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LONG-RUN TRENDS IN HOURS: A MODEL

by

Guillaume Vandenbroucke

Contact information:
Guillaume Vandenbroucke
Department of Economics
University of Southern California
Los Angeles, California 90089-0253, USA.
(213) 740-2134, Fax: (213) 740-8543
E-mail: vandenbr@usc.edu

Long-Run Trends in Hours: A Model *

Guillaume Vandenbroucke†
University of Southern California

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Abstract

During the 20th century the average number of hours worked per worker in the United States declined, and the distribution of hours across wage deciles narrowed. Coincidentally, the wage distribution narrowed and then widened again. The first half of the century is investigated. The hypothesis proposed is that (i) Home production of leisure services creates an incentive to work less on the market as wages increase and the price of leisure goods decrease (ii) The development of education accounts for the narrowing of the wage structure and, henceforth, of the distribution of hours.

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†Department of Economics, University of Southern California, 3620 S. Vermont Ave. KAP 324F, Los Angeles, CA 90089-0253, USA. Email: vandenbrATusc.edu.

1 *Introduction*

Over the course of the last two hundred years, in the United States, the length of the workweek declined significantly. The welfare implications of such trend are not captured by the growth of output per capita. Figure 1 represents average weekly hours in manufacturing and the non-farm business sector: The workweek declined, on average, at a rate of 0.37% per year between 1830 and 2004.

Figure 1 seems, on the surface, a contradiction to the usually accepted stationarity of hours. Such stationarity has been crucial in macro economic modelling, and researchers provided empirical evidence to support it. One must, of course, distinguish between hours per worker and hours per person. The latter is affected by participation rates which, for women, have increased considerably over the 20^{th} century, resulting in an increase of hours per women whereas hours per female workers declined (see Ríos-Rull (1993)). For male, both hours per worker and hours per person declined. The object of this research is the behavior of hours per worker.

Is the steady decline observed in Figure 1 an artifact of aggregation? For instance, women who usually have a shorter workweek (see McGrattan and Rogerson (2004), Table 2) are increasingly represented in the labor force. Consider then the workweek for male workers in manufacturing, represented in Figure 2. Note the sharp decline in the first half of the century and the relative stationarity after World War II. Figure 2 suggests that the workweek exhibits little to no trend in the second half of the century. As a matter of fact, McGrattan and Rogerson (2004) report that average weekly hours per worker exhibit a ∪-shape between 1950 and 2000 with, on average, a rate of change of -0.09% per year. They emphasize that this pattern is common to many subgroups of the population. During the pre-1950 period, however, the workweek declined unambiguously. Thus, this research investigates the behavior of hours per worker in the US economy during the period 1900-1950.

A few point a worth noticing. First, the workweek in various sectors, represented in Figure 3: From the 1870s until the 1950s, the length of the workweek declined in most sectors. Second, Figure 4 represents weekly hours per person for the United States and a set of industrialized countries: The reduction in hours is a widespread phenomena.[1] Third, non-market hours, i.e., hours of housework, also declined significantly over the course of the 20th century. This fact has been well documented. See for instance Greenwood, Seshadri, and Yorukoglu (2005). Fourth, workers take more vacations during the course of a given year: Lebergott (1976) reports that 6% of non-farm workers took vacations in 1901, 60% in 1950 and 80% in 1970. Finally the welfare implications of the reduction in the workweek are magnified by the fact that households spend a larger fraction of their lifetime out of the marketplace: The age of retirement has been declining for about a century. See Kopecky (2005) for a presentation of the facts and a model of the retirement decision.

Beside the trend in the average, the distribution of hours among different groups of the population has been far from stationary. For instance, McGrattan and Rogerson (2004) document important reallocations of hours among demographic groups for the post-World War II period. For the period of interest here, that is 1900-1950, such exercise is hard to reproduce. Indications exist, though, that there has been a remarkable change in the distribution of hours per wage decile. Costa (1998) emphasizes that workers at the bottom of the wage distribution experienced the biggest decline in hours between the 1890s and the 1990s. For instance, a worker in the lowest decile of the wage distribution in the 1890s works an average of 11 hours a day. In 1991, the same worker spends 8 hours a day at work. Over the same period, workers in the top decile do not experience a significant change in the amount of time at work. This finding is summarized by Figure 5.[2] It can be argued, from Fig-

[1] Figure 4 shows hours per person, so it should be taken as an indication that the same phenomena is likely to have occurred in more countries than just the US, not as evidence of it.

[2] Does the distribution of daily hours translates into a similar distribution for weekly or annual hours? Did the 1890 low-wage worker have a long day at work because of a shorter week? Costa presents evidence that such is not the case. For instance, those who reported

ure 5, that the dispersion of hours followed a non-monotonic path: Sometime between 1970 and the 1990s the distribution reversed and high-wage earners work more hours than low-wage earners. This has already been pointed out in the literature. See, for instance, Ríos-Rull (1993). Unfortunately, data do no exist to assess when, between 1890 and 1973, the minimum dispersion is attained. Table 1 reports summary statistics for the distribution of hours of men and women. Such changes in the distribution of hours per wage decile

	1870	1970	1990
Men	1.22	1.07	0.92
Women	1.26	0.98	0.81

Table 1: Ratio of Daily Hours Worked of 10th to 90th Wage Decile.

has important welfare implications. On this sole ground, one could infer that workers at the bottom of the wage distribution benefited more from growth than those at the top: Their hours dropped faster.

Two questions are asked in this research: First, what caused the change in hours per worker during the period 1900-1950? Second, assuming as Figure 5 suggests that its distribution narrowed over the same period, what caused this narrowing? The explanations proposed hinges on the following mechanisms.

- First, productivity growth in the market place led to an increase in real wages and, therefore, to a reduction in hours. This deserves additional explanations. It is commonly accepted, in macroeconomics, that the wealth and substitution effects of an increase in wages offset each other, leaving labor supply invariant to market productivity. This modelling practice is usually motivated by the relative stationarity of hours observed since the end of World War II. As stated above, however, hours have displayed a trend at least until the middle of the 20$^{\text{th}}$ century.

  Many type of preferences imply that the income effect of an increase

Sunday at work where more likely to have longer hours in the 1890s. Likewise for those who reported no reduction or increase in Saturday hours. Similarly, workers with 3 months of unemployment in a year worked less per day than workers with no unemployment during the year.

in wages dominates the substitution effect. Introducing a "subsistence level" of consumption in an otherwise standard logarithmic utility is a an example of such preferences. As the wage rate increases, the amount of work needed to meet the subsistence level diminishes, and leisure time increases. A second channel does not impose restrictions on preferences. It requires the introduction of a notion of home-production, though. More precisely, it draws on a distinction made by Mincer (1962) between work, work at home and leisure. It also uses the idea of Becker (1965) that some commodities are produced at home by combining time and some other goods. Consider then the home-production of "leisure services." Leisure services are enjoyed, for instance, from a bicycle ride in the country, time spent reading a book, listening to the radio, exercising in a fitness club, etc... Beside time, the production of leisure services requires another input which can be purchased on the market: a leisure good (bicycles, books, radios, golf passes...). A specificity of leisure is that leisure-time and leisure-goods can be thought off as complements: As the wage rate increases and the price of leisure goods decreases, both as a result of technological progress, a household can purchase more leisure goods. Complementarity implies then that leisure-time must increase. Observe that, through the same mechanism, households at the bottom of the income distribution work more than those at the top, since they cannot afford as much of the leisure-good.

What are the facts supporting the second mechanism exposed here? Between 1900 and 2000, the relative price of consumption expenditures is divided by two while the recreation's share of expenditure rises from 3 to 8%.[3] Whaples (1990) and Lebergott (1993), indicate that the period of fastest increase for this share is the 1920s. So it seems, on the surface, that households have been producing more leisure services in the home. Observe also Figure 6: Hours worked for male workers line up remarkably well with the relative price of recreation goods. In particular, the slowdown in hours, observed in the second half of the century is common

[3]See Lebergott (1996) and U.S. Bureau of the Census (2003).

	1890	1940	1950	2000
Ratio of wages: 90 / 10 decile	2.81	2.15	1.50	2.15

Table 2: Changes in the Wage Structure for Male Production Workers in Manufacturing.

to both series until the 1980s.

- Second, wage growth has not been the same for high-skill and low-skill workers. Goldin and Katz (2001) present evidence that the wage distribution was narrower in 1950 than in 1890 and that, from 1950 until 2000, it widened. Interestingly, the wage distribution in 2000 is as dispersed as it was at the end of the great depression. In comparison to the late nineteenth century, the wage distribution today is less dispersed than it used to be. Summary statistics for the distribution of wages are presented in Table 2. What caused the contraction of the wage distribution during the first half of the 20$^{\text{th}}$ century? One explanation is the rise of education: The fraction of a generation graduating from high school rose from 6% to 60% between 1900 and 1950 – see Figure 7.[4] The resulting flow of skilled workers into the market place slow down the skilled-wage growth and compressed the distribution of wages. The hypothesis is that low-wage earners reduced their hours faster because they experienced faster wage growth....

