

Valuing Lost Home Production for Dual-Earner Households

Christopher House, John Laitner, Dmitriy Stolyarov
University of Michigan

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Abstract

We use micro data on income and wealth to estimate the value of foregone home production by working married women. Our key finding is that the costs of women's labor force participation are modest: for every dollar that a married woman earns in the market, she incurs an average cost of 30 cents in lost home production.

This estimate implies that observed changes in women's wages are not enough to explain increased participation, and non-economic discrimination could have played a significant role in keeping married women at home in the past.

Using additional data on the allocation of time between market work and home production, we estimate the GDP bias associated with home production to be between 20 and 30 percent of measured GDP. Women's transition into the labor force decreases GDP bias associated with home production and causes a decline in the measured national saving rate. However, because the estimated decline is small, the explanation for the falling national saving rate must be elsewhere.

1 Introduction

One of the biggest changes in the U.S. economy in the twentieth century has been the dramatic increase in the labor force participation of women. Married women played a prominent role in this change: Goldin [1990, p.10] shows that while the participation rate for all women in the market labor force grew from 19% in 1890 to 60% in 1990, the participation rate of married women rose substantially more, from 5% in 1890 to nearly 60% in 1990. In 2002, the labor force participation rate for married women was 67%. The increased presence of married women in the labor force has obvious benefits: women now earn much more income than they did in the past. The costs of increased female labor supply are not as easy to measure. Working women, especially women with children, necessarily spend less time doing housework and other types of home production. The foregone value of time at home reduces the net benefit to working in the market. Thus when married women switch from home production to market production, the measured increase in income overstates the net benefit of working. However, because the value of a working woman's foregone home production is not measured directly, the true costs of giving up her time at home are unknown.

We estimate the costs associated with women’s labor force participation by comparing savings decisions of single-earner and dual-earner married couples. A household in which the wife works spends some of its income on goods and services that substitute for home production. Purchases of day care, cleaning services, restaurant meals, and so forth leave the household with less resources available for savings. As a result, we expect dual earner households to systematically consume a larger fraction of their measured income. We can then estimate the cost of working in the market from the differences in savings rates among single- and dual-earner couples.

We use micro data on income and retirement wealth from the Health and Retirement Study (HRS) to construct detailed measures of households’ lifetime earnings and wealth at retirement. We compare the data with predictions from a life-cycle savings model that incorporates home production. In the absence of measurement problems, retirement savings should depend on the total amount of household earnings and should not depend on whether the income was earned by the husband or the wife. The data suggests, however, that a wife’s earnings contribute only seventy percent as much as her husband’s earnings to the household’s retirement wealth. This is consistent with the hypothesis that there are costs associated with having married women enter the workforce. While these costs are economically significant, they are modest – for every dollar she earns in the market, a married woman incurs an average cost of just thirty cents of lost home production.

We use our estimate of the cost of market participation for married women to address three issues. First, we use additional data on time allocation between market work and work at home to construct an estimate of the gross domestic product that includes home production. We find that national accounts understate GDP by roughly 20 percent due to the omission of home production by married women. The GDP bias is substantial, because employed married women still spend 40 percent of their productive time on home production. Our estimate for the bias is somewhat lower than those in the previous studies that conjectured an understatement of GDP by almost fifty percent. This is because our specification allows for decreasing returns in home production, and our estimates suggest that the average product of labor at home is lower than its marginal product. Therefore, on average, time at home is valued at less than a woman’s market wage. Second, our analysis suggests that changing social norms and reductions in gender discrimination may have played a greater role than improving economic conditions did in explaining the increase in female labor force participation. Finally, increased labor force participation by married women makes GDP bias smaller, because some goods previously produced at home are now purchased on the market. As a larger fraction of GDP is measured by the national accounts, the national saving rate will appear to drop over time. However, because costs of labor force participation are modest, the GDP bias does not change much even after significant increase in labor force participation. Consequently, the estimated drop in the saving rate that can be attributed to the shift away from home production is small, and the explanations for the decline in the national savings rate must be found elsewhere. All of these results stem from the same basic finding: the cost of participating in the labor market for married women is relatively small compared to their market earnings.

