

# *Treatment of Uncertainty in Integrated Assessment Modeling*

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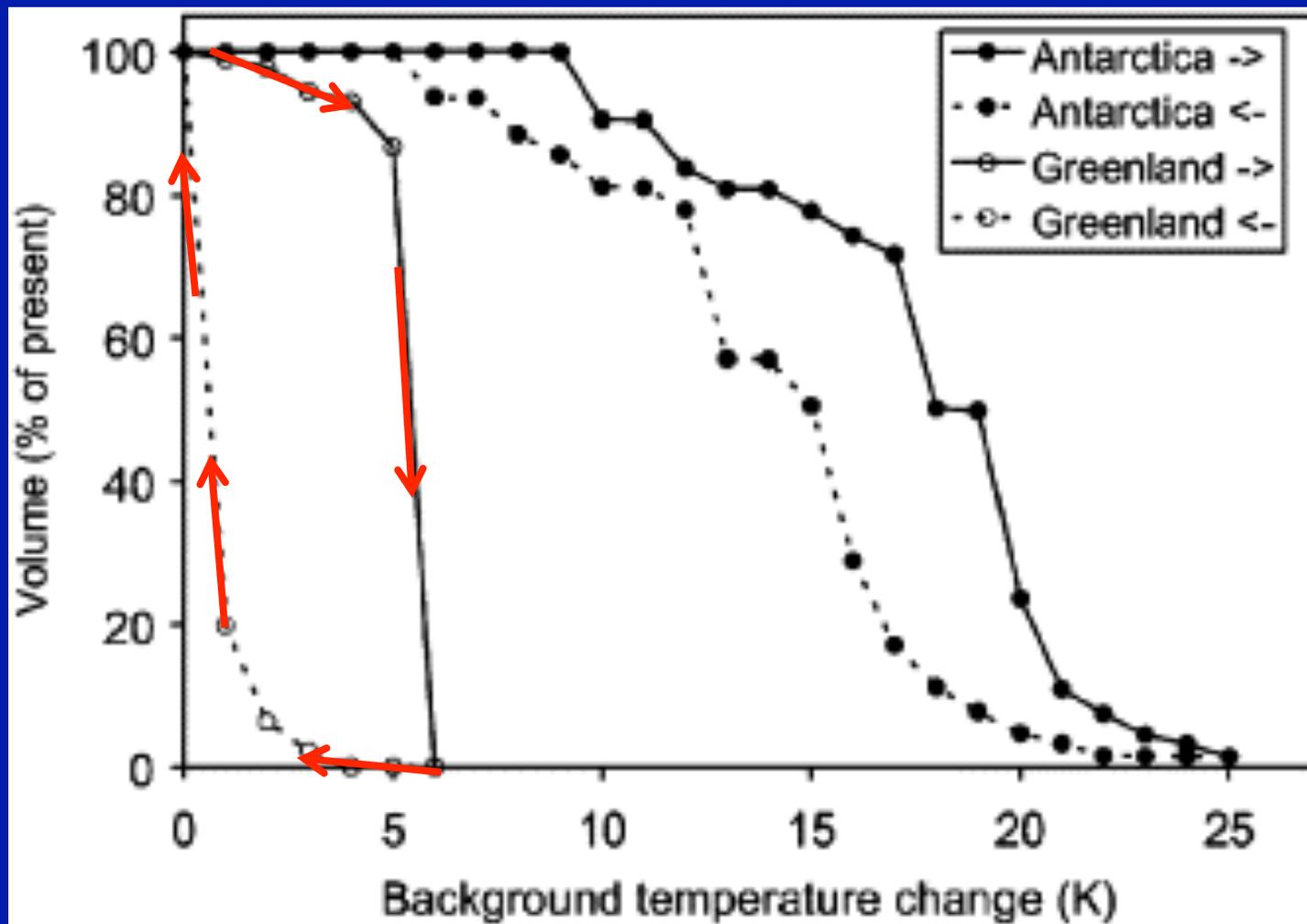
# Outline of lecture

1. Treatment of uncertainty in integrated assessment models (IAM)
2. Results for DICE and RICE models

# *I. Dilemmas of uncertainty*

- Uncertainty and the potential for catastrophic impacts are a major concern in global warming science and policy today.
- Some major risks include:
  - Reversal of North Atlantic deepwater circulation
  - Melting of Greenland and West Antarctic ice sheets
  - Abrupt climate change
  - Ocean carbonization
- How can we model risk and uncertainty in IAMs?

# Hysteresis loops for Ice Sheets and the “Tipping Point”



Frank Pattyn, “GRANTISM: Model of Greenland and Antarctica,” *Computers & Geosciences*, April 2006, Pages 316-325<sup>4</sup>

## Approach to Uncertainty: Monte Carlo/Subjective Probability Uncertainty Analysis

1. The approach we have used is to combine modeling, subjective probability theory, and Monte Carlo sampling.
2. Begin with a structure like DICE model equations. Can represent schematically as follows:

$$(1) \quad y_t = H(z_t; \theta)$$

where

$y_t$  = the endogenous variables (output, emissions, etc.)