Let us take stock of the discussion above. It suggests that two periods should be considered separately: pre- and post-1950. First, hours per worker decline unambiguously before 1950, but exhibit less of a trend afterwards. Second, and as far as changes in the wage distribution are concerned, 1950 seems again to be a turning point: Until then, the wage distribution narrowed. Afterwards, it widens. Thus, in the remaining of the paper, a model is developed in an attempt to measure the quantitative relevance of the mechanisms just described. The period under consideration is 1900-1950. Section 2 presents the model economy and an analysis of some of its properties. Section 3 presents

[4] High School enrollment increased from 10 to 76% of a generation during the same period.

a quantitative exercise conducted to assess the importance of the explanation proposed. Section 4 concludes.

2 *The Model*

2.1 Economic Environment

The economy is described by an overlapping-generations model where time is discrete and indexed by $t = 0, 1, \ldots, \infty$. Agents are alive for three periods. One as a child and two as an adult. Only adults are economically active.

Each generation is of size 1. Households are indexed by the variable $\lambda \in \mathbb{R}_+$, which represents the level of their market ability, i.e., one unit of time of household λ is worth λ efficiency units of labor. The ability level λ is distributed across agents according to the cumulative distribution function Λ. Agents can choose to purchase education during the first period of their life. The cost of education is two-fold. First there is a fixed consumption cost, e, which must be paid while attending school. Second there is a time cost: The time spent at school during the first period of life, τ. An educated agent becomes a skilled worker on the market. The market wage per efficiency unit of skilled labor is called w^s. An agent who decides to not attend school is an unskilled worker. The market wage per efficiency unit of unskilled labor is called w^u.

There are three sectors. One hires skilled, unskilled labor and capital to produce the consumption good c_m. Capital depreciates at rate $1-\delta$. The rental rate for capital is denoted by $r - \delta$, thus the gross interest rate is r. A second sector produces a durable good called g and sells it at price p. The technology used here requires input of the final consumption good. Third, households produce leisure services using time and the leisure good g. Thereafter, leisure services might also be referred to as home production. Leisure services are not traded on the market.

2.2 Households

Preferences are defined over streams of consumption of the market good, $\{c_{1m}, c_{2m}\}$ and leisure services, $\{c_{1n}, c_{2n}\}$. They are represented by the following utility function

$$\sum_{i=1,2} \beta^{i-1} \left[\phi \ln \left(c_{im}\right) + (1-\phi) \ln \left(c_{in}\right)\right]$$

Where $\beta \in (0,1)$ is the subjective discount factor and $\phi \in (0,1)$.

Leisure services are produced using time and the leisure good. The household technology is described by the production function H.

$$H(g,\ell) = \left(\gamma g^{\theta} + (1-\gamma)\ell^{\theta}\right)^{1/\theta},$$

where $\theta < 1$ and $\gamma \in (0,1)$. The variable g represents inputs of the leisure good and ℓ represents leisure time. The elasticity of substitution between g and ℓ is $1/(1-\theta)$. Observe that $\operatorname{sign}\left(H_{g\ell}\right) = -\operatorname{sign}(\theta)$. Thus, when $\theta < 0$, g and ℓ are Pareto-Edgeworth complements: The marginal utility of leisure time is an increasing function of the quantity of leisure goods used.

Let us first consider the problem of a household with ability λ who decides to purchase education. He solves:

$$V^{s}(\lambda) = \max_{c_{1m}, c_{2m}, \ell_1, \ell_2, g_1, g_2} \sum_{i=1,2} \beta^{i-1} \left[\phi \ln \left(c_{im}\right) + (1-\phi) \ln \left(c_{in}\right)\right] \qquad \text{P(1)}$$

subject to

$$\begin{aligned} c_{1m} + \frac{c_{2m}}{r} + pg_1 + \frac{pg_2}{r} + e &= w^{s}\lambda h_1 + \frac{w^{s}\lambda h_2}{r} \\ c_{in} &= H\left(g_i, \ell_i\right), \ i = 1,2, \\ \ell_1 &= 1 - \tau - h_1, \\ \ell_2 &= 1 - h_2. \end{aligned}$$

The first constraint is the intertemporal budget constraint of the household. The second imposes that leisure services are produced and consumed at home. The third imposes that the total time available for market, h_1, or leisure time, ℓ_1, is $1-\tau$ during the first period of life, i.e., the agent spends a fraction τ of his time acquiring skills. During the second period of life, total time is 1 (fourth constraint).

Turning to a household deciding to not attend school, the maximization problem reads

$$V^u(\lambda) = \max_{c_{1m},c_{2m},\ell_1,\ell_2,g_1,g_2} \sum_{i=1,2} \beta^{i-1} \left[\phi \ln(c_{im}) + (1-\phi)\ln(c_{in})\right] \qquad \mathrm{P}(2)$$

subject to

$$\begin{aligned} c_{1m} + \frac{c_{2m}}{r} + pg_1 + \frac{pg_2}{r} &= w^u \lambda h_1 + \frac{w^u \lambda h_2}{r} \\ c_{in} &= H(g_i, \ell_i), \ i = 1,2, \\ \ell_i &= 1 - h_i, \ i = 1,2. \end{aligned}$$

The constraints faced by the uneducated agent are similar to that of the educated agent. The only difference the wage rate and the absence of costs due to education.

What makes this model a household production model is the constraint $c_{in} = H(g_i, \ell_i), \ i = 1,2$, in problems P(1) and P(2). Classic references on macroeconomic models with household production are Benhabib, Rogerson, and Wright (1991), Ríos-Rull (1993) and Greenwood, Rogerson, and Wright (1995). In the model presented here the household production function is not subject to time-saving technological progress. This can be viewed as justifying that the home good be called "leisure." One can also think of it as a definition of "leisure."

Proposition 1 *Assume that $\theta < 0$. Then, a skilled worker's optimal decisions for hours, h_1 and h_2, are decreasing in the wage rate w^s and increasing in the* *price of the intermediate good p. The unskilled worker's choices for hours is increasing in* p/w^u*.*

Proof: See appendix.

Proposition 1 states the condition under which technological progress leads to a reduction in hours spent on the market. To see this, suppose that technological progress propels an increase in the wage rates w^s and w^u, and a fall in p, the price of the leisure good. Proposition 1 states that if leisure time ℓ and the leisure good g are Pareto-Edgeworth complements (in utility terms), i.e., if $\theta < 0$, then hours worked decrease as the real wage rates are increasing. The underlying mechanism is as follows: On the one hand, as the wage rate increases, the time spent out of market activities becomes more costly and the household has an incentive to work more. This is the usual substitution effect. On the other hand, the increase in wages combined with the decrease in p leads to an increase in the purchase of g. Thus, the marginal utility of ℓ increases, and the household decides to spend more time at home. This effect comes in addition to the usual income effect which would, otherwise, be offset by the substitution effect. [5]

For a skilled agent, there is an additional channel through which the wage rate affects labor supply: Remember that the tuition, e, is paid in consumption goods. Thus, an educated agent must work on the market to finance it. When the wage rate increases, the time one needs to spend working on the market to purchase education is reduced. The educated agent uses part of the spared time to afford more leisure.

Proposition 2 *Assume* $\theta < 0$*. Then, conditional on an agent being educated, the higher the ability level, the higher the non-market time. Likewise among non-educated households: Those with the highest ability level spend more time out of the market.*

[5] In the case of the unskilled worker, the condition $\theta < 0$ is necessary and sufficient for hours to increase with p/w^u. For the skilled worker, this condition is sufficient, but not necessary. As can be seen from the appendix, when $\theta > 0$, it is possible, depending on parameter values, that h_1 decreases with w^s.

Proof: See appendix.

The mechanism at work behind Proposition 2 is the same as the one described in Proposition 1. It is important to note, though, that this mechanism does not imply that non-educated workers work more than educated workers. It is possible that an educated worker spends more time at work than an uneducated worker.

Proposition 1 shows that, qualitatively speaking, the model is in line with one feature of the US data: Real wages and market hours move in opposite directions. Whether this mechanism is quantitatively relevant is to be discussed in the following section. It is not clear a priori, however, that the model generates a contraction in the distribution of hours. As mentioned above, educated household have an incentive to reduce their hours that uneducated household do not have. Thus, potentially, the distribution of hours across skill and unskilled workers can evolve in any direction. Whether the distribution contracts is therefore a quantitative question, which will be addressed in the next section.

