Overhead Slides for

Chapter 20

of

Fundamentals of Atmospheric Modeling

by

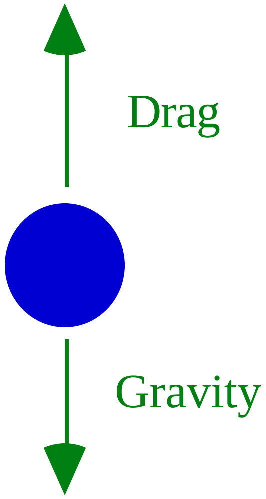
Mark Z. Jacobson

Department of
Civil & Environmental Engineering
Stanford University
Stanford, CA 94305-4020

January 30, 2002

Sedimentation

Fig. 20.1. Vertical forces acting on a particle



Drag force for Stokes flow

$$F_D = 6\pi r_i \eta_a V_{f,i} \qquad (20.1)$$

Drag force for Slip flow

$$F_D = \frac{6\pi r_i \eta_a V_{f,i}}{G_i} = \frac{6\pi r_i \eta_a V_{f,i}}{1 + \mathrm{Kn}_{a,i}\left[\mathrm{A}' + \mathrm{B}'\exp\left(-\mathrm{C}'\mathrm{Kn}_{a,i}^{-1}\right)\right]} \qquad (20.2)$$

Gravitational force

$$F_G = \frac{4}{3}\pi r_i^3\left(\rho_p - \rho_a\right)g \qquad (20.3)$$

Sedimentation

Solve for estimated fall velocity

$$V_{f,i}^{est} = \frac{2r_i^2\left(\rho_p - \rho_a\right)g}{9\eta_a}G_i \qquad (20.4)$$

Small particles

Less resistance to motion ---> diffusion and sedimentation velocity enhanced at small particle sizes

Large particles

Sedimentation velocity decreases due to physical properties effect

Physical Properties Correction

Estimate Reynolds number from estimated fall velocity.

$$\mathrm{Re}_i^{est} = 2r_i V_{f,i}^{est} / \nu_a \qquad (20.5)$$

Recalculate Reynolds number for three flow regimes

- slip flow around a rigid sphere («1-20 μm diameter)
- continuum flow around a rigid sphere (20 μm - 1 mm)
- continuum flow around equilibrium-shaped drop (1 - 5 mm)

$$\mathrm{Re}_i^{final} = \begin{cases} 2r_i V_{f,i}^{est} / \nu_a & \mathrm{Re}_i^{est} < 0.01 \\ G_i \exp\left(B_0 + B_1 X + B_2 X^2 + \ldots\right) & 0.01 < \mathrm{Re}_i^{est} < 300 \\ N_P^{1/6} G_i \exp\left(E_0 + E_1 Y + E_2 Y^2 + \ldots\right) & \mathrm{Re}_i^{est} > 300 \end{cases}$$

$$(20.6)$$

X and Y = functions of physical properties (*e.g.*, temperature, pressure, air density, air viscosity, particle density, surface tension, and gravity)

$$X = \ln\left[\frac{32 r_i^3 \left(\rho_p - \rho_a\right)\rho_a g}{3\eta_a^2}\right] \qquad (20.7)$$

$$Y = \ln\left[\frac{4}{3} \mathrm{N}_{Bo} \mathrm{N}_P^{1/6}\right] \qquad (20.7)$$

Physical Properties Correction

Physical property number

$$N_p = \frac{\sigma_{w/a}^3 \rho_a^2}{\eta_a^4 \left(\rho_p - \rho_a\right) g} \tag{20.8}$$

Bond number

$$N_{Bo} = \frac{4 r_i^2 \left(\rho_p - \rho_a\right) g}{\sigma_{w/a}} \tag{20.8}$$

Final fall velocity recalculated from final Reynolds number

$$\boxed{V_{f,i} = V_{f,i}^{final} = \mathrm{Re}_i^{final} \nu_a \big/ 2 r_i \qquad (20.9)}$$

Sedimentation Times

Table 20.2. Time for particles to fall 1 km in the atmosphere due to sedimentation.