$z_t$  = exogenous and non-stochastic variables (financial meltdown, land-based emissions, etc.)

$\theta = [\theta_1, \dots, \theta_n]$  = uncertain parameters (including functional forms)

H = model structure

3. We then develop subjective probabilities for major parameters,  $f(\theta)$ .
4. Final step is Monte Carlo simulation to obtain distribution of important endogenous variables (T, Y, SCC, ...)

## *Uncertainty analysis in DICE model*

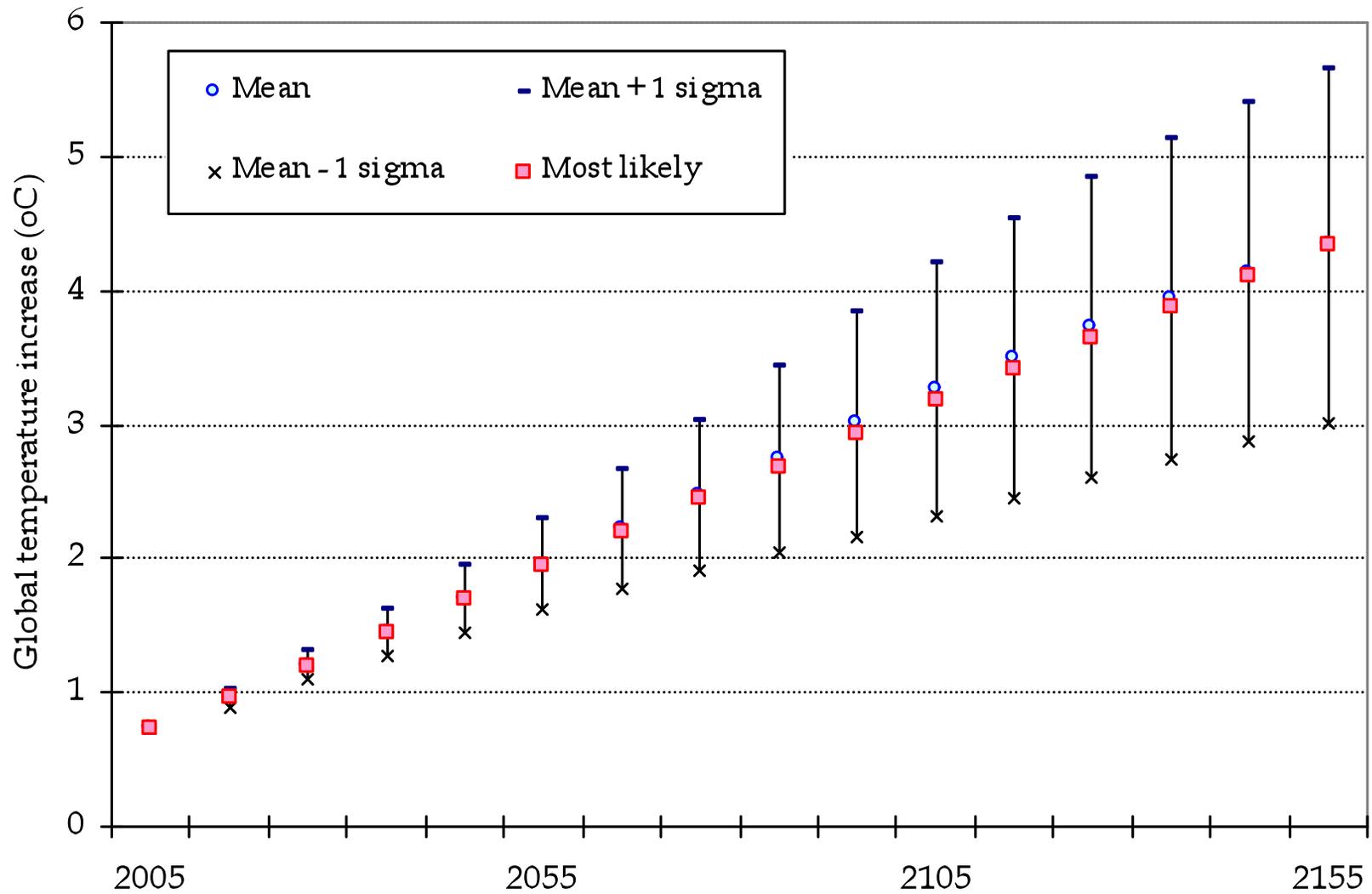
### Implementation in DICE-2007

- For DICE model, we examine 100 random runs, where these are baseline for 8 uncertain parameters.
- Then fit a “response-surface model” that estimates major outcomes as function of uncertain parameters
- These provide information about what the uncertainties are for major variables.