The paper is set out as follows: Section 2 presents our econometric model and a simple life-cycle model with home production that underlies the econometric specification. Section 3 describes the methodology and the data. Section 4 presents a detailed econometric spec-

ification and regression results. Section 5 discusses the implications of our estimates, and Section 6 concludes.

2 Model

This section presents a regression equation that captures the connection between savings and the measurement of income and derives this equation from a simple life-cycle model with home production.

2.1 Econometric model

Our econometric model stems from the idea that working married women have to replace their time away from home with additional purchases of market goods that substitute for their home production. This could be day care, cleaning services, restaurant meals, etc. Let $\theta \in (0, 1)$ be a fraction of a woman's income that is spent on purchases of such substitutes. When $\theta = 1$, every dollar earned in the market has to be spent to replace the lost home production, and women should be indifferent between working at home and working in the market. If $\theta = 0$, the value of a woman's domestic work is zero and increases in market income accurately reflect increases in the household's total available resources. If $\theta > 0$, the necessity to replace the lost home production implies that dual-earner couples will systematically save a smaller fraction of their measured income than single-earner households. Then at retirement a dual-earned couple will accumulate less assets than a single-earner household with the same income path. Put differently, $\theta > 0$ means that the average propensity to save out of female earnings should be less than the average propensity to save out of male earnings. This reasoning leads to a regression equation

$$\ln(NW_i) = \ln(\kappa) + \ln(Y_i^M + (1 - \theta) \cdot Y_i^F) + \epsilon_i \quad (1)$$

where NW_i is net worth at retirement for household i and Y_i^M and Y_i^F are present values of male and female lifetime earnings, respectively. Parameter κ is the fraction of of lifetime resources set aside for retirement, which depends on retirement age, average propensity to save at different ages and the rate of return on assets.

Before turning to our estimation strategy, we introduce a life-cycle model with home production that leads to the regression equation in (1).

2.2 A Life-cycle Model with Home Production

Consider a household consisting of a man and a woman. The man works full time, supplies labor inelastically and earns after-tax income y^M . The woman takes her after-tax wage rate w as given and optimally allocates her time between working in the market and working at home. If a woman works $l \in [0, 1]$ hours in the market, her market income is $y^F = wl$, but the household has to spend $Al^{1/\theta}$ to compensate for the lost home production (the household may have to pay for day-care, hire a nanny, get a cleaning service and so on). We assume that $\theta \in (0, 1)$, so that the cost of being away from home is increasing and convex

in l . Given our normalization of l , we can interpret A as the total output of a full-time housewife. The wife chooses l to maximize her contribution to household's net income:

$$\max_l (wl - Al^{1/\theta})$$

and optimally,

$$Al^{1/\theta} = \theta wl = \theta y^F. \quad (2)$$

This says that a constant fraction θ of every dollar earned in the market is spent on purchasing home goods and the rest is available for consumption. This fraction is fixed even as women's wages change. As the wage w rises, women spend more time working in the market. However, the marginal cost of foregone home production also rises. These two effects exactly offset and the woman's average propensity to spend on home production substitutes is constant. For this household, the income available for purchases of consumption goods is market earnings net of home production expenses. Assuming that the man earns income y^M and does not do home production, the household's net income is $y^M + y^F(1 - \theta)$.

The time allocation model above showed that the income earned by the wife in the market came at a cost. The cost was the reduced value of home production. Households will take this cost into account when they make their life-cycle savings decision.

Consider a household that lives from age $s = 0$ to $s = T$. Let x_s denote household expenditure at age s . When the woman works, some of the household's income must be used to replace foregone home production. The household only gets utility from market goods other than home production substitutes. Let c_s denote current consumption net of home production expenses at age s . With the optimal allocation of time given by (2),

$$c_s = x_s - \theta y_s^F.$$

Both the wife and the husband work until an exogenous retirement age $R < T$. The man works full time, and his exogenous income at age s is y_s^M . The woman can either work in the market or at home and chooses her allocation optimally. At age $s \geq R$ both y_s^M and w_s^F drop to zero.

