{\text{out}}) + Z_{\text{MDM}}(d_{\text{in}})} \right|^2.$$

We observe that there is very good agreement between  $\bar{R}$  and the exact reflection coefficient  $R$  calculated using FDFD. This agreement suggests that the concept of characteristic impedance for MDM waveguides is indeed valid and useful. The deviation between  $\bar{R}$  and  $R$  at large values of  $d_{\text{in}}/d_{\text{out}}$  is due to the fact that  $d_{\text{in}}$  is not very small compared to the wavelength and the quasistatic approximation therefore breaks down. We found that similar deviations are observed for PEC parallel-plate waveguides. Such deviations decrease at longer wavelengths in both the PEC and MDM waveguide cases.

As final remarks, we note that the two-dimensional model that we used should accurately describe the case of a subwavelength slit in a thick metal film. We also expect that the impedance concept can be generalized to three-dimensional MDM waveguides when the dielectric layer thickness is much smaller than the wavelength. Finally we note that, even though the choice of metal affects the propagation length of MDM waveguides,<sup>12</sup> our conclusions on bends and splitters are valid regardless of the choice of metal.

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