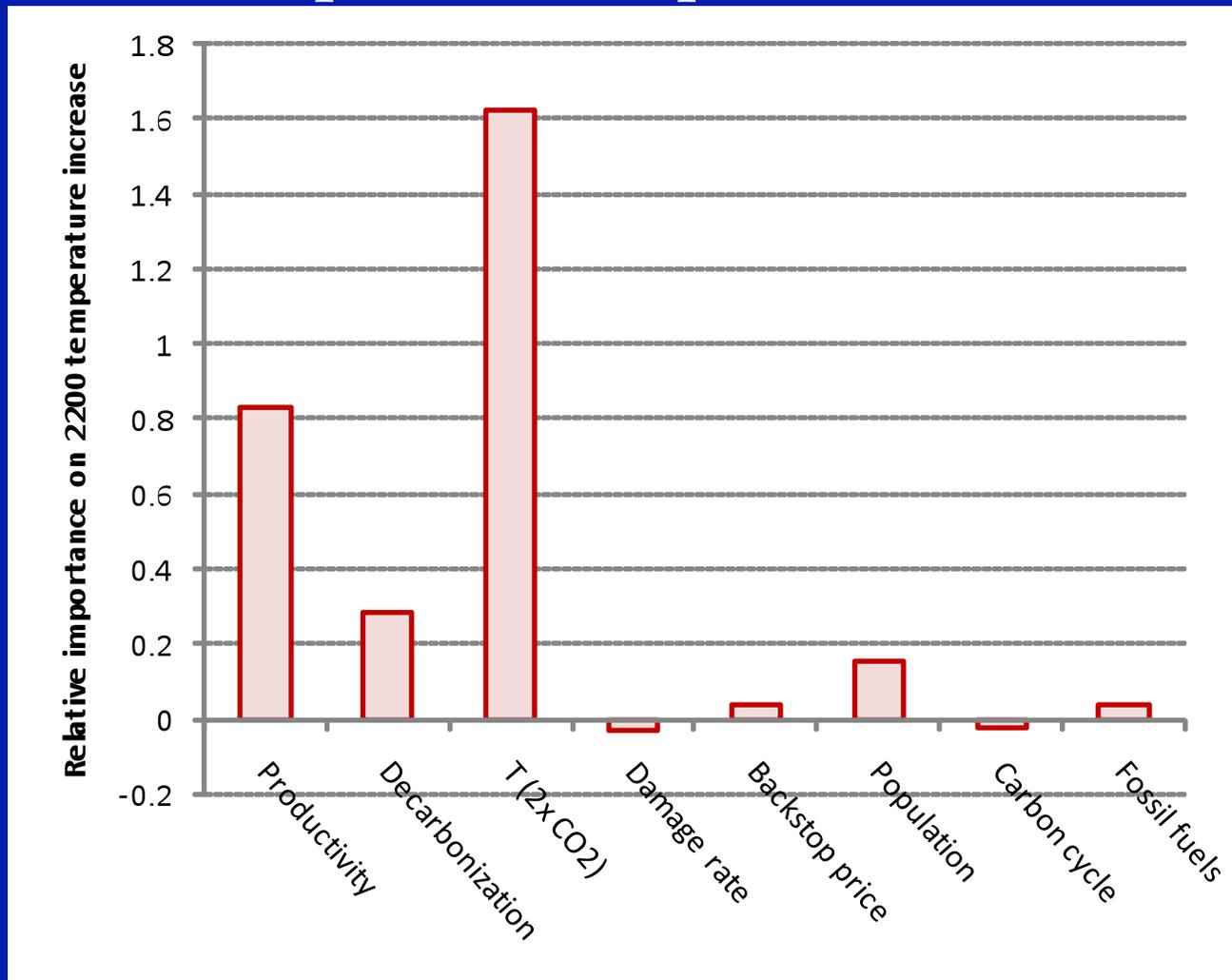
## Major uncertain variables

| Variable  | Mean    | Standard deviation |
|---|---------|--------------------|
| Rate of growth of total factor productivity (% per year)                          | 0.0092  | 0.004              |
| Rate of decarbonization (% per year)  | -0.0070 | 0.002              |
| Equilibrium temperature-sensitivity coefficient (°C per CO <sub>2</sub> doubling) | 3.00    | 1.11               |
| Damage parameter (intercept of damage equation)                                   | 0.0028  | 0.0013             |
| Price of backstop technology (\$ per ton C removed at 100 % removal)              | 1,170   | 468                |
| Asymptotic global population (millions)   | 8,600   | 1,892              |
| Transfer coefficient in carbon cycle  | 0.189   | 0.017              |
| Total resources of fossil fuels (billions of tons of carbon)                      | 6,000   | 1,200              |