The household chooses a time path for net consumption c to solve:

$$\max_c \int_0^T e^{-\rho s} \cdot u(c_s) ds, \quad (3)$$

subject to:

$$\dot{a}_s = r \cdot a_s + y_s^M + y_s^F(1 - \theta) - c_s, \quad (4)$$

$$a_0 = 0 \quad \text{and} \quad a_T \geq 0,$$

Here r is the real interest rate and ρ is the subjective time discount rate.

The following proposition shows that when the utility has a constant elasticity of intertemporal substitution, the ratio of wealth at retirement to lifetime income is a constant. All proofs are in the appendix.

Proposition 1: If u is isoelastic $u(c) = \frac{c^\gamma}{\gamma}$ then, for all $s \geq R$,

$$\frac{NW_s}{Y_s^M + (1 - \theta) Y_s^F} = \kappa_s \quad (5)$$

Where

$$\eta = -r + \frac{r - \rho}{1 - \gamma} \quad (6)$$

and

$$Y_s^M = \int_0^T e^{-r \cdot (t-s)} \cdot y_t^M dt \quad \text{and} \quad Y_s^F = \int_0^T e^{-r \cdot (t-s)} \cdot y_t^F dt,$$

$$NW_s = a_s = \int_s^T e^{-r \cdot (t-s)} c_t dt \quad \text{and} \quad \kappa_s \equiv \frac{\int_s^T e^{\eta t} dt}{\int_0^T e^{\eta t} dt}$$

Proposition 1 establishes a simple relationship between observable earnings (Y^M, Y^F) and the household's net worth (NW) at retirement. This relationship plays a central role in our econometric specification in the following section.

2.3 Discussion of the model

HETEROGENEITY. Our specification allows a great deal of heterogeneity among households — i.e., men can have different earning abilities, y_{it}^M ; women can have different market opportunities w_{it}^F ; and women can have different replacement costs for their home production, A_{it} . The model, therefore, predicts that some married women will work in the labor market more hours than others: market hours rise with the ratio w_{it}^F/A_{it} . We could elaborate our utility function to incorporate differing numbers of children at different ages (e.g., Laitner and Silverman [2005]), and A_{it} could be higher when more children are at home. The role of children remains a topic for future work.

ENDOGENOUS RETIREMENT. Although we assume that retirement age is exogenously given, future work will endogenize it (see, for instance, Laitner and Silverman [2005]). In the meantime, our regression analysis below does take the possible endogeneity of R_i into account.

TIME-ALLOCATION CONSTRAINTS. Our model does not explicitly consider a participation margin, as each household chooses $l > 0$ if $w > 0$. Empirically, many married women choose $l = 0$. Presumably, this is because there are some fixed costs of having a market job at all. If these fixed costs are in terms of time (such as commuting) or money (e.g. an extra car or a business wardrobe), they must be subtracted from household's lifetime resources, and the proportionate relationship between retirement assets and present value of lifetime earnings in Proposition 1 will no longer hold. However, the fixed costs of labor force participation will reduce the net income of dual earner couples in the same way as purchases of home production substitutes. Then our empirical strategy is still valid, but the estimate of θ must be re-interpreted as a measure of tangible costs of labor force participation, not necessarily in terms of lost home production alone.

It is equally plausible that there are substantial non-monetary costs of a wife going to work that are related to changing the household organization pattern and re-defining the

roles of spouses in the family. If, in fact, the fixed costs of participation are in terms of *utility*, some households will choose to be traditional with $l = 0$, and others will choose to be dual-earner couples where women choose an optimal time allocation $l > 0$. Proposition 1 will hold for either type of households, and then the proportionate relationship between retirement assets and lifetime wealth in regression equation (1) will be exact. However, in this case our method will measure only tangible costs of labor force participation, and not the non-monetary barriers to entry of the labor force.

