

## **Chapter 7**

# **ERRATA TO 2ND PRINTING OF *ANGULAR MOMENTUM***

The following errata have been collected from many sources to which we express our sincere gratitude. Special thanks go to Melissa A. Hines, Alexei Buchachenko, Shan Tao Lai, Andrew J. Orr-Ewing, and Yo Fujimura who communicated numerous corrections.

**pg. 10** In Eq. (1.57) for the expression for  $Y_{2,\pm 1}(\theta, \phi)$  change  $16\pi$  to  $8\pi$  in the square root sign.

**pg. 36** In Eq. (45) change  $-$  sign to  $+$  sign in front of  $\frac{1}{b}(\frac{\pi}{2})$ .

**pg. 36** In Eq. (46) replace  $f[b(x), y]$  by  $f[x, b(x)]$  and replace  $f[a(x), y]$  by  $f[x, a(x)]$ .

**pg. 36** In the two lines below Eq. (46) replace  $b/k$  by  $1/k$ .

**pg. 38** Replace twice  $k$  by  $\kappa$  in the expression in the line below Eq. (58).

**pg. 39** In the ordinate of Fig. 5 replace  $E_0 - E$  by  $(E_0 - E)$ .

**pg. 43** In the third line below Eq. (2.1) replace Eq. (1.16) by Eq. (1.8).

**pg. 66** In line 9 replace “total angular momentum” by “total orbital angular momentum.”

**pg. 90** In both Eq. (3.77) and Eq. (3.78) replace superscript  $\dagger$  by superscript  $*$ , and replace  $d_{MM'}^J(\theta)$  by  $d_{M'M}^J(\theta)$ .

**pg. 91** In line 1 replace  $\dagger$  by  $*$  and omit “transpose.”

**pg. 99** Fig. 3.6 is misdrawn;  $\mathbf{J}_2$  should connect to  $\mathbf{J}_1$  and  $\mathbf{J}_3$ .

**pg. 99** In the second to last line on the page change “Figures 1.1, 1.2, 1.3, and 2.2” to read “Figures 1.1, 2.1, 2.2, and 2.3.”

**pg. 101** In the exponent of  $(-1)$  in Eq. (3.111) change  $J_1 - M_1$  to  $J_1 + M_1$ .

**pg. 101** In the exponent of  $(-1)$  in Eq. (3.112) change  $J_1 - M_1'$  to  $J_1 + M_1'$ .

**pg. 117** In the left side of Eq. (2) replace  $P_{JM}(\theta)$  by  $P_{JM}(\theta)d\Omega$ .

**pg. 144** In Eq. (4.3) replace  $\langle j_{12} j_3 j | j_1 j_{23} j' \rangle$  by  $\langle j_{12} j_3 j' | j_1 j_{23} j \rangle$ .

**pg. 149** In the second row of the  $9-j$  symbol appearing once in Eq. (4.21) and twice in Eq. (4.22) interchange  $j_3$  and  $j_4$ .

**pg. 152** Rewrite rules 1 and 2 to read:

1. To add an arrow pointing toward or to drop an arrow pointing away from a particular  $jm$  pair, multiply the diagram by  $(-1)^{j+m}$  and change the sign of  $m$  in the diagram.
2. To add an arrow pointing away or drop an arrow pointing toward a particular  $jm$  pair, multiply the diagram by  $(-1)^{j-m}$  and change the sign of  $m$  in the diagram.

**pg. 164** In the graphical diagram at the bottom of the page omit the central line  $j_{1234} = 0$  and its label and add two lines connecting the  $+$  and  $-$  nodes; label these lines by  $j_{12}$  and  $j_{34}$ .

**pg. 167** In Eq. (4.58) in the graph in the first line at the top of the page change from  $+$  to  $-$  the node on the right and the node on the bottom. Then in the graph in the second line of this equation, change from  $+$  to  $-$  the node on the top and the central node. Finally, remove the phrase starting with “where ...”