The decision to purchase education is made at the beginning of life. A household with ability λ solves

$$\max\left\{V^s(\lambda), V^u(\lambda)\right\}. \tag{P(3)}$$

Proposition 3 *If*

$$\frac{w^s}{w^u} > \left(\frac{w^s}{w^u}\right)^*, \tag{2.1a}$$

where $(w^s/w^u)^* \equiv (1-\tau r/(1+r))^{-1/\phi}$, *then there exists* λ^* *such that* $V^s(\lambda^*) = V^u(\lambda^*)$, *and the optimal education decision of an agent with ability* λ *is:*

$$\begin{cases} \textit{Purchase education} & \textit{if } \lambda \geq \lambda^*, \\ \textit{Do not} & \textit{if } \lambda < \lambda^*. \end{cases}$$

Proof: See appendix.

Proposition 3 states the condition under which there exists a marginal agent who is indifferent between purchasing education or not. The intuition underlying the proof is as follows: First, notice that Condition (2.1a) implies $w^s > w^u$. Since education is costly, there must be a wage premium if any agent is to purchase education. Second, an agent with very low ability does not purchase education because his income, even if paid the wage w^s, is too low to allow him to pay the fixed cost e. Third, under Condition (2.1a), an agent with very high ability purchases education. The existence of a marginal agent follows. Figure 8 represents the value functions of an educated and an uneducated agent, and illustrates the construction of λ^*.

Condition (2.1a) deserves additional explanations. It is a condition under which very able agents (i.e., $\lambda \to \infty$) decide to purchase education. Why would such agents not necessarily do so? One can grasp the intuition behind Condition (2.1a) by writing it as

$$\frac{w^s\left(\frac{1+r}{r}-\tau\right)}{w^u\left(\frac{1+r}{r}\right)} > \left(\frac{w^s}{w^u}\right)^{1-\phi}. \tag{2.1b}$$

(See appendix for the derivation.) The left-hand side of this condition is the factor by which a very able agent's wealth changes if he purchases education. First, observe that the fixed cost of education, e, does not enter here since, for very able agents, it is negligible. But the time cost, τ, is not. By deciding to purchase education, agents do not necessarily increase the present value of their income, for they must also work less during their youth.

So, how large should the skill premium be, so that education increases wealth enough to induce a very able agent to attend school? Figure 9 represents Condition (2.1b), and the schooling decision of such agent. Observe first that, for values of the skill premium below the threshold $(w^s/w^u)^{**}$, the present value of wages decreases when receiving education. Facing such prices, a very able agent would not choose to attend school. Notice also that, as the time cost of education decreases, i.e., $\tau \to 0$, the threshold $(w^s/w^u)^{**}$ converges to 1: Now, the existence of a skill premium implies that educated agent are richer.

For a very able agent to decide to attend school, it is not enough, though, that the skill premium induces an increase in wealth. This transpires from Figure 9 since $(w^s/w^u)^* > (w^s/w^u)^{**}$. The difference between the critical premium $(w^s/w^u)^*$, above which agents attend school, and the threshold $(w^s/w^u)^{**}$, above which educated agents are richer, depends on preferences through ϕ. When the weight of market consumption in utility, ϕ, is large, the household is willing to attend school for a small increase in wealth. This is due to the fact that market consumption can be purchased only with the market wage, which is enhanced by education. If, on the contrary, market consumption is less valued by the household (i.e., ϕ is small) the benefit of education is lower. This is due to the fact that leisure can be produced at home and is not affected by education. In such case, the household requires a larger increase in wealth to decide to attend school.

2.3 Production

2.3.1 Consumption-good Sector

Output y is produced by a single firm using a constant-returns-to-scale production function $F(k, s, u)$, where k represents the input of capital services, s represents efficiency units of skilled labor and u represents efficiency units of unskilled labor. The specification adopted for F is

$$y = F(k, s, u) = \left[\mu\left(z_u u\right)^{\sigma} + (1-\mu)\left[\alpha\left(z_k k\right)^{\rho} + (1-\alpha)\left(z_s s\right)^{\rho}\right]^{\sigma/\rho}\right]^{1/\sigma}.$$

With this formulation, the parameters σ and ρ govern elasticities of substitution between factors and μ and α govern factor shares.[6] The elasticity of substitution between capital and skilled labor is $1/(1-\rho)$. The elasticity of substitution between capital and unskilled labor is $1/(1-\sigma)$. Note that the elasticity of substitution between the two types of labor is also $1/(1-\sigma)$. The parameters satisfy $\sigma, \rho < 1$ and $\alpha, \mu \in (0, 1)$. When $\sigma > \rho$, the elasticity of

[6] This specification is closed to the one used in Krusell, Ohanian, Ríos-Rull, and Violante (2000).

substitution between capital and unskilled labor is larger than between capital and skilled labor: There is capital-skill complementarity. The variables z_u, z_s and z_k are productivity terms affecting skilled and unskilled labor and the capital stock.

The maximization program of the firm is

$$\max_{k,s,u} \left\{ F\left(k, s, u\right) - \left(r - \delta\right) k - w^s s - w^u u \right\}, \qquad \text{P(4)}$$

The optimality conditions associated with this program imply a skill premium given by

$$\frac{w^s}{w^u} = \frac{\left(1-\mu\right)\left(1-\alpha\right)}{\mu} \left[\alpha \left(\frac{z_k k}{z_s s}\right)^{\rho} + 1 - \alpha\right]^{\sigma/\rho - 1} \left(\frac{z_s}{z_u}\right)^{\sigma} \left(\frac{u}{s}\right)^{1-\sigma}$$

Three ratios govern the skill premium. First, it increases with the relative scarcity of skilled workers, u/s. Second, when $0 < \sigma < 1$, i.e., the elasticity of substitution between unskilled labor and the capital-skilled labor aggregate is larger than 1, the skill premium is increasing with the relative efficiency of skilled labor, z_s/z_u. Finally, when $\sigma > \rho$, i.e., capital is more complementary with skilled than with unskilled labor, an increase in the efficiency units of capital per efficiency units of skilled labor, $\left(z_k k\right) / \left(z_s s\right)$, raises the premium.

2.3.2 Intermediate-good Sector

Let the function G describe the technology used to produce g. The only production factor needed in this sector is the final consumption good:

$$G(x) = z_g x$$

Here, x represents inputs of the consumption good and z_g is the state of technology. The maximization program of the firm is

$$\max_{x} \left\{ pG(x) - x \right\}. \qquad \text{P(5)}$$

The optimality condition associated with this program dictates that $p = 1/z_g$.

2.4 Equilibrium

An allocation is a list of functions specifying market consumption, hours and leisure good consumption for each level of ability λ, for skilled and unskilled agents, young and old. An allocation also specifies quantities of factors used in the production of the consumption good and the leisure good. Finally, an allocation must specify the education choice of agents. The latter is summarized by the critical ability parameter λ^*. The equilibrium allocation must solve households and firms' problems given prices. Furthermore, prices must clear the market for skilled and unskilled worker, as well as the market for the consumption good, the leisure good and capital. Equilibrium conditions are detailed in the following definition.

Definition 1 (Equilibrium) *A competitive equilibrium is an allocation for young skilled agents,* $\{c^s_{1m}(\lambda), \ell^s_1(\lambda), g^s_1(\lambda)\}$ *and old skilled agents* $\{c^s_{2m}(\lambda), \ell^s_2(\lambda), g^s_2(\lambda)\}$, *an allocation for young unskilled agents* $\{c^u_{1m}(\lambda), \ell^u_1(\lambda), g^u_1(\lambda)\}$ *and old unskilled* $\{c^u_{2m}(\lambda), \ell^u_2(\lambda), g^u_2(\lambda)\}$, *factors allocation* $\{k, s, u, x\}$ *for firms, prices* $\{w^s, w^u, r, p\}$ *and a critical ability level* λ^*, *such that*

1. *The allocation* $\{c^s_{1m}(\lambda), c^s_{2m}(\lambda), \ell^s_1(\lambda), \ell^s_2(\lambda), g^s_1(\lambda), g^s_2(\lambda)\}$ *solves Problem P(1) given prices.*
2. *The allocation* $\{c^u_{1m}(\lambda), c^u_{2m}(\lambda), \ell^u_1(\lambda), \ell^u_2(\lambda), g^u_1(\lambda), g^u_2(\lambda)\}$ *solves Problem P(2) given prices.*
3. *The schooling decision solves Problem P(3) for each agent* λ, *given prices.*
4. *The allocation* $\{k, s, u\}$ *solves Problem P(4) given prices.*
5. *The allocation* $\{x\}$ *solves Problem P(5) given prices.*
6. *The markets for skilled and unskilled labor clear, or*

$$\int_{\lambda^*} \lambda(1 - \tau - \ell^s_1(\lambda))d\Lambda + \int_{\lambda^*} \lambda(1 - \ell^s_2(\lambda))d\Lambda = s,$$

$$\int^{\lambda^*} \lambda(1-\ell_1^u(\lambda))d\Lambda + \int^{\lambda^*} \lambda(1-\ell_2^u(\lambda))d\Lambda = u.$$

7. *The intermediate good market clears, or*

$$\int_{\lambda^*} \sum_{i=1,2} g_i^s(\lambda)d\Lambda + \int^{\lambda^*} \sum_{i=1,2} g_i^u(\lambda)d\Lambda = G(x)$$

8. *The resource constraint is given by*

$$c + x + e(1-\Lambda(\lambda^*)) + k = y + \delta k,$$

where $e(1-\Lambda(\lambda^*))$ *is the quantity of the final good used to pay the fixed cost of education, and* c *represents aggregate consumption of the market good:*

$$c = \int_{\lambda^*} \sum_{i=1,2} c_{im}^s(\lambda)d\Lambda + \int^{\lambda^*} \sum_{i=1,2} c_{im}^u(\lambda)d\Lambda.$$

Walras's law ensures that the capital market clears.