Instead of fixed costs, one might want to impose exogenous constraints on market hours

$$l_{it}^F \in [\underline{l}, \bar{l}] \subset (0, 1).$$

The idea of a lower limit $l > 0$ might be that there are fixed costs to — for both an employee and an employer — so that labor force participation with very short hours almost never occurs. This paper ignores such a possibility — in part because very low female earnings have little weight in determining θ in equation (1).

The idea of an upper bound might be that employers often must, by statute, pay overtime wages for a workweek exceeding 40 hours and are reluctant to do so. In practice, however, a worker might take a second job. What seems at least as likely is that an ambitious employee might work extra hours per week “off the clock” in order to secure future salary raises, promotions, etc.¹

Perhaps a more plausible obstacle constraining time allocation is as follows: due to coordination problems with other workers, efficiency costs to frequent startups, and advantages to finishing tasks promptly without requiring many workers to incur learning costs for the same project, employers might in practice offer a higher wage rate to full-time employees than to part-time workers. In other words, for some \underline{l} , we might have

$$w_{it}^F = \begin{cases} \underline{w}_{it} & \text{if } l < \underline{l} \\ \bar{w}_{it} & \text{if } l \geq \underline{l} \end{cases}$$

This could, for instance, lead a woman who would otherwise prefer a market job for 25 hours per week to select a 40-hour per week job instead (because of a lower wage rate for the former), and it would complicate our solution. Our data set (see below) does not provide information on work hours (before 1992), and we leave this issue as a topic for future research.

3 Data

We use data from the Health and Retirement Study (HRS) to predict lifetime earnings of men and women, and household net worth. We assume a gross-of-tax real interest rate of 5 percent per year.² Our income tax rate, $\tau = .15$, comes from government spending on goods

¹A dynamic model of human capital, including experience, accumulation is beyond the scope of this paper and remains a topic for future research. See, for example, Ryder *et. al.* [1976] and Attanasio *et. al.* [2004].

²Suppose that we calibrate our real interest rate from a ratio of factor payments to capital over the market value of private net worth. For the numerator, NIPA Table 1.13 gives corporate business income, indirect taxes, and total labor compensation. The first less the other two is a measure of corporate profits (net of depreciation); the ratio of profits to profits plus labor remuneration is “profits share.” Multiply

and services less indirect taxes.³ Thus, we set $r = .05 \cdot (1.0 - .15) = 0.0425$. This paper's focus is married couples.

3.1 Household Net Worth

We use the original survey cohort from the HRS, consisting of households in which the respondent is age 51-61 in 1992.⁴ In each survey wave (i.e., 1992, 1994, 1996, 1998, 2000, 2002), the HRS obtains a complete inventory of each household's assets (including own home, other real estate, automobiles, bank accounts, stocks and bonds, equity in own business, and equity in insurance) as well as debts. Prior to retirement assets include value of defined contribution pension accounts but not the capitalized value of future defined benefit pension rights; thus, we restrict our analysis to the net worth of retired couples. After retirement, the HRS asks households about their pension and Social Security Benefit flows — collecting information separately for husbands and wives on up to three pensions, three annuities, and Social Security Benefits.⁵

For our regression in Section 4, we exclude couples with either member under age 50, with males over 74, and with females over 80.⁶ The median male retirement age 1990-2000 is 62, and we restrict our sample to males who retire at ages 56-68 and to couples with 6 years or less difference in age (see below).

the latter times corporate and noncorporate business income plus nonprofit-institution income, less indirect taxes. Add the income of the household sector (see NIPA Table 1.13) less indirect taxes and labor remuneration. Finally, reduce the numerator by personal business expenses (brokerage fees, etc. from NIPA Table 2.5.5, rows 61-64). For the denominator, use U.S. Flow of Funds household and private non-profit institution net worth (Table B.100, row 19), less government liabilities (Table L106c, row 20). Average the net sum at the beginning and end of each given year. The average ratio 1952-2003 of the numerator to the denominator is .0504.