**pg. 174** In Eq. (4.65) change  $\mathbf{j}_2$  to  $\mathbf{j}$ .

**pg. 174** In the first line below Eq. (4.67) change  $\mathbf{j}_{23}$  to  $|\mathbf{j}_{23}|$ .

**pg. 174** In Eq. (4.67) add a minus sign before

$$\frac{d}{dt}(\mathbf{j}_{23} \cdot \mathbf{j}).$$

**pg. 174** In Eq. (4.69) change  $(\mathbf{j}_1 \times \mathbf{j})$  to  $(\mathbf{j}_{12} \times \mathbf{j})$ .

**pg. 176** In Eq. (4.71) change  $j_9$  to  $j_7$  in the last 6- $j$  symbol on the right side of this equation.

**pg. 178** In the third line from the bottom, change (see Application 4) to (see Eq. (14) of Application 4).

**pg. 179** In three lines below Eq. (5.8) change Eq. (5.9) to Eq. (5.8).

**pg. 181** In the line below Eq. (5.14) insert the factor  $(-1)^{k+j-j'}$  in front of  $(2j+1)^{1/2}$ .

**pg. 183** In the second line of Eq. (5.25) replace  $\langle 3 || L || 3 \rangle$  by  $\langle 3 || L^{(1)} || 3 \rangle$ .

**pg. 187** In Eq. (5.38) change  $\sum_{q'}$  to  $\sum_{q',q'_1}$ .

**pg. 188** In Eq. (5.46) in the first and second lines replace  $\sum_q$  by  $\sum_{q,q'}$ .

**pg. 189** In the line above Eq. (5.50) replace Eq. (4.21) by Eq. (4.20).

**pg. 190** In the last line of the first paragraph replace “the gradient of the electric field” by “the gradient of the gradient of the electric field.”

**pg. 194** In Eq. (5.67) add  $+2j_1$  to the exponent of  $(-1)$ .

**pg. 194** In Eq. (5.69) add  $\delta_{jj'}$  to the last line on the right side.

**pg. 195** In Eq. (5.70) add on the right side  $\delta_{jj'}$ .

**pg. 201** In Eq. (5.85) replace  $\psi(\alpha_e J_e M_e; t = 0)$  by  $\psi(\alpha_e J_e; t = 0)$ .

**pg. 201** In Eqs. (5.86), (5.87), and (5.88) replace  $\psi(\alpha_e J_e M_e; t)$  by  $\psi(\alpha_e J_e; t)$ .

**pg. 204** In the last line of this page remove the parentheses about  $\hat{\mathbf{e}}_a$  and  $\hat{\mathbf{e}}_a^*$ .

**pg. 209** In Eq. (5.118) insert a minus sign before the right side.

**pg. 209** In two lines below Eq. (5.118) insert a minus sign before  $e(1, -1)$ .

**pg. 216** In Eq. (5.133) replace  $\dagger$  by  $*$ .

**pg. 217** In Eq. (5.134) and in Eq. (5.136) change  $\dagger$  to  $*$  twice in each equation.

**pg. 222** In Eq. (13) second line from bottom of page change  $j'$  to  $j$ .

**pg. 224** In right side of Eq. (17) change  $-1$  to  $1$ .

**pg. 228** Below Eq. (23) it should read: In the general case our system is in a *mixed state* which is represented by a density operator  $\rho$  that is an incoherent superposition of a number of pure states  $|\psi^{(i)}\rangle$  with statistical weights  $W^{(i)}$ .

$$\rho = \sum_i W^{(i)} |\psi^{(i)}\rangle \langle \psi^{(i)}| \quad (24)$$

where

$$\sum_i W^{(i)} = 1. \quad (25)$$

**pg. 228** In the line below Eq. (25) change “in” to “In.”

**pg. 229** In Eq. (27) insert “ $\langle JM | \rho | JM' \rangle =$ ” before the expression on the right side of this equation.