3 Quantitative Analysis

In this section, the quantitative analysis of the model is carried out. The nature of the experiment is as follows. First the model parameters are calibrated to match some key features of the US economy in 1900. In particular, the skill premium, the fraction of skilled workers, the ratio of hours between skilled and unskilled, the relative price of leisure goods, the consumption to output ratio, the labor share of income and the real interest rate from a steady state of the model economy will match their counterparts in the US economy of 1900.

Second, technological progress is allowed and another steady state is computed. Think of this second steady state as a snapshot of the US economy in 1950. Between the two steady states, the productivity term z_g increases as dictated by the data. Similarly, the time cost of education, τ rises to take into account the increased length of formal schooling required to qualify as a skilled-worker. The productivity terms z_k, z_s, z_u and the cost of education e are calibrated such that the second steady state matches some key features of the US economy in 1950. In particular, it will display the same skill premium, fraction of skilled, labor share of income and output growth relative to 1900, as the US economy does. Note that it is important that the 1950-steady state is constrained to exhibit the same output relative to 1900 than in the actual US economy. With this constraint, one can argue that the 1950's values for the productivity parameters are sensible. For the same reason, the labor share of income is constrained to stay at its 1900 level. (There is no obvious trend in this statistics in the US economy.) This imposes further discipline on the changes in z_k, z_s and z_u.

Third, two questions are asked: (i) How much of the observed decline in average hours worked, between 1900 and 1950, is accounted for by the mechanisms at work in the model? (ii) How did the dispersion of the distribution of hours worked change? Figure 5 is silent about the distribution of hours in 1950. Thus the second question cannot be phrased as a comparison between model and data. It is reasonable, though, to argue that the contraction in the distribution of hours was underway between 1900 and 1950.

3.1 Calibration – First Steady State

Consumption Good Technology – The parameters to choose are $\rho, \sigma, \delta, z_k, z_s$ and z_u. Normalize the productivity parameters to one: $z_{k,00} = z_{s,00} = z_{u,00} = 1.0$. Following Greenwood, Seshadri, and Vandenbroucke (2005), the annual depreciation rate for capital is set to 4.7%. A model period is chosen to represents 20 years, thus $\delta = (1 - 0.047)^{20}$. Krusell, Ohanian, Ríos-Rull, and Violante (2000) estimate the elasticity parameters for the production function used here. They find $\sigma = 0.40$ and $\rho = -0.50$, those values are used in the following experiment. The choice of the weights α and μ is described below.

Leisure Good Technology – Only one parameter needs to be chosen here: z_g, the productivity term for this sector. This term commands the relative price of the leisure good. Thus, data on the relative price of recreational expenditures are used to find $z_{g,00}$. From Owen (1969) works, one can compute that the relative price of leisure goods in 1900 is 1.26 when it is normalized to 1 in 1950. Consistently, set $z_{g,00} = 1/1.26$ and $z_{g,50} = 1.0$.[7]

Education – Maddison (1987) reports that the average length of formal education in the US, circa 1900, is about 6 years. This number increases to 9.5 years in 1950. Remember a model period is 20 years, thus τ is set to $\tau_{00} = 6/20$ and $\tau_{50} = 9.5/20$ for 1900 and 1950, respectively. The fixed cost of education, e, is chosen according to a procedure described in details below.

Household's preference and technology – The parameter ϕ is set to 0.36, the calibrated value used by Cooley and Prescott (1995). This value corresponds roughly to the middle of the range of values for ϕ one can find in the literature (see Laitner and Silverman (2005).) Set the share parameter γ to 1/2, i.e., time and the leisure good have the same productivity. The choice of the discount factor β and the elasticity parameter θ are discussed below.

Remaining parameters – The choice of the remaining parameters is now discussed. They are: the discount factor β, the fixed cost of education, e_{00}, the share parameters in the production function, α and μ, the elasticity parameter in the household technology θ and, finally, the distribution of ability Λ. The choice of Λ is simplified to one single parameter by imposing $\ln \lambda \sim N(1, S^2)$. Define the vector $a = (\beta, e_{00}, \alpha, \mu, \theta, S)$. The objective is to find a value for a such that the model economy satisfies the following restrictions:

1. The measure of wage dispersion in the model is w^s/w^u. This is matched to the ratio of wages of the 90^{th} to 10^{th} decile in the US economy of 1900, thus $w^s/w^u = 2.81$.

2. The consumption to output ratio corresponds to the US output share of non-durable consumption and services: $c/y = 0.60$.

[7] See Kopecky (2005) for details.

3. The labor share of income corresponds to the US data: $(w^s s + w^u u)/y = 0.67$.

4. The mass of skilled workers corresponds to the fraction of 17 years-old high-school graduates as reported in U.S. Bureau of the Census (2003): $1 - \Lambda(\lambda^*) = 0.064$.

5. The real interest rate is 6.9% per year (the average return on capital over the period 1954-1992, computed by Cooley and Prescott (1995).) Thus $r^{1/20} = 1.069$.

6. Finally, the distribution of hours worked corresponds to the US data. What is the model's counterpart to daily hours worked by households at the top and the bottom of the wage distribution? For an unskilled worker, the time spent on the market is $1-\ell_1^u(\lambda)$ when young and $1-\ell_2^u(\lambda)$ when old. In equilibrium, there is a measure $\Lambda(\lambda^*)$ of such agents. Thus, conditional on the fact that agents are unskilled (at the bottom of the distribution,) the average time spent on the market is

$$d^u = \frac{1}{\Lambda(\lambda^*)}\left[\int^{\lambda^*}(1-\ell_1^u(\lambda))d\Lambda + \int^{\lambda^*}(1-\ell_2^u(\lambda))d\Lambda\right]. \tag{3.2}$$

A young skilled workers spends $1-\tau-\ell_1^s(\lambda)$ units of time on the market. This does not correspond, however, to the notion of daily hours. The time cost, τ, is generally paid at the beginning of one's youth. In other words, the agent's time endowment is $1-\tau$, conditional on being skilled. His average time spent on the market is therefore $(1-\tau-\ell_1^s(\lambda))/(1-\tau)$. An old skilled worker has one unit of time. Thus, conditional on being skilled, the average market time is

$$d^s = \frac{1}{1-\Lambda(\lambda^*)}\left[\int_{\lambda^*}\left(\frac{1-\tau-\ell_1^s(\lambda)}{1-\tau}\right)d\Lambda + \int_{\lambda^*}(1-\ell_2^s(\lambda))d\Lambda\right]. \tag{3.3}$$

The data show that, in 1900, the ratio d^u/d^s is about 1.22 for male workers.

Market technology	$\delta = (1-0.047)^{20}$, $\sigma = 0.40$
	$\rho = -0.50$, $\alpha = 0.13$, $\mu = 0.13$
Household technology	$\theta = -0.48$, $\gamma = 0.50$
Preferences	$\phi = 0.36$, $\beta = 0.98^{20}$
Cost of education	$\tau_{00} = 6/20$, $e_{00} = 9.30$
Distribution of Abilities	$\ln \lambda \sim N(1, 2.41^2)$
Productivity terms	$z_{k,00} = z_{s,00} = z_{u,00} = 1.0$, $z_{g,00} = 1.0/1.26$

Table 3: Calibration – Baseline Economy

Define the function $M_{00}(a) : \mathbb{R}^6 \to \mathbb{R}^6$ as

$$M_{00}(a) = \begin{bmatrix} w^s/w^u - 2.81 \\ c/y - 0.60 \\ (w^s s + w^u u)/y - 0.67 \\ 1 - \Lambda(\lambda^*) - 0.064 \\ r^{1/20} - 1.069 \\ d^u/d^s - 1.22 \end{bmatrix}$$

and solve for a the system of 6 equations in 6 unknown

$$M_{00}(a) = 0.$$

The results are presented in Table 3. Define y_{00} to be output in the model economy given the calibration described in Table 3. The calibrated value of θ implies an elasticity of substitution between time and the leisure good of 0.67.