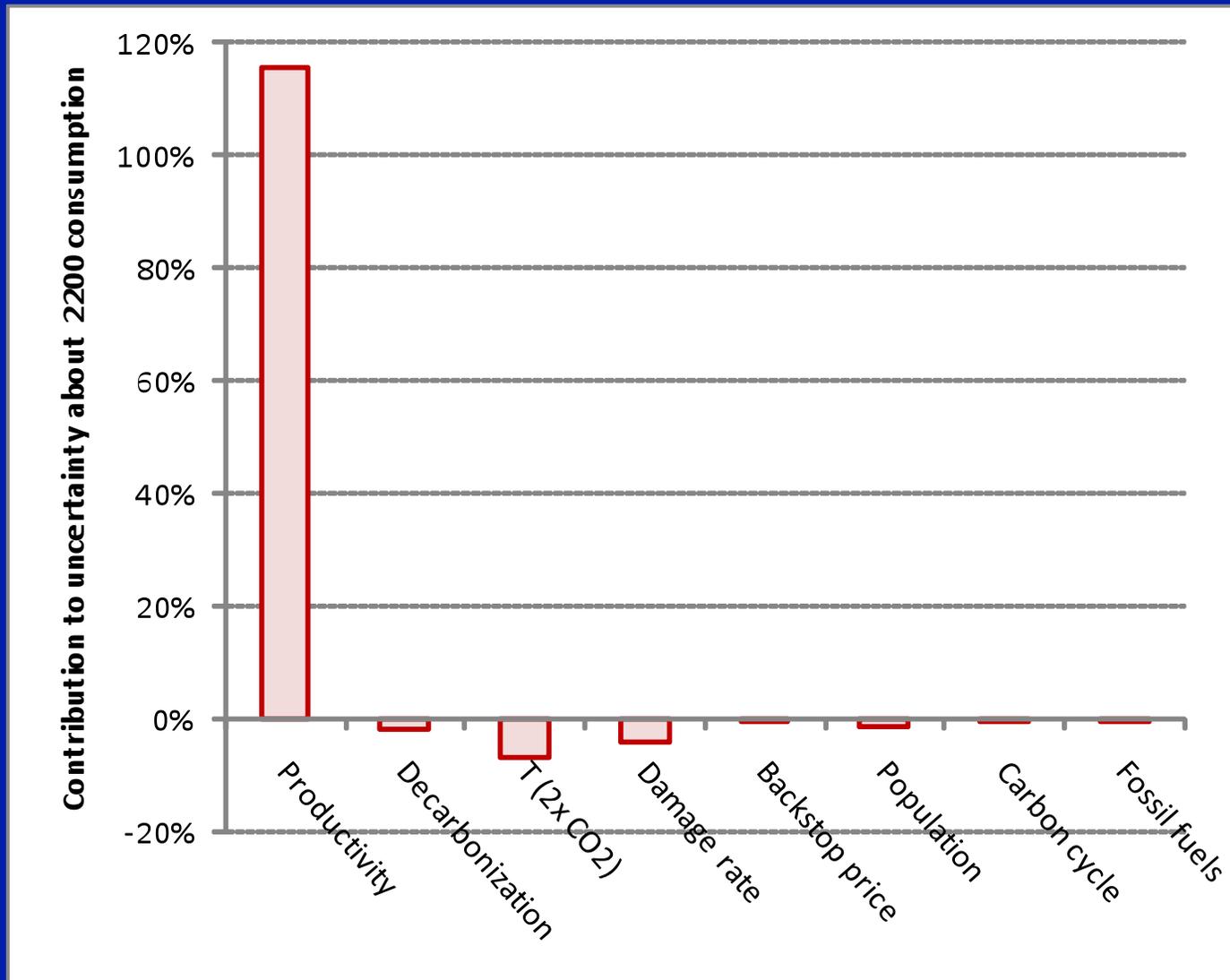
## Uncertainty for future temperature: DICE 2007 baseline