**pg. 229** In the sixth line from the bottom of the page, change  $a_M$  to  $a_M^{(i)}$ .

**pg. 230** Change the sentence in line 13 to read: A pure state represents a completely ordered ensemble, whereas the mixed state that is uniform is in a state of maximum disorder.

**pg. 230** Change the sentence in line 17 to read: Then for a pure state  $S = 0$ , whereas for a state of maximum disorder  $S = \ln(2J + 1)$ .

**pg. 232** Add “cosine of the” before “angle” in part J.

**pg. 237** In the exponent of  $(-1)$  in Eqs. (62) and (63) change  $M'$  to  $M$ .

**pg. 238** In Eqs. (67) and (69) insert 3 in front of  $O_0^{(1)}(J_i)$  and after  $A_0^{(2)}$  insert the factor

$$\left[ \frac{5J_i(J_i + 1)}{(2J_i + 3)(2J_i - 1)} \right].$$

**pg. 238** In Eq. (68) change  $=$  to  $\propto$ .

**pg. 239** In the right side of Eq. (71) change  $A_0^{(2)}(J_f)$  to  $A_0^{(2)}(J_i)$ .

**pg. 239** In the right side of Eq. (72) it should read:

$$\left[ 1 - \frac{1}{(J_f + 1)^2} \right]^{1/2}.$$

**pg. 239** In the right side of Eq. (73) change  $(2J_f + 3)$  to  $(2J_f + 1)$ .

**pg. 239** In the right side of Eq. (76) make it read:  $\left[ 1 - \frac{1}{J_f^2} \right]^{1/2}$ .

**pg. 239** In the right side of Eq. (77) change  $(2J_f - 1)$  to  $(2J_f + 1)$ .

**pg. 240** In the Eqs. (79) and (80) drop the subscript  $q$  on  $J$  and the subscript 0 on  $I$ .

**pg. 240** In the right side of Eq. (81) insert the factor  $[(2F' + 1)(2F + 1)]^{1/2}$  inside the summation sign.

**pg. 241** In Eq. (84) add a minus sign in front of the factor  $i(E_{F'} - E_F)t/\hbar$  that appears in the first and third lines of this equation.

**pg. 250** In the fourth line of Eq. (22) add a minus sign in front of  $(5)^{1/2}$ .

**pg. 271** Replace the paragraph starting with “Eq. (6.63) is ...” by “Note that Eq. (6.58), (6.59), and (6.60) are valid for integer and half-integer  $J$  and  $K$ , whereas Eq. (6.63) holds true only for integral  $J$  and  $K$ .”

**pg. 280** In Eq. (6.99) insert  $i$  in front of  $(E_j - E_k)$ .

**pg. 281** In line 2 change Eq. (6.97) to Eq. (6.98).

**pg. 281** In Eq. (6.103) in the first line for  $\langle j | \mathcal{G}_3 | k \rangle$  change  $(E_k - E_\alpha)$  to  $(E_k - E_\beta)$ .