3.2 Second Steady State

Consider now the US in 1950. Average weekly hours worked are about 70% of what they were in 1900. The wage distribution is narrower, compared with 1900, since a worker in the top decile has a wage that is now 50% above that of a worker in the bottom decile (c.f., 181% in 1900.) A larger fraction of the workforce is now educated: About 60% of a generation graduates from high school and the number of high school-graduates moving on to college increased, thus the length of formal education has risen (from 6 years to 9.5 on average). Finally, output per person is 2.25 times its 1900 value.[8]

What are the factors that brought about such changes? Technological progress is likely to have played a part in this transformation. To assess the extent to which this is so, let the relative price of leisure good be given by $1/z_{g,50}$ and the time cost of eduction be given by τ_{50}. Define $M_{50}(b) : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ as

$$M_{50}(b) = \begin{bmatrix} w^s/w^u - 1.50 \\ (w^s s + w^u u)/y - 0.67 \\ 1 - \Lambda(\lambda^*) - 0.60 \\ y_{50}/y_{00} - 2.25 \end{bmatrix}$$

where $b = (z_{k,50}, z_{s,50}, z_{u,50}, e_{50})$ and solve for b the system of 4 equations in 4 unknown

$$M_{50}(b) = 0.$$

Observe that, between the first and the second steady state, the model economy is constrained to exhibits just as much growth as observed in the actual US economy. The percentage of an age class receiving formal education is also replicated as well as the fall in the skill premium and the labor share of income is held constant to 0.67. The resulting change in average hours worked and its distribution are reported in Table 4. The model predicts that average hours worked decline and that their dispersion shrinks. Quantitatively, average hours worked decline by 17% vis à vis a 27% decline in the US data. Thus the model accounts for 63% of the trend in hours through the mechanisms described above. Note again that logarithmic preferences are used. Thus the decline in hours is not due to a wealth effect but to the substitution of leisure time for market time.

How much do the driving variables change between 1900 and 1950? Table 4 shows that both the capital and skilled-labor specific technological parameters increase by 150 and 105% respectively. The experiment suggest that the

[8]This number is obtained by comparing the HP-trend of output per capita in 1950 and 1900.

		1900	1950
Output	Model	1	2.25
	US Economy (normalized)	1	2.25
Average Hours	Model	1.20	0.99
	US Economy (normalized)	1.20	0.87
Dist. of hours: d^u/d^s	Model	1.22	1.15
	US Economy	1.22	-
	z_k	1.0	2.47
Driving variables	z_s	1.0	2.08
	z_u	1.0	0.16
	z_g	1.0/1.26	1.0
	e	9.30	-0.57

Table 4: Results – Baseline Economy

productivity of unskilled labor drops significantly: from 1.0 to 0.16. There is an identification issue here, though. The drop in z_u could be due to a change in the share parameter μ, i.e., the importance of unskilled labor in the production function, versus the skilled-labor-capital composite. The extreme case is that where only μ adjusts, and z_u remains constant. The 1950-steady state would then correspond to $\mu = 0.13 \times 0.16^{\sigma} = 0.06$, vis à vis 0.13 for the 1900-steady state.[9] Observe that the change in the fixed cost of education needs to be quite large to account for the increase in the share of educated households. In fact the cost of education e is now slightly negative.

Figure 10 represents the labor supply of young households per ability level, for each steady-state. Consider the 1900-steady state. The fraction of time a young adult spends on the market is decreasing in his ability level (see Proposition 2). As already mentioned, an educated households do not necessarily spend less time on the market than an uneducated households. In fact, the marginal household would work more if he chose to purchase education than if not. In the 1950-steady state, each agent works less. Observe that those

[9]The same is true for the terms z_s and z_k. Suppose that z_k does not change, but that the share α adjusts. Then, the 1950-steady state requires that $\alpha = 0.13 \times 2.47^{\rho} = 0.08$ vis à vis 0.13 in the 1900-steady state.

agents deciding to switch their status, from non-educated to educated, decrease their hours less than agents maintaining their status. This is due to the extra cost imposed on them by education. Thus, the bulk of reduction in hours come from agents with ability level low enough such that they would not become educated, or high enough so hat they already purchased education in the 1900-steady state.

One can disentangle the effect of rising wages from that of the declining price of leisure goods. Under the calibration used above, compute the steady state of the model with all driving variables, but z_g, set to their 1950 values. Set z_g to its 1900 value. Such an experiment generates a 13% decline in average hours worked. Compare this with the 17% declined obtained in the baseline calibration. The ratio of hours unskilled to skilled workers remains virtually unaffected. Thus, it is the increase in wage rates that accounts for the bulk of the decline in hours and the wage compression.

3.3 Non-logarithmic Preferences

Contemplate a slight change in preferences: Suppose now that agents order streams of consumption and leisure in line with

$$\sum_{i=1,2} \beta^{i-1} \left[\phi \ln\left(c_{im} - \bar{c}\right) + (1-\phi) \ln\left(c_{in}\right)\right]$$

where $\bar{c}$ stands for a subsistence level of consumption. Under this specification, households need to work a minimum amount of time in order to simply meet their subsistence level. During the first period of their life, each must work a minimum of $\bar{c}/w^s$ if skilled and $\bar{c}/w^u$ if unskilled. Observe that, as the wage rate increases, this requirement vanishes, i.e., the household can allocate more time to non-market activities. How does this new mechanism affect the results derived above?

To answer this question, the same exercise as described above is conducted for various values of $\bar{c}$. The results are reported in Table 5. As one could

	Baseline $\bar{c}=0$	$\bar{c}=$ 0.1	$\bar{c}=$ 0.15	US Economy
% Change in total hours	-17%	-18%	-19%	-27%
d^u/d^s (1950)	1.15	1.29	1.34	1900: 1.22 1970: 1.07

Table 5: Change in Hours in the economy with subsistence consumption

have expected, the introduction of the subsistence level strengthens the effect of wages on hours. For instance, in the case of $\bar{c} = 0.15$, the model accounts for 70% of the observed decline in hours. The model, however, predicts that hours of skilled workers declined faster than for unskilled workers. Thus, the predicted distribution of hours widens whereas, over this period, it presumably contracted in the US data. How can this happen? Remember that the exercise conducted here is the same as the one described above. Thus the ratio w^s/w^u is reduced between the initial and the final steady state. Remember, however, that when the wage rate is increasing, skilled workers reduce their hours for two sperate reasons. First, they purchase more leisure time and leisure goods. Second, they can have more free time once they have worked just enough to cover the fixed cost of education. Therefore, and as mentioned earlier, it is not guaranteed that a slower wage growth leads to a slower reduction in hours for skilled workers.

4 Conclusion

The length of the workweek declined significantly during the first half of the 20$^{\text{th}}$ century and exhibits little to no trend after 1950. During the same period, the distribution of hours across wage deciles narrowed: Those who used to work the longest week in 1900, i.e., workers at the bottom of the wage distribution, experienced the fastest decline in hours. The explanation proposed here relies on (i) Household production of leisure services: As leisure goods become more affordable, the marginal benefit of leisure time increases, driving household out of the market; and (ii) The change in the wage distribution: As more educated households enters the labor force the wage growth for skilled worker slows down and, therefore, they reduce their hours at a slower pace than unskilled workers do.

The model developed above incorporates those effects and is used to analyze the 1900-1950 period. The quantitative analysis is conducted such that the model economy is similar to the US economy in several important respect: (i) they share the same growth of output per capita over the period of interest; (ii) the movements in the skill premium are the same and (iii) the fraction of educated household is matched. The driving forces at work are technological progress and a reduction in the cost of education. The model accounts for more than 60% of the reduction in the workweek and generate a significant contraction of the distribution of hours. Most of those effects are due to rising real wages.

New directions of research naturally arise from this work. For instance, non-market time can be divided between leisure time and housework. Incorporating such a distinction into a model could allow one to investigate the second half of the 20$^{\text{th}}$ century. One can think of a household where two commodities are produced: leisure services and home consumption. The former can be modelled along the line developed here. The later would be subject to labor saving technological progress. Potentially, such a model would allow one to generate a decline in the workweek and an increase in female labor force participation.