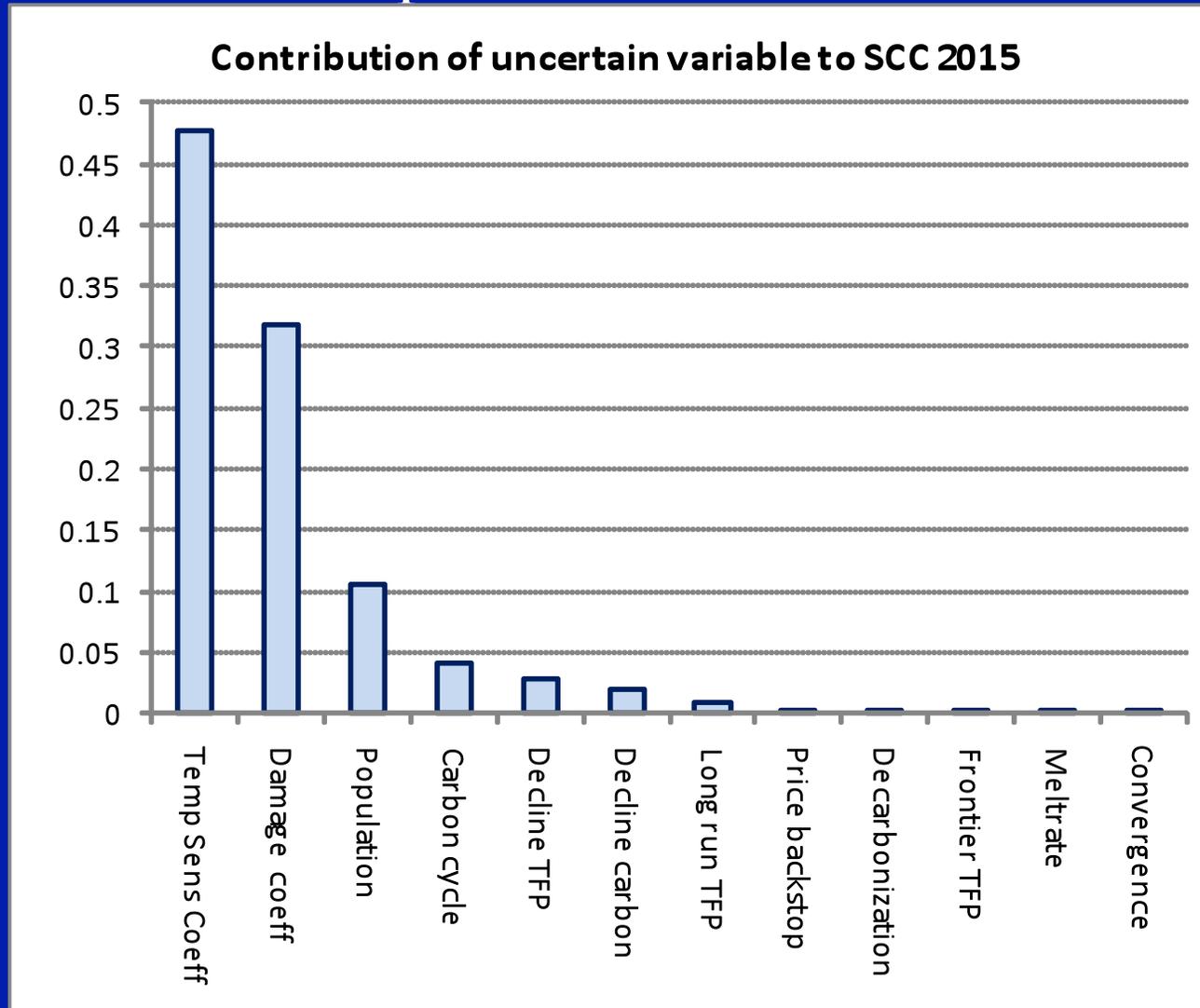
# Normalized contribution of uncertain variable: impact on temperature, 2200