³Dividing by national income, the average over 1952-2003 is 14.28%/year.

⁴Age in this case refers to the age of the household's "principal respondent." For a couple, the "principal respondent" can be either male or female — and the survey incorporates the principal respondent's spouse.

⁵For the first two pensions, and the first two annuities, the HRS collects data on whether the flow is real or nominal, and on whether the flow carries survivorship rights. Our calculations assume a nominal interest rate three percent higher than our real rate.

⁶As explained below, our model assumes males die when they turn 75, and females when they turn 81. Very young retirees are presumably independently wealthy or disabled — both outside the scope of our model.

Variable	All FINR: ^a	Retired couples: ^b		
	HRS Net worth	HRS Net worth	Add Private Pensions	Add SSB
Minimum	-4,494,000	-41,000	-31,000	39,000
Lower Quartile	21,000	91,000	149,000	252,000
Median	76,000	198,000	281,000	398,000
Upper Quartile	194,000	392,000	488,000	588,000
Maximum	5,842,000	4,615,000	4,808,000	4,935,000
Mean	193,000	314,000	389,000	494,000
Coeff. of Variation	3.1	1.3	1.1	0.9
Nbr. of obs.	40024	795	795	795
Nbr. of households	7612	388	388	388

a. Sample is households of all “financial respondents” with valid net worth from original sample HRS.

b. Sample is intersection of married couple with both spouses alive, retirement age 56-68, 9-24 years of education for both spouses, husband’s current age 50-74; wife never worked or retired, with linked Social Security earnings record, wife’s current age 50-80. See text.

Table 1. Distribution of Household Net Worth: 1984 dollars

Table 1 presents details on our sample’s net worth. Clearly private pensions and Social Security Benefits are important components of net worth at ages near retirement.

3.2 Male lifetime earnings

The HRS links to their Social Security Administration (SSA) earnings histories participants who sign permission waivers. An individual’s “earnings history” includes annual data on his/her number of quarters with Social Security coverage and his/her Social Security taxable earning amount. Each history runs 1951–91. The HRS survey waves for 1992, 1994, 1996, 1998, 2000, and 2002 collect previous year’s earnings (but see below) and hours for all individuals. Two great advantages of the SSA linked data are that (i) even for a 61 year old in 1992, the SSA history includes earnings back to age 20, and (ii) the SSA data provides administrative-record quality observations. Disadvantages are (a) the Social Security System does not cover all jobs; (b) Social Security records do not include work hours;⁷ and, (c) histories are right-censored in some cases because they only track earnings up to the year’s statutory maximum subject to the Social Security tax.⁸

⁷Although the histories do annually report quarters of coverage, the definition of a “covered quarter” changes over time and is not directly related to work hours (e.g., Social Security Administration [2002, tab 2.A7]).

⁸The SSA also provides linked W2 tax reports annually 1980-91. Although the W2 records are right-censored for confidentiality, the upper limit is substantially higher than the Social Security earnings cap — \$125,000 for earnings under \$250,000, \$250,000 for earnings under \$500,000, and \$500,000 for earnings above that amount. In practice, we assume right-censoring at \$125,000 for all W2 amounts at or above \$125,000. The W2 amounts include non-FICA earnings — and separately identify the latter. They omit some tax

Our procedure for characterizing male lifetime earnings is as follows. We are willing to assume that men typically work full time. Our SSA linked earnings histories do not cover all jobs (especially in early years) and are subject to right-censuring in a large number of cases; thus, from all available positive (see below) male earnings observations, we estimate an earnings dynamics model, using a random-effects panel data specification and maximum likelihood methodology that takes account of censoring, and we use the model to impute male earnings prior to 1992. After 1992 (when part-time work prior to retirement might occur), we employ earnings from the survey. Appendix I provides a detailed explanation.