A1 Data Sources

- Figure 1: The data from 1830 to 1890 is from Whaples (1990), Table 2.1, pp. 33–35. The data for 1869 to 1946 are from Kendrick (1961), Tables A-IV, pp. 305-307 and A-X, pp. 311-313. The data from 1947 to 2004 are from Bureau of Labor Statistics, series PRS85006023: Average Weekly Hours in non-farm business. The last two series are spliced together in 1947.
- Figure 2: Whaples (1990), Table 2.1, pp. 33–35.
- Figure 3: Kendrick (1961), Table A-IX, p. 310.
- Figure 4: Maddison (1987), Table A9.
- Figure 5: Costa (1998), Table 2.
- Table 2: The numbers presented are derived as follows. (i) 1890 and 1940 - Goldin and Katz (2001), Table 2.1; (ii) 1950 - From Goldin and Margo (1992), Table 1, one can derive that the wage ratio in 1950 is approximately 70% of what is was in 1940; (iii) 2000 - Goldin and Katz (2001), Figure 2.1, show that the dispersion in 2000 is approximately the same as in 1940.
- Figure 6: Whaples (1990), Table 2.1, pp. 33–35. and Kopecky (2005).
- Figure 7: U.S. Bureau of the Census (2003).

A2 Proof of Propositions 1 and 2

Consider the problem of a skilled worker, Problem P(1). It writes

$$\max_{\ell_1,\ell_2,g_1,g_2,c_{2m}} \left\{ \phi \ln \left(\lambda w^s (1-\tau) + \frac{\lambda w^s}{r} - \lambda w^s \ell_1 - \frac{\lambda w^s \ell_2}{r} - p g_1 - \frac{p g_2}{r} - \frac{c_{2m}}{r} - e \right) \right.$$
$$\left. + \frac{1}{\theta} (1-\phi) \ln \left(\gamma g_1^\theta + (1-\gamma) \ell_1^\theta \right) + \beta \phi \ln (c_{2m}) + \beta \frac{1}{\theta} (1-\phi) \ln \left(\gamma g_2^\theta + (1-\gamma) \ell_2^\theta \right) \right\}.$$

The first order conditions are

$$\ell_1 \quad : \quad \phi \frac{\lambda w^s}{c_{1m}} = (1-\phi) \frac{(1-\gamma)\ell_1^{\theta-1}}{\gamma g_1^\theta + (1-\gamma)\ell_1^\theta} \tag{A2.4}$$

$$g_1 \quad : \quad \phi \frac{p}{c_{1m}} = (1-\phi) \frac{\gamma g_1^{\theta-1}}{\gamma g_1^\theta + (1-\gamma)\ell_1^\theta} \tag{A2.5}$$

$$\ell_2 \quad : \quad \phi \frac{\lambda w^s / r}{c_{1m}} = \beta (1-\phi) \frac{(1-\gamma)\ell_2^{\theta-1}}{\gamma g_2^\theta + (1-\gamma)\ell_2^\theta} \tag{A2.6}$$

$$g_2 \quad : \quad \phi \frac{p/r}{c_{1m}} = \beta (1-\phi) \frac{\gamma g_2^{\theta-1}}{\gamma g_2^\theta + (1-\gamma)\ell_2^\theta} \tag{A2.7}$$

$$c_{2m} \quad : \quad \phi \frac{1/r}{c_{1m}} = \beta \phi \frac{1}{c_{2m}} \tag{A2.8}$$

From (A2.4)-(A2.5) and (A2.6)-(A2.7) it transpires that

$$\frac{\ell_1}{g_1} = \frac{\ell_2}{g_2} = \left(\frac{\gamma}{1-\gamma} \frac{\lambda w^s}{p} \right)^{1/(\theta-1)}. \tag{A2.9}$$

Thus (A2.4) and (A2.6) write

$$\phi \frac{\lambda w^s}{c_{1m}} = (1-\phi) \frac{\ell_1^{-1}}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)} \left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}}$$

and

$$\phi \frac{\lambda w^s / r}{c_{1m}} = \beta (1-\phi) \frac{\ell_2^{-1}}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)} \left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}}$$

implying

$$\frac{\ell_2}{\ell_1} = \frac{g_2}{g_1} = \beta r. \tag{A2.10}$$

Using Equations (A2.8)-(A2.10), the budget constraint writes

$$c_{1m}\left(1+\beta\right) = \lambda w^s \left(1-\tau\right) + \frac{\lambda w^s}{r} - e - \ell_1 \left(1+\beta\right) \lambda w^s \left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)} + \left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}\right),$$

and, therefore, the first order condition (A2.4) becomes

$$\frac{\lambda w^s \left(1-\tau\right) + \frac{\lambda w^s}{r} - e - \ell_1 \left(1+\beta\right) \lambda w^s \left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)} \left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}\right)}{\lambda w^s \left(1+\beta\right)} = \frac{\phi}{1-\phi} \ell_1 \left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)} \left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}\right)$$

rearranging,

$$\ell_1 \left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)} \left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}\right) = \frac{1-\phi}{1+\beta} \left(\frac{1+r}{r} - \frac{e}{\lambda w^s} - \tau\right). \tag{A2.11}$$

For a skilled agent, time spent at home during the first period of life, ℓ_1, is defined by equation (A2.11) as a function of p, w and r. When $\theta < 0$, the term $\theta/(\theta-1)$ is positive. Then, as p increases, the term in parenthesis on the left-hand side of (A2.11) increases, but the right-hand side remains constant. Thus ℓ_1 must decrease, i.e., ℓ_1 is a decreasing function of p. As w^s increases, the term in parenthesis on the left decreases, but the right-hand side increases. Thus ℓ_1 must increase, i.e., ℓ_1 is an increasing function of w. Observe, from Equation (A2.10) that $\ell_2 = \beta r \ell_1$. Hence, ℓ_2 is increasing in w and decreasing in p. Similarly, one concludes that ℓ_1 and ℓ_2 are increasing in λ, the ability level of the household. The proposition follows from the definition of ℓ_1 and ℓ_2 for the skilled agent.

Let us turn to the decision of the unskilled agent. Comparing problems P(1) with P(2) leads to the conclusion that the solution to P(1) when $e = \tau = 0$

is the solution to P(2). Thus, for an uneducated agent, time spent at home during the first period of life is defined by

$$\ell_1\left(1+\left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)}\left(\frac{p}{\lambda w^u}\right)^{\theta/(\theta-1)}\right)=\frac{1-\phi}{1+\beta}\frac{1+r}{r}. \tag{A2.12}$$

Notice that for the uneducated agent the time spent at home is a function of p/w^u and r. One can see that, when $\theta < 0$, an increase in p/w^u leads to a decrease in ℓ_1. Likewise, ℓ_2 is decreasing in p/w^u and both ℓ_1 and ℓ_2 are increasing in λ. The proposition follows from the definition of ℓ_1 and ℓ_2 for the unskilled agent. ■

A3 Proof of Proposition 3

Consider an educated agent. His value function is

$$V^s(\lambda)=\phi\ln(c_{1m})+(1-\phi)\ln(c_{1n})+\beta\left(\phi\ln(c_{2m})+(1-\phi)\ln(c_{2n})\right),$$

or, using Equations (A2.8)-(A2.11)

$$\frac{V^s(\lambda)}{1+\beta}=\phi\ln(c_{1m})+(1-\phi)\ln(\ell_1)+(1-\phi)\frac{1}{\theta}\ln\left(1+\left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)}\left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}\right)$$
$$+\frac{\beta}{1+\beta}\ln(\beta r)+(1-\phi)\frac{1}{\theta}\ln(1-\gamma)$$

Plugging (A2.11) into the budget constraint leads to the following expression for c_{1m}:

$$c_{1m}=\frac{\lambda w^s}{1+\beta}\phi\left[\frac{1+r}{r}-\left(\frac{e}{\lambda w^s}+\tau\right)\right].$$

Finally

$$\frac{V^s(\lambda)}{1+\beta} = (1-\phi)\ln(1-\phi)+\ln\left(\frac{1}{1+\beta}\right)+\ln\left(\frac{1+r}{r}-\frac{e}{\lambda w^s}-\tau\right)+\phi\ln(\lambda w^s\phi)$$
$$+\frac{\beta}{1+\beta}\ln(\beta r)+(1-\phi)\frac{1}{\theta}\ln(1-\gamma)$$
$$+(1-\phi)\left(\frac{1}{\theta}-1\right)\ln\left(1+\left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)}\left(\frac{p}{\lambda w^s}\right)^{\theta/(\theta-1)}\right).$$