**pg. 282** In Eq. (6.105) in its first line, replace twice  $\mathcal{H}_R$  by  $\mathcal{H}_1$ .

**pg. 284** In Eq. (6.112) replace  $\pi$  by  $\pi^4$ .

**pg. 285** Eq. (6.117) should read:

$$\begin{aligned}
S(J'K'; J''K'') &= 3 \sum_{M', M''} \left| \left[ \left( \frac{2J'+1}{8\pi^2} \right) \left( \frac{2J''+1}{8\pi^2} \right) \right]^{1/2} \int D_{M'K'}^{J'} D_{0K'-K''}^{1*} D_{M''K''}^{J''} d\Omega \right|^2 \\
&= 3 \sum_{M', M''} \left| \frac{[(2J'+1)(2J''+1)]^{1/2}}{8\pi^2} \left[ \int D_{M'K'}^{J'*} D_{0K'-K''}^1 D_{M''K''}^{J''} d\Omega \right]^* \right|^2 \\
&= 3 \sum_{M', M''} \left| \left( \frac{2J''+1}{2J'+1} \right) \langle J''M'', 10 | J'M' \rangle \langle J''K'', 1K' - K'' | J'M' \rangle \right|^2 \\
&= 3 \frac{2J''+1}{2J'+1} \langle J''K'', 1K' - K'' | J'K' \rangle^2 \sum_{M', M''} \langle J''M'', 10 | J'M' \rangle^2 \\
&= 3 \frac{2J''+1}{2J'+1} \langle J''K'', 1K' - K'' | J'K' \rangle^2 \sum_M \left( \frac{2J'+1}{3} \right) \langle J''M'', J' - M | 10 \rangle^2 \\
&= (2J''+1) \langle J''K'', 1K' - K' | J'K' \rangle^2 \\
&= (2J'+1)(2J''+1) \begin{pmatrix} J'' & 1 & J' \\ K'' & K' - K'' & -K' \end{pmatrix}^2
\end{aligned}$$

**pg. 287** In Eq. (6.123) insert the phase factor  $(-1)^{J'-1+K''}$  in front of the 3- $j$  symbol inside the double summation.

**pg. 294** In Eq. (6.142) change  $\omega$  to  $\omega$  in the middle term.

**pg. 297** In last line insert after rotation: for  $M = J$ .

**pg. 298** In line 2 insert after rotation: for  $M = J$ .

**pg. 303** Change Eq. (34) to read:

$$|\psi(F_2)\rangle = a_J \left| {}^2\Pi_{\frac{3}{2}}vJ \right\rangle - b_J \left| {}^2\Pi_{\frac{1}{2}}vJ \right\rangle.$$

**pg. 303** Change Eq. (35) to read:

$$|\psi(F_1)\rangle = b_J \left| {}^2\Pi_{\frac{3}{2}}vJ \right\rangle + a_J \left| {}^2\Pi_{\frac{1}{2}}vJ \right\rangle.$$

**pg. 304** In the second sentence change  $F_1$  and  $F_2$  to  $F_2$  and  $F_1$ .

**pg. 306** In line 13 change former to latter.

**pg. 306** Change Eq. (38) to read:

$$\sigma_v(yz) |n \Lambda\rangle = (-1)^s |n - \Lambda\rangle.$$

**pg. 306** Change Eq. (39) to read:

$$\sigma_v(yz) |S \Sigma\rangle = (-1)^s |S - \Sigma\rangle.$$

**pg. 306** In the first line above Eq. (40) change  $xz$  to  $yz$  and change  $\chi \rightarrow -\chi$  to  $\chi \rightarrow \pi - \chi$ .

**pg. 306** Change Eq. (40) to read:

$$Y_{L\Lambda}(\theta, \chi) \rightarrow Y_{L\Lambda}(\theta, \pi - \chi) = (-1)^\Lambda Y_{L\Lambda}^*(\theta, \chi) = Y_{L-\Lambda}(\theta, \chi).$$

**pg. 307** In line 3 change  $\sigma_v(xz), y \rightarrow -y$  to  $\sigma_v(yz), x \rightarrow -x$ .

**pg. 307** In line 5 change  $y$  to  $x$  and change  $C_2(y)$  to  $C_2(x)$ .

**pg. 307** In line 6 change  $C_2(y)$  to  $C_2(x)$ .

**pg. 307** Change Eqs. (41) and (42) by replacing  $(0, \pi, 0)$  by  $(\pi, \pi, 0)$ ; the right side of Eq. (41) is  $e^{i\pi/2} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ ; and the right side of Eq. (42) is  $e^{i\pi/2} \left| \frac{1}{2}, \frac{1}{2} \right\rangle$ .