# Normalized contribution of uncertain variable: impact on per capita consumption, 2200



# Normalized contribution of uncertain variable: Impact on SCC 2015\*

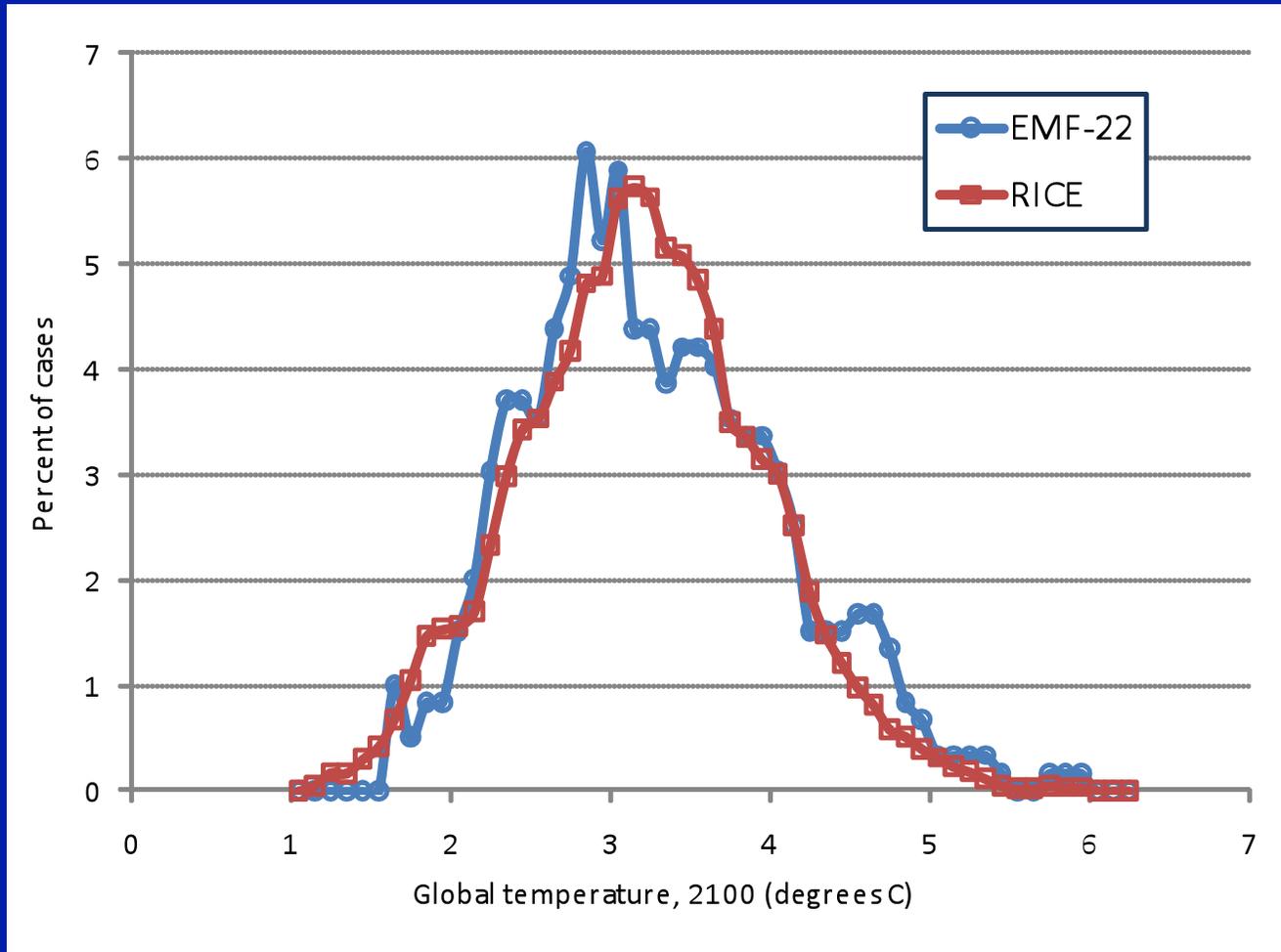


\*Note that elasticity of consumption and rate of time preference are not included.

## Uncertainty estimates for the T(2100)

- Looked at two different approaches:
  - 1. Used the approach above of Monte Carlo for RICE-2011
  - 2. Used a combination of uncertainty of economic and climate models
- Latter requires some explanation:
  - Took 12 models from EMF-22 reference cases
  - Took 17 climate models from IPCC AR4 with simple parameterization
- Looked only at CO<sub>2</sub> concentrations
- Results surprisingly similar (next slide)

# Histogram of T(2100) under two approaches



## *The macro risk question: theory*

Should we be paying a large risk premium on high-climate scenarios? The idea is that we would invest more than the certainty equivalent to prevent the high-climate outcome.

Earlier studies generally assume yes. These approaches applied a risk-averse utility function to the damages. (This was the approach in Nordhaus-Boyer 2000 and Stern 2007.)

This is a partial-equilibrium approach. The correct approach would be to apply modern risk analysis in the consumption capital asset pricing model.

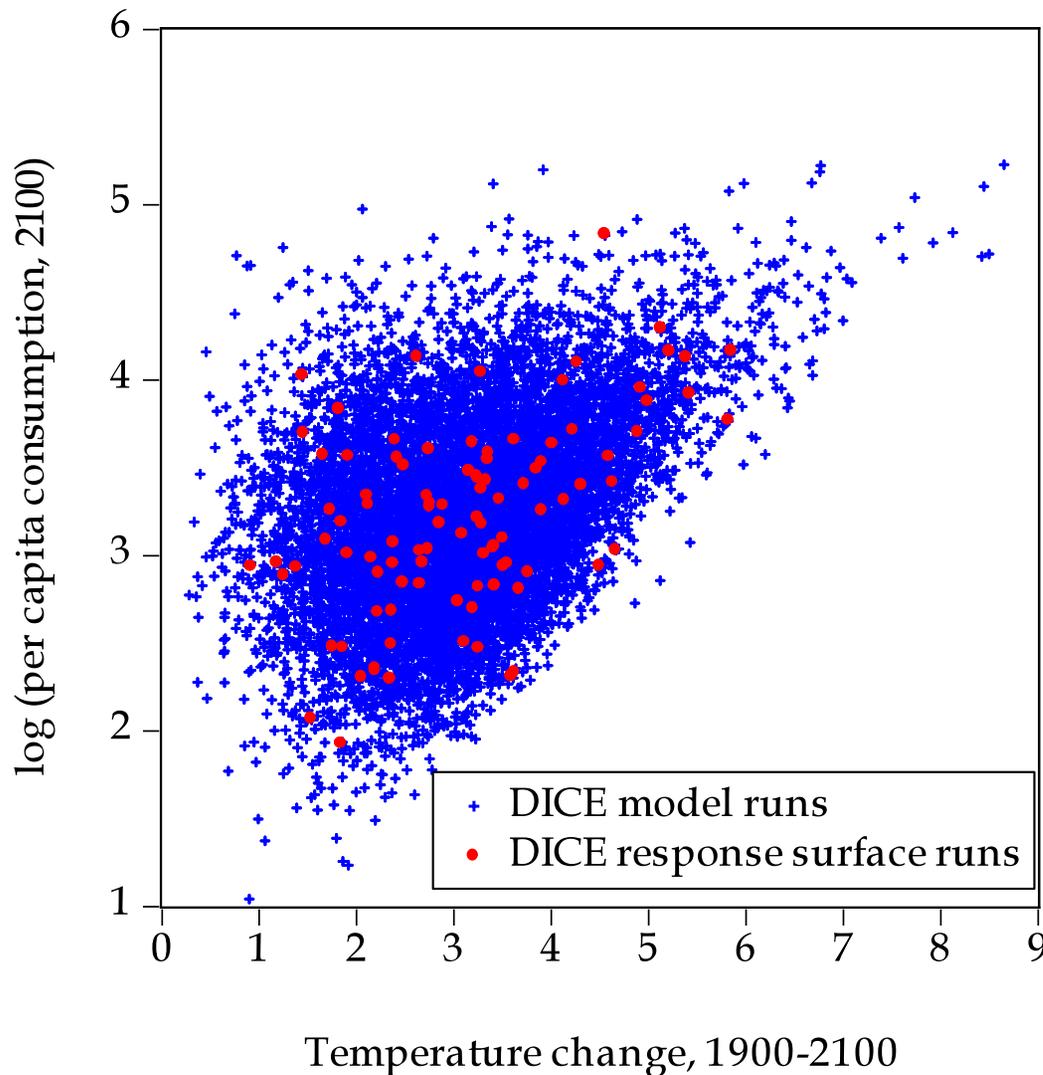
In this framework, want to pay a risk premium  $\leftrightarrow$  the high-climate outcome is positively correlated with high MU consumption (or negatively correlated with consumption).