Similar calculations lead to

$$\frac{V^u(\lambda)}{1+\beta} = (1-\phi)\ln(1-\phi)+\ln\left(\frac{1}{1+\beta}\right)+\ln\left(\frac{1+r}{r}\right)+\phi\ln(\lambda w^u\phi)$$
$$+\frac{\beta}{1+\beta}\ln(\beta r)+(1-\phi)\frac{1}{\theta}\ln(1-\gamma)$$
$$+(1-\phi)\left(\frac{1}{\theta}-1\right)\ln\left(1+\left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)}\left(\frac{p}{\lambda w^u}\right)^{\theta/(\theta-1)}\right),$$

for the value of an uneducated agent. Observe first that, when $\theta < 0$, $1/\theta - 1 < 0$, therefore V^s and V^u are increasing in λ. Consider the difference between the value functions

$$\frac{V^s(\lambda)}{1+\beta}-\frac{V^u(\lambda)}{1+\beta} = \ln\left(\frac{1+r}{r}-\frac{e}{\lambda w^S}-\tau\right)-\ln\left(\frac{1+r}{r}\right)+\phi\ln\left(\frac{w^s}{w^u}\right)$$
$$+(1-\phi)\left(\frac{1}{\theta}-1\right)\ln\left(\frac{1+\left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)}\left(\frac{p}{\lambda w^S}\right)^{\theta/(\theta-1)}}{1+\left(\frac{1-\gamma}{\gamma}\right)^{1/(\theta-1)}\left(\frac{p}{\lambda w^U}\right)^{\theta/(\theta-1)}}\right)$$

Define $\underline{\lambda}$ such that $(1+r)/r - e/(\underline{\lambda} w^s) - \tau = 0$, and observe that $\underline{\lambda} > 0$ and that $lim_{\lambda\to\underline{\lambda}^+} V^s(\lambda) = -\infty$, whereas $V^u(\underline{\lambda})$ is finite:

$$\lim_{\lambda\to\underline{\lambda}^+}\frac{V^s(\lambda)}{1+\beta}-\frac{V^u(\lambda)}{1+\beta} < 0.$$

Observe also that

$$\lim_{\lambda\to\infty} \frac{V^s(\lambda)}{1+\beta} - \frac{V^u(\lambda)}{1+\beta} = \ln\left(\frac{1+r}{r} - \tau\right) + \phi \ln\left(\frac{w^s}{w^u}\right) - \ln\left(\frac{1+r}{r}\right).$$

This limit is positive as long as

$$\frac{w^s(1-\tau) + w^s/r}{w^u + w^u/r} > \left(\frac{w^s}{w^u}\right)^{1-\phi},$$

or

$$\frac{w^s}{w^u} > \left(1 - \tau\frac{r}{1+r}\right)^{-1/\phi}.$$

By continuity there exists $\lambda^* \in (\underline{\lambda}, +\infty)$ such that $V^s(\lambda^*) = V^u(\lambda^*)$. Furthermore, for $\lambda > \lambda^*$, $V^s(\lambda^*) > V^u(\lambda^*)$, and for $\lambda < \lambda^*$, $V^s(\lambda^*) < V^u(\lambda^*)$. See Figure 8 for an illustration. ■

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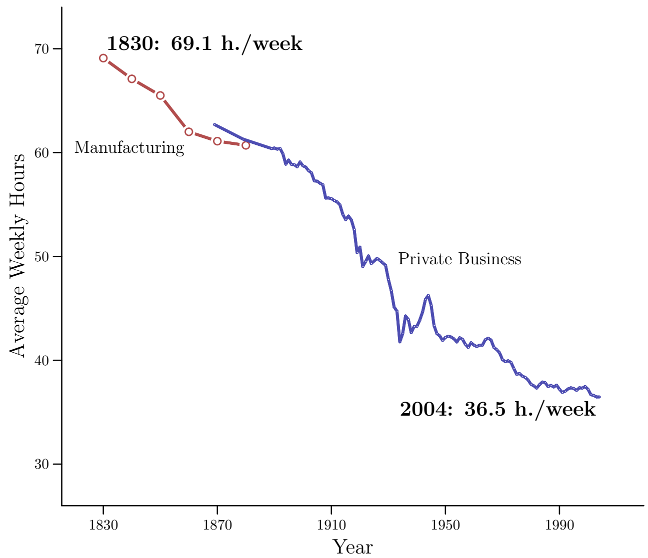

Figure 1: Average Weekly Hours – United States, 1830-2002.

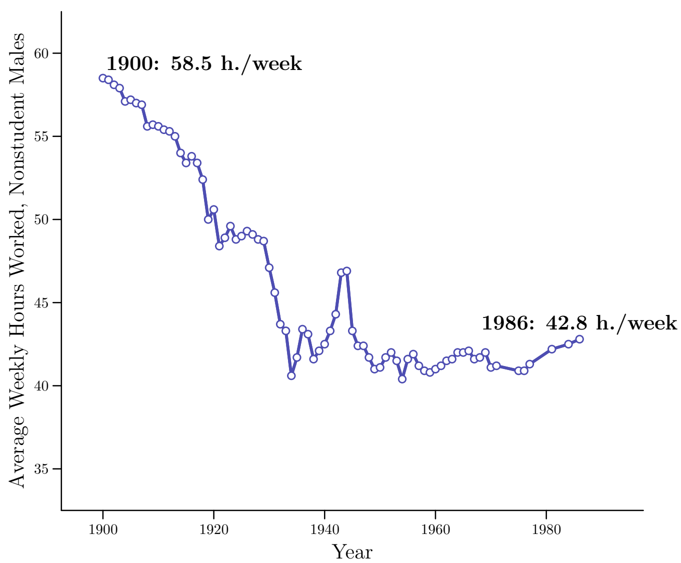

Figure 2: Average Weekly Hours for Non-student Males in Manufacturing – United States, 1900-1986.

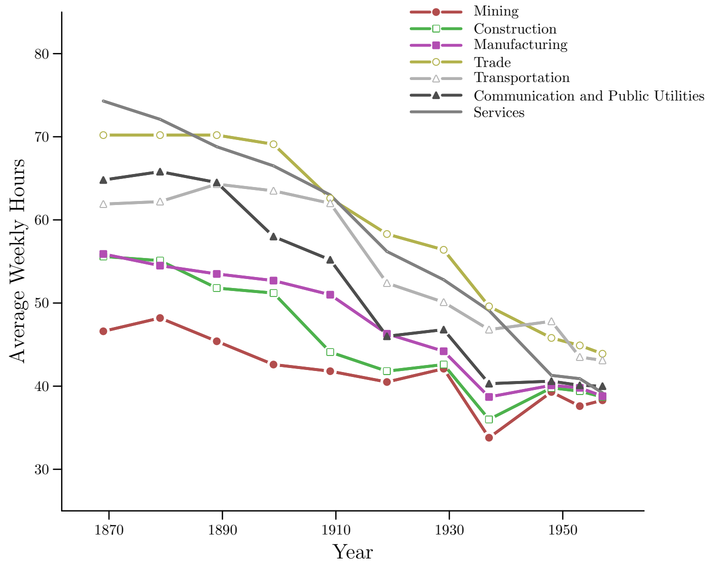

Figure 3: Average Weekly Hours in Various Sectors – United States, 1869-1957.

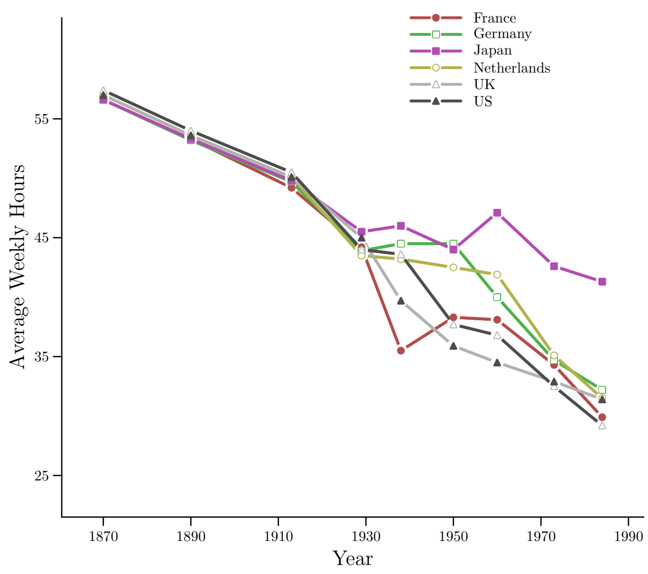

Figure 4: Average Weekly Hours – Various Countries, 1870-1990

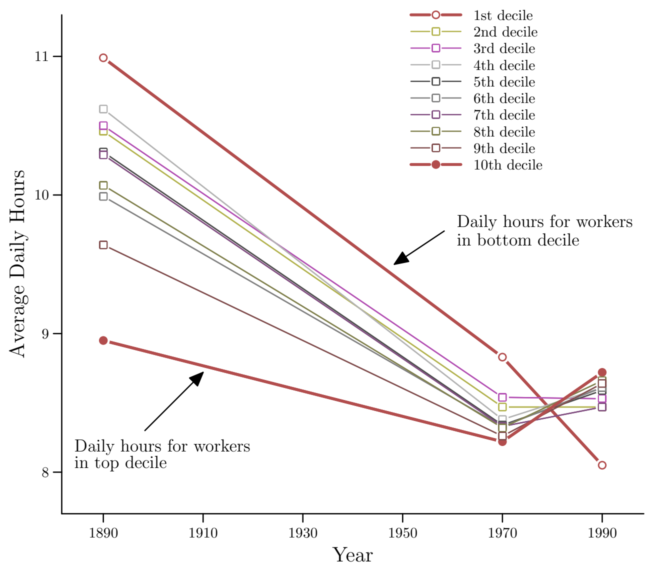

Figure 5: Daily Hours per Wage Decile – United States, 1890s, 1973 and 1991.