Particle Diameter (μm)	Time to Fall 1 km (s)	Particle Diameter (μm)	Time to Fall 1 km (s)
0.02	228 y	5.0	14.5 d
0.1	36 y	10.0	3.6 d
0.5	3.2 y	20.0	23 h
1.0	328 d	100.0	1.1 h
2.0	89 d	1000.0	4 m
4.0	23 d	5000.0	1.8 m

Dry Deposition

Dry deposition

- Gases or particles contact a surface and stick to or react with the surface

Gas dry deposition velocity

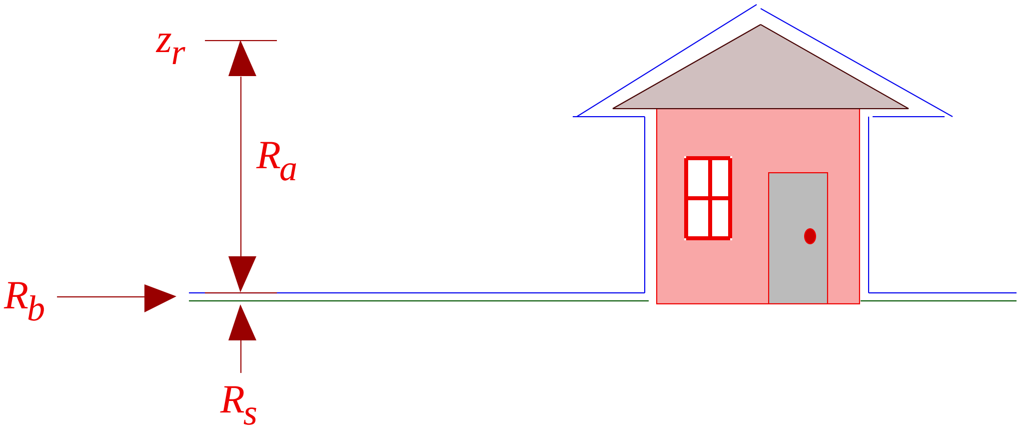
$$V_{d,gas} = \left(R_a + R_b + R_s\right)^{-1} \qquad (20.10)$$

Particle dry deposition velocity

$$V_{d,part,i} = \left(R_a + R_b + R_a R_b V_{f,i}\right)^{-1} + V_{f,i} \; . \qquad (20.11)$$

Dry Deposition Resistances

Fig. 20.2. Locations where gas dry deposition resistances apply. Figure not to scale.



Dry Deposition Resistances

Aerodynamic resistance

$$R_a = \frac{\int_{z_{0,q}}^{z_r} \varphi_h \frac{\mathrm{dz}}{z}}{ku_*} \qquad (20.12)$$

Resistance to diffusion in laminar sublayer

$$R_b = \ln\left(\frac{z_{0,m}}{z_{0,q}}\right)\frac{\left(\mathrm{Sc}/\mathrm{Pr}\right)^{2/3}}{ku_*} \qquad (20.14)$$

Particle Schmidt number

$\mathrm{Sc}_{pi} = \nu_a / D_{pi}$

Gas Schmidt number

$\mathrm{Sc}_q = \nu_a / D_q$

Prandtl number

$\mathrm{Pr} = \eta_a c_p / \kappa_d$

Surface Resistance

Surface resistance due to biological interactions

$$R_s = \left(\frac{1}{R_{stom} + R_{meso}} + \frac{1}{R_{cut}} + \frac{1}{R_{conv} + R_{exp}} + \frac{1}{R_{canp} + R_{soil}} \right)^{-1} \tag{20.15}$$

Stomatal resistance

$$R_{stom,q} = R_{min} \left[1 + \left(\frac{200}{S_f + 0.1} \right)^2 \right] \left[\frac{400}{T_{a,c}\left(40 - T_{a,c}\right)} \right] \frac{D_v}{D_q} \tag{20.16}$$

Leaf mesophyll resistance

$$R_{meso,q} = \left(\frac{H_q^*}{3000} + 100 f_{0,q} \right)^{-1} \tag{20.17}$$

Resistance to deposition on leaf cuticles

$$R_{cut} = R_{cut,dry,q} = R_{cut,0}\left(10^{-5} H_q^* + f_{0,q}\right)^{-1} \tag{20.18}$$

Surface Resistance

Resistance of gases due to buoyant convection

$$R_{conv} = 100\left(1 + \frac{1000}{S_f + 10}\right)\frac{1}{1 + 1000 s_t} \tag{20.19}$$

Resistance due to deposition on bark, other exposed surfaces

$$R_{exp,q} = \left(\frac{10^{-5} H_q^*}{R_{exp,\mathrm{SO_2}}} + \frac{f_{0,q}}{R_{exp,\mathrm{O_3}}}\right)^{-1} \tag{20.20}$$

In-canopy resistance

$$R_{canp} = \frac{b_c h_c \mathrm{LAI}}{u_*} \tag{20.21}$$

Resistance due to soil and leaf litter at ground surface

$$R_{soil,q} = \left(\frac{10^{-5} H_q^*}{R_{soil,\mathrm{SO_2}}} + \frac{f_{0,q}}{R_{soil,\mathrm{O_3}}}\right)^{-1} \tag{20.22}$$