Begin with a utility function,  $U(C)$ . Examine the expected utility in year  $t$  (say 2100). This is:

$$E(U) = \int_{\text{SOW } i} p^i U[C(\boldsymbol{\theta}^i)]$$

In this,  $p^i$  are probabilities, and  $\boldsymbol{\theta}^i = [\theta_1^i, \dots, \theta_8^i]$  are uncertain parameters, and  $i$  are uncertain states of the world (SOW). In general, we apply risk premia to SOWs where  $U'[C(\boldsymbol{\theta}^i)]$  or the MU of consumption is high. The question is, when do these occur?

# Consumption and 2100 Temperature

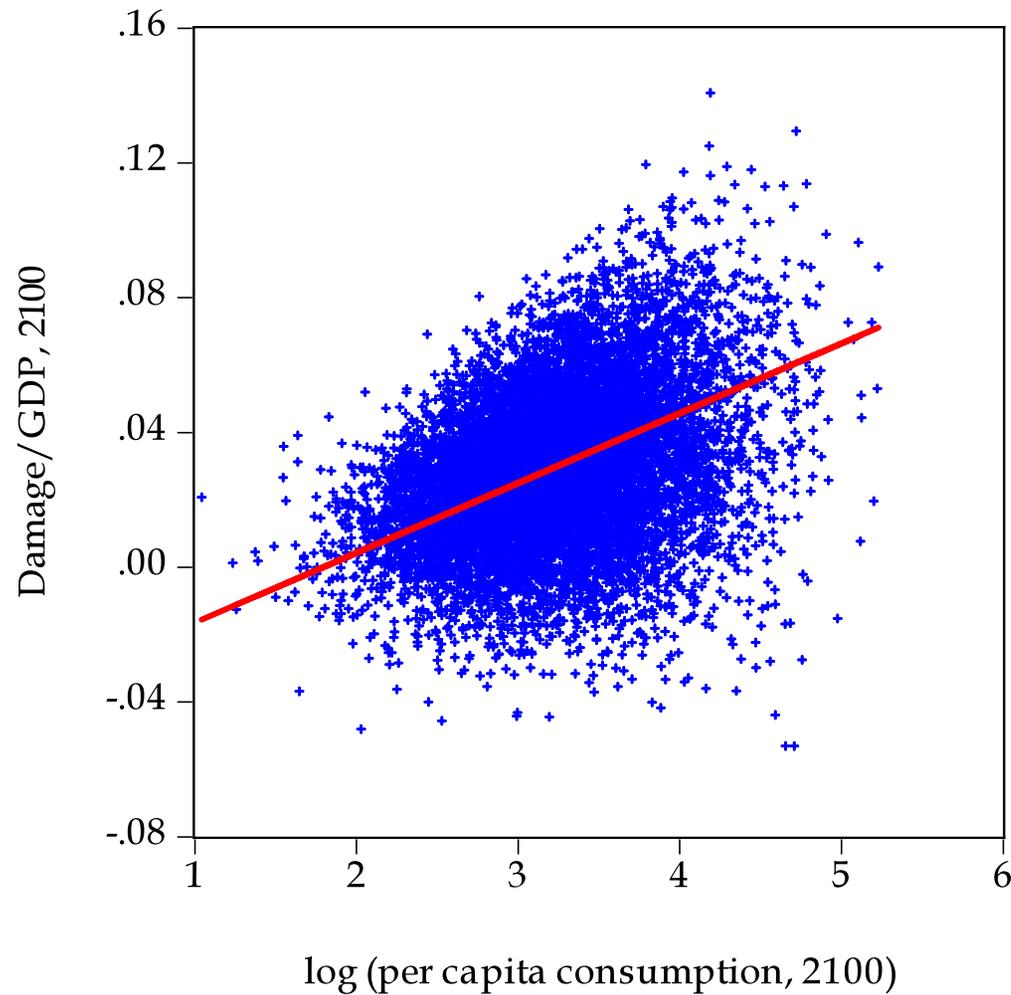


The red dots are 100 runs of DICE model with random draw of uncertain parameters

The blue dots are 10,000 runs of response-surface model.