Since our data registers take home pay, we multiply earnings at each age by the year's ratio of NIPA total compensation to NIPA wage and salary accruals. Then we subtract employee and employer Social Security taxes and income taxes at our constant rate of .15.

Variable	Large Sample ^a	Restricted Sample ^b
Minimum	199,000	448,000
Lower Quartile	965,000	1,062,000
Median	1,309,000	1,403,000
Upper Quartile	1,750,000	1,802,000
Maximum	11,181,000	7,432,000
Mean	1,482,000	1,557,000
Coeff. of Variation	1.5	0.5
Nbr. of obs.	2692	388
Nbr. of households	2692	388

a. Sample is males with 9-24 years of education, age 50-74, at least one earnings observation

b. Sample is one male per household from column 2 of Table 1.

Table 2. Distribution of lifetime male earnings (present value at age 50): gross of benefits; net of taxes; 1984 dollars (NIPA PCE deflator); HRS household weights

Table 2 presents information on the distribution of male lifetime earnings. Column 1 includes all men who were married at one time, had at least one annual earnings figure that passed our screens (see Appendix I), had 9-24 years of education and a valid retirement age (see Appendix I), and had a retirement age between the ages of 50 and 69. Column 2 further restricts the sample to men who have at least one earnings observation and appear in Table-1 households.

3.3 Female lifetime earnings

We are unwilling to assume that women necessarily work in the labor force full time until retirement. As for men, we estimate a conventional earnings dynamics equation and use

deferred pension amounts. Although they also omit self-employment earnings, they identify Social Security measures of the latter. In practice, an individual may have multiple jobs, and we add the corresponding W2 amounts.

it to impute replacements for right-censored observations. We take all other observations, including zeros, directly from the data. Appendix I provides details.

Since we are not imputing substitutes for zeros (or very low reported earnings figures), we make an additional correction to the data as follows. We re-estimate our earnings dynamics equation using all positive earnings values (even the lowest). If a respondent reported that she had previous years of non-FICA employment and the number of years, we impute such earnings using the second earnings dynamics equation. See Appendix I for details.

Finally, as in the case of males, we multiply earnings at each age by the year's ratio of NIPA total compensation to NIPA wage and salary accruals. Then we subtract employee and employer Social Security taxes and income taxes at our constant rate of .15.

Variable	Large Sample ^a	Restricted Sample ^b	
		PV wife age 50	PV husband age 50
Minimum	0	0	0
Lower Quartile	133,000	186,000	165,000
Median	329,000	338,000	310,000
Upper Quartile	610,000	584,000	510,000
Maximum	2,969,000	1,386,000	1,365,000
Mean	421,000	410,000	338,000
Coeff. of Variation	0.9	0.7	0.7
Nbr. of obs.	2582	388	388
Nbr. of households	2582	388	388

a. Sample is males with 9-24 years of education, age 50-74, never worked or linked Social Security earnings record, valid retirement age.

b. Sample is one female per household from column 2 of Table 1.

c. Sample is one female per wave from column 2 of Table 1.

Table 3. Distribution of present value of female lifetime earnings: gross of benefits; net of taxes; 1984 dollars (NIPA PCE deflator); HRS household weights

Table 3, column 1, presents information on the distribution of lifetime earnings for women with 9-24 years of education, a usable retirement age, linked SSA data (or a survey response indicating no labor force participation prior to 1992), and a retirement age 50-69. As with men, all figures are present values at respondent age 50 and 1984 dollars. Column 2 restricts the sample as in Table 1. Column 3 recomputes column-2 present values at age 50 for the husband (see below).

4 Regression outcomes

We estimate θ from equation (5) using our HRS data, including linked Social Security earnings records. We use a subsample consisting of married couples with both spouses surviving

yet retired. The latter enables us to compute the capitalized values of household private pensions, as described in Section 3. We assume that all households have the same parameters θ , γ , and ρ ; hence, θ and η are the same for all families.