One-sided leaf area index (LAI)

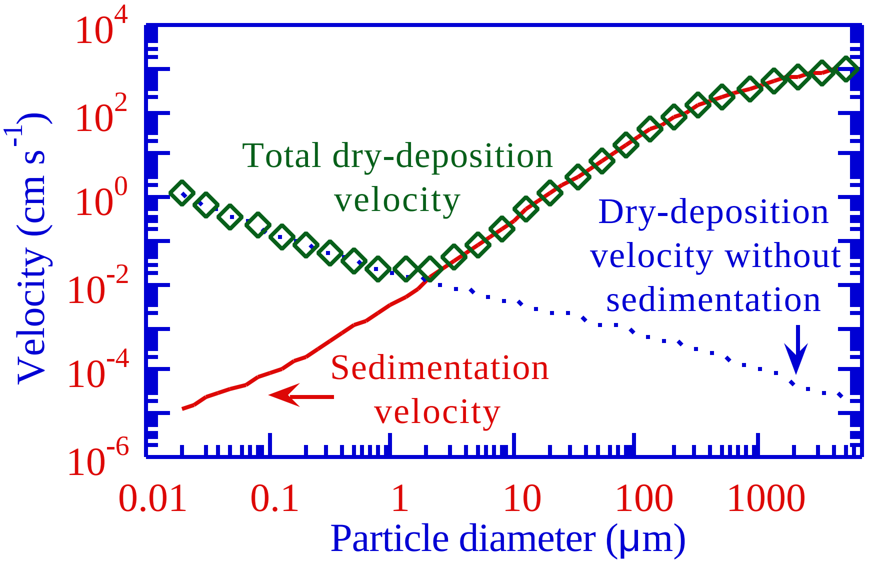
Integrate foliage area density from surface to height h_c

Foliage area density

Area of plant surface per unit volume of space. Thus, the LAI measures canopy area density

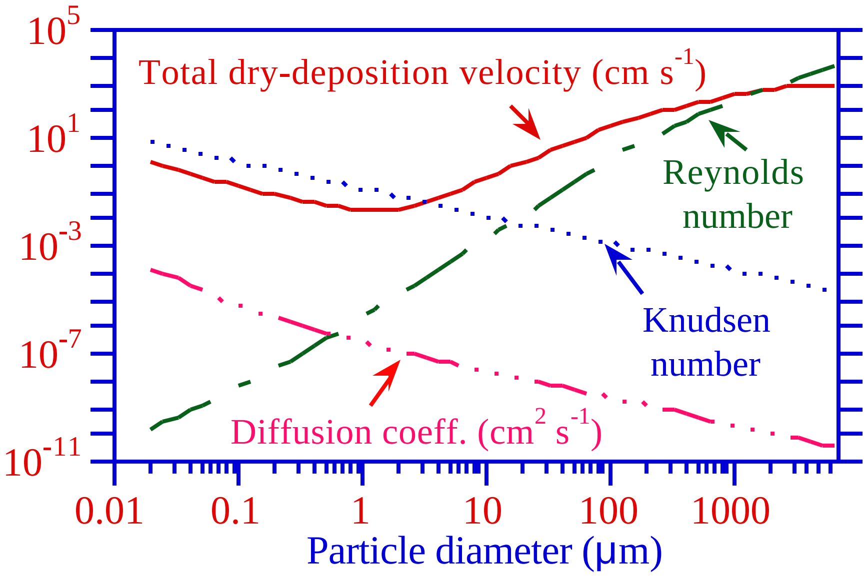
Dry Deposition and Sedimentation Velocities

Fig. 20.3. Calculated sedimentation, dry deposition, and total deposition velocities vs. particle diameter.



Dry Deposition Velocity and Associated Parameters

Fig. 20.4. Plot of total deposition velocity near the surface, Reynolds number, Knudsen number for air, and particle diffusion coefficient as a function of particle diameter.



Gas Dry Deposition Velocities

Figs. 20.5 a and b. Gas dry deposition velocities versus surface resistance when (a) $z_{0,m}$ = 3 m and (b) $z_{0,m}$ = 0.01 m, and when molecular weigh and wind speed were (i) 10 g mole^{-1}; 10 m s^{-1}, (ii) 130 g mole^{-1}; 10 m s^{-1}, (iii) 10 g mole^{-1}; 0 m s^{-1}, and (iv) 130 g mole^{-1}; 0 m s^{-1}, respectively. Also, z_r = 10 m, $T(z_r)$ = 288 K, $T(z_{0,m})$ = 290 K, and $p_a\ (z_r)$ = 999 mb.