Key result is that per capita consumption is positively correlated with temperature.

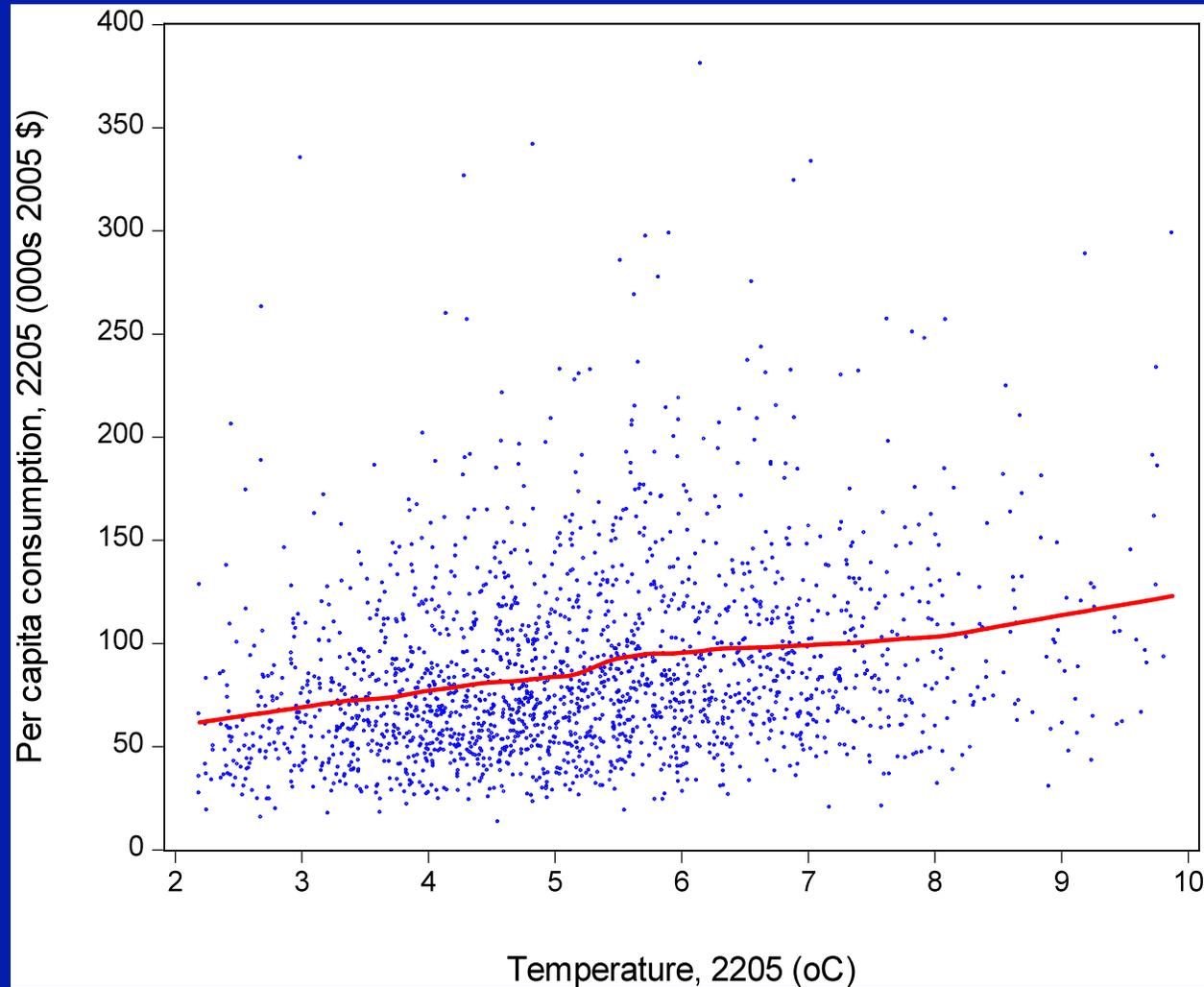
# Consumption and damage ratio



The blue dots are 10,000 runs of response-surface model.

Key result is that damage ratio is high when per capita consumption is high.

# Temperature and p.c. consumption (RICE-2011)



Update using  
RICE-2011  
model

## *Implications for Risk Premium*

Study suggests that the most important uncertainty in long-run is growth in productivity. High climate damages are associated with high growth rates of TFP.

- So good economic news = bad climate news in baseline.
- Think of the \$2500 car.

But this also means that the worst climate cases are ones in which the world is *rich*, which is a situation where we are more likely to be able to afford more costly climate abatement.