**pg. 307** Change Eq. (43) to read:

$$\sigma_v(yz) \left| \frac{1}{2}, \sigma \right\rangle = (-1)^{1/2} \left| \frac{1}{2}, -\sigma \right\rangle.$$

**pg. 307** In line 9 of the second paragraph the sentence should read: On reflection we interchange  $\alpha$  and  $\beta$  in the uncoupled state so that  $\Sigma \rightarrow -\Sigma$ , and we pick up a phase factor  $(-1)^x$ , where  $x$  equals the number of electrons divided by two, i.e.,  $x = n/2$ . Then omit the sentence beginning with “Since...”

**pg. 307** In Eq. (45) change  $\sigma_v(xz)$  to  $\sigma_v(yz)$ , and change  $(-1)^{S-\Sigma}$  to  $(-1)^S$ .

**pg. 308** In the third line below Eq. (46) change  $x \rightarrow x, y \rightarrow -y$ , and  $z \rightarrow z$  to read:  $x \rightarrow -x, y \rightarrow y$ , and  $z \rightarrow z$ .

**pg. 308** In the fourth line below Eq. (46) change  $\pi - \chi$  to  $-\chi$  and change  $C_2(y)$  to  $C_2(x)$ .

**pg. 308** In the fifth line below Eq. (46) change Eq. (6.59) to Eq. (6.60).

**pg. 308** In Eq. (47) change  $J - \Omega$  to  $-J$ .

**pg. 308** In Eq. (48) in the second line change  $\Lambda + s$  to  $s$ ,  $S - \Sigma$  to  $S$ , and  $J - \Omega$  to  $-J$ .

**pg. 312** Replace  $a_{n\Omega}$  by  $a_{n\Sigma\Omega}$  in Eq. (65) and twice in the second line from the bottom.

**pg. 312** In the third line from the bottom of the page, change  $\delta_{0\Lambda}$  to  $\delta_{0,\Lambda}$ .

**pg. 313** Change Eq. (66) to read:

$$S(J'; J) = (2J' + 1)(2J + 1) \times \left| \sum_{\Omega'} \sum_{\Omega} a_{n'\Sigma'\Omega'}(p'^{\pm}) a_{n\Sigma\Omega}(p^{\pm}) \delta_{\Sigma,\Sigma'} (-1)^{J'-1+\Omega} \begin{pmatrix} J & 1 & J' \\ \Omega & \Omega' - \Omega & -\Omega' \end{pmatrix} \right|^2.$$

**pg. 316** In reference 27, change  $C_2(x)\sigma_v(yz)$  to  $C_2(y)\sigma_v(xz)$ .

**pg. 316** Update the last line of reference 29 to read: M. H. Alexander et al. *J. Chem. Phys.* **89**, 1749 (1988).

**pg. 317** Reference 31 should be rewritten to read:

As Alexander and Dagdigian [29] show, the electron distribution in the  $F_1 e \Lambda$ -doublet level is oriented preferentially in the plane of rotation and in the  $F_1 f \Lambda$ -doublet level oriented preferentially perpendicular to the plane of rotation along  $\mathbf{J}$  for a single filled  $\pi$  orbital. The opposite applies to  $F_2 \Lambda$ -doublet levels. For a  $\pi^3$  configuration, the preferences are reversed. The treatment outlined in F is an approximation in which the integration over only  $\chi$  is considered. A more complete treatment in which the integration over  $\theta$  and  $\phi$  are included modifies Eq. (50).

**pg. 320** In Eq. 16 replace  $\psi_{LKM}(\Omega_0)$  by  $\psi_{LKM}(\Omega)$  and replace  $\psi_{LKM}(\Omega)$  by  $\psi_{LKM}(\Omega_0)$ .

**pg. 325** Eq. (A-3) should have the signs in front of  $\nu$  in the parentheses in the denominator  $-, -, -, +, +$  so that it agrees with Eq. (2.25).