For household i of age s , with $s \geq R_i$, equation (5) is

$$\frac{NW_{is}}{Y_{is}^M + (1 - \theta) \cdot Y_{is}^F} = \frac{\int_s^T e^{\eta t} dt}{\int_0^T e^{\eta t} dt}. \quad (7)$$

Note that for the denominator on the left-hand side we need both Y^M and Y^F in present value at the same date — i.e., household age s . Henceforth, we take a household's age to be the husband's age. It is convenient to have all lifetime earnings at a common household age, namely, 50; so, we use the fact that

$$\frac{NW_{is} \cdot e^{-r \cdot (s-50)}}{Y_{i,50}^M + (1 - \theta) \cdot Y_{i,50}^F} = \frac{NW_{is}}{Y_{is}^M + (1 - \theta) \cdot Y_{is}^F}.$$

Similarly, we rewrite the right-hand side of (5) as follows:

$$\frac{\int_s^T e^{\eta t} dt}{\int_0^T e^{\eta t} dt} = \frac{e^{\eta s} \int_s^T e^{\eta(t-s)} dt}{\int_0^T e^{\eta t} dt} = \frac{e^{\eta(s-50)} \int_0^{T-s} e^{\eta v} dv}{e^{-\eta \cdot 50} \int_0^T e^{\eta t} dt}.$$

Since the denominator of this term is independent of i and s , we fold it into the regression constant. It is easy to perform the integration in the numerator — i.e.,

$$\int_0^{T-s} e^{\eta v} dv = \frac{e^{\eta(T-s)} - 1}{\eta}.$$

The latter expression creates problems near $\eta = 0$ — at $\eta = 0$, one needs to evaluate the ratio using L'Hospital's rule. To avoid such computational complexities, we evaluate the integral numerically (using trapezoidal integration with 1000 equal intervals).

Standard recent mortality tables for the U.S. imply an average male life span of 74 years and an average female life span of 80 years. We simply assume certain life spans of these amounts. There remains an issue of which life span to use in computing years left to live (i.e., $T - s$) in the numerator integral of $\kappa(s, \eta)$. Our resolution is based on the “equivalent adults” consumption rule implicit in the Social Security System: the System's benefit formula assumes that a retired household with two adults needs 150 percent as many dollars for expenditure as a single-adult household. If the wife in our analysis is age s^F when her husband is age s , if $\phi(s, i) \equiv \min\{75 - s, 81 - s^F\}$, and if $\psi(s, i) \equiv \max\{75 - s, 81 - s^F\}$, the numerator that we ultimately employ is

$$e^{\eta(s-50)} \cdot \left[\frac{3}{2} \cdot \int_0^{\phi(s,i)} e^{\eta v} dv + \int_{\phi(s,i)}^{\psi(s,i)} e^{\eta v} dv \right].$$

This paper's actual regression equation is

$$\begin{aligned} \ln(NW_{is} \cdot e^{-r(s-50)}) &= \beta_0 + \ln(Y_{i,50}^M + (1 - \beta_1) Y_{i,50}^F) + \beta_2(s - 50) + \\ &\ln\left(\frac{3}{2} \cdot \int_0^{\phi(s,i)} e^{\eta v} dv + \int_{\phi(s,i)}^{\psi(s,i)} e^{\eta v} dv\right) + \sum_{j=3}^{18} \beta_j X_{isj} + \mu_i + \varepsilon_{is} \end{aligned} \quad (8)$$

Table 4. Nonlinear Least Squares and Nonlinear Feasible Generalized Least Squares Regression Estimates: Coefficient (Standard Deviation)