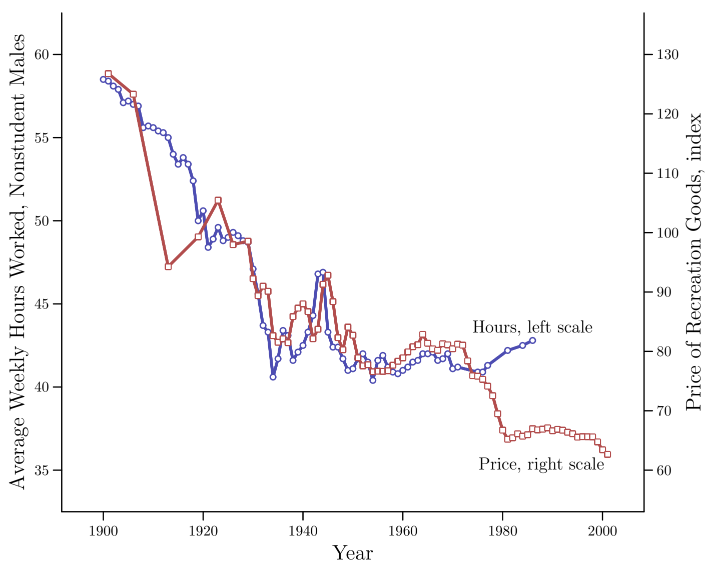

Figure 6: The Relative Price of Recreation Goods and the Workweek of Non-students Males – United States, 1900-2000.

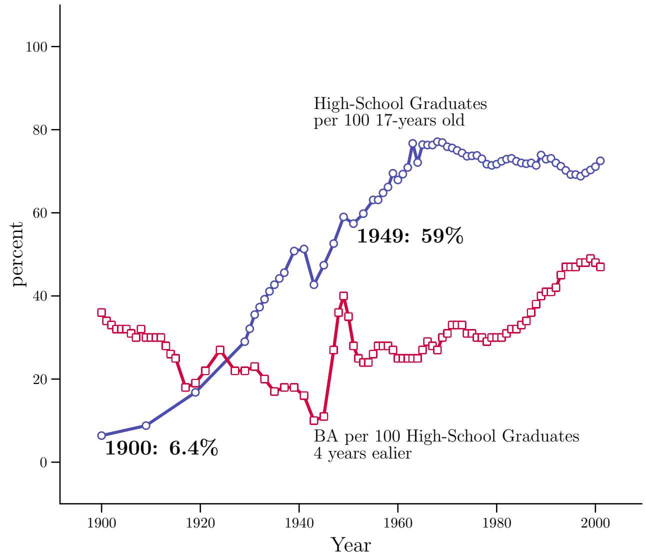

Figure 7: Number of High-School Graduates per 100 17-years Old and Number of BA Degrees Conferred per 100 High-School Graduates 4 Years Before – United States, 1900-2000.

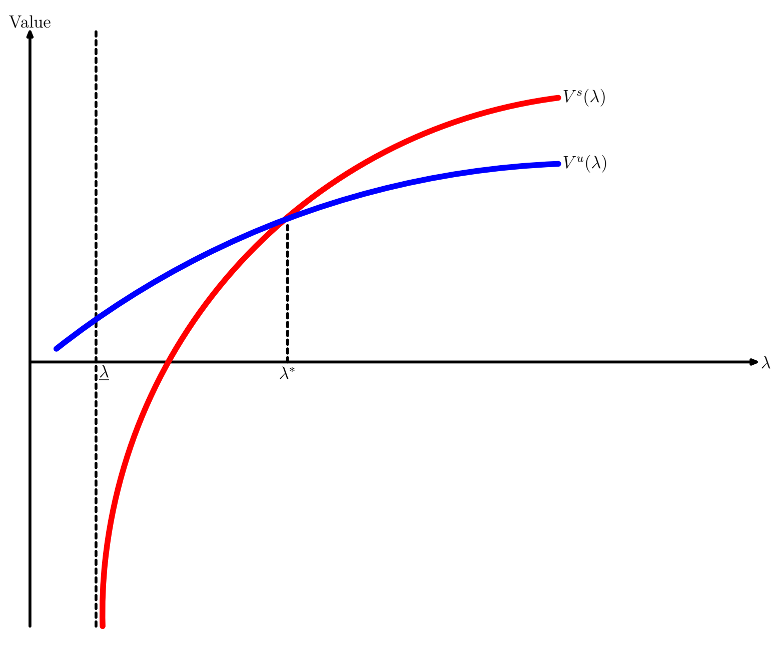

Figure 8: Value Functions of a Skilled and an Unskilled Agent.

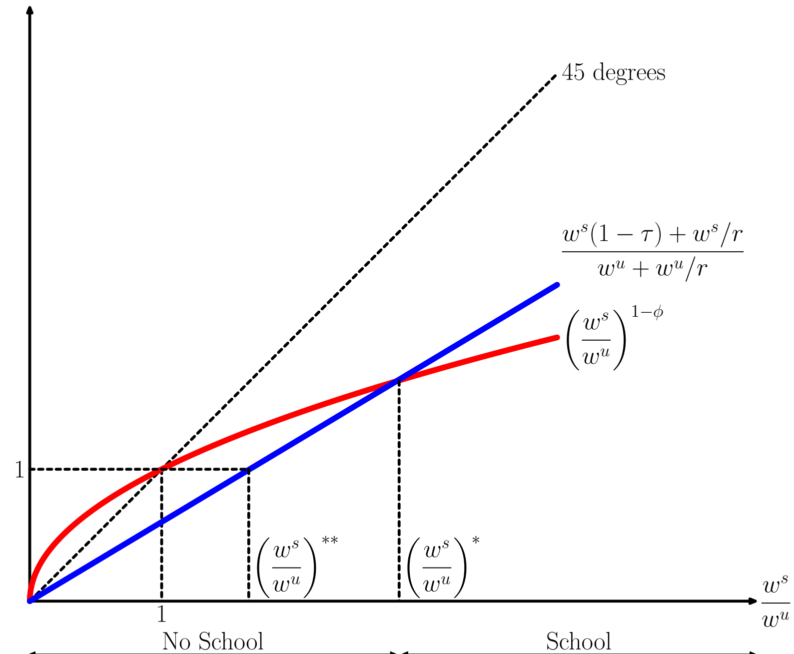

Figure 9: Schooling Decision of a Very Able Agent.

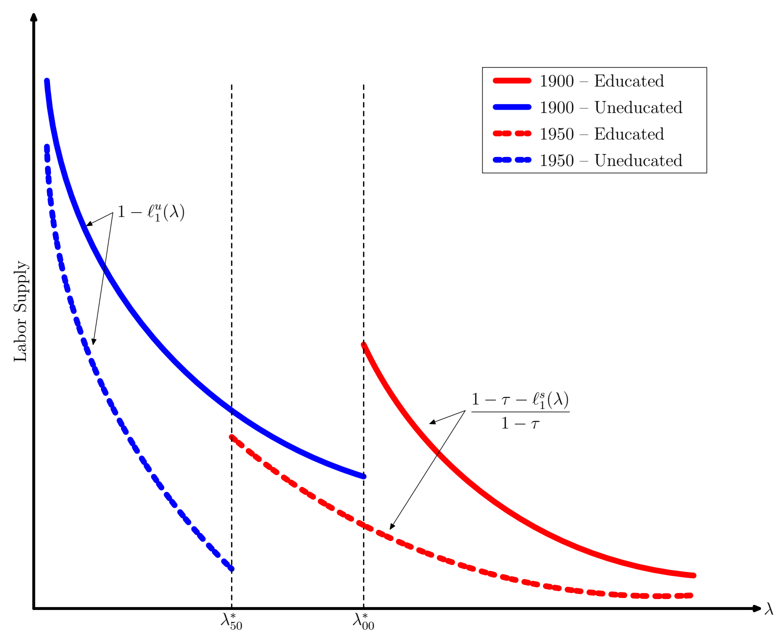

Figure 10: Labor Supply of Young Households.