Variable	Omit Retirement Dummies		Include Retirement Dummies	
	NLLS	FGLS NLLS	NLLS	FGLS NLLS
CONSTANT	-4.9398 (0.3220)	-5.0175 (0.3832)	-4.9436 (0.3256)	-5.0761 (0.3826)
THETA	0.3546 (0.1296)	0.3023 (0.1645)	0.3277 (0.1309)	0.2658 (0.1652)
ETA	-0.0093 (0.0134)	-0.0065 (0.0160)	-0.0139 (0.0136)	-0.0087 (0.0161)
DUM 1994	-0.2927 (0.1697)	-0.3186 (0.1495)	-0.2774 (0.1683)	-0.3117 (0.1488)
DUM 1996	-0.1293 (0.0916)	-0.1942 (0.0779)	-0.1151 (0.0910)	-0.1884 (0.0779)
DUM 1998	-0.1178 (0.0600)	-0.1244 (0.0479)	-0.1148 (0.0595)	-0.1247 (0.0479)
DUM 2002	-0.0111 (0.0491)	-0.0134 (0.0398)	-0.0163 (0.0487)	-0.0160 (0.0398)
RET AGE 57			0.1695 (0.0839)	0.2062 (0.1043)
RET AGE 58			0.0999 (0.0945)	0.1503 (0.1161)
RET AGE 59			0.2143 (0.0866)	0.2020 (0.1018)
RET AGE 60			-0.0197 (0.0758)	-0.0048 (0.0970)
RET AGE 61			0.2170 (0.0659)	0.1952 (0.0831)
RET AGE 63			0.0678 (0.0678)	0.0490 (0.0838)
RET AGE 64			0.2768 (0.0943)	0.1562 (0.1183)
RET AGE 65			0.2724 (0.0862)	0.2658 (0.1034)
RET AGE 66			0.2487 (0.1264)	0.2457 (0.1424)
RET AGE 67			0.0385 (0.5654)	0.0348 (0.5615)
RET AGE 68			-0.0469 (0.2553)	-0.0146 (0.2921)
Addendum				
OBSERVATIONS	795	795	795	795
NO. HOUSEHOLDS	388	388	388	388
MEAN SQ ERROR	0.3243	0.3247	0.3169	0.3187
R^2	0.3274	0.3266	0.3520	0.3483

where $\beta_0, \dots, \beta_{18}$ are parameters to be estimated; where β_1 estimates θ and β_2 estimates η ; and where X_{isj} is a time dummy for HRS wave 1994, 1996, 1998, or 2002 (2000 being omitted since the equation has a separate constant β_0 , and there are no observations from the 1992 wave in our subsample) or a time dummy for age of male retirement.⁹ The median age for male retirement is about 62, and we restrict our sample to households with husbands retiring at ages 56-68. We also restrict our sample to couples with husbands and wives no more than six years apart in age.¹⁰

The assumptions of our model imply that we expect an estimate, say, $\widehat{\beta}_1 \in (0, 1)$. Similarly, (6) implies

$$\beta_2 = -r + \frac{r - \rho}{1 - \gamma}.$$

We assume $r = .0425$. The existing literature frequently sets $\rho \approx .01$ and $\gamma = 0$ to -1 , and we therefore expect $\beta_2 \in [-0.01, -0.03]$.

Table 4 presents our regression results.¹¹ Columns 1 and 3 present nonlinear least squares regression outcomes; columns 2 and 4 present results for nonlinear feasible generalized least squares — correcting for the error correlations across observations for the same household from different survey waves. Appendix II outlines the steps for the FGLS estimation.

The outcomes should be viewed as preliminary at this stage. The estimates of theta fall in the range .26–.36. Assuming asymptotic normality, a 95 percent confidence for theta is (-0.02, 0.62) based on column 2, and it is (-0.06, 0.59) based on column 4. For η , a 95 percent confidence interval is (-0.04, 0.02) based on column 2, and the same based on column 4. In all cases, these overlap the set of values that we expected. The estimated intervals are, however, somewhat wide, and future work will seek to enlarge the sample (by including HRS data collected in 2004, for example) and to include additional covariates.

5 Implications

5.1 The Cost of Increased Female Labor Supply

Our most surprising finding is the small magnitude of our estimate of θ . Table 4 implies that as a wife enters the labor market, her household’s loss of home production services is only about 30 percent as much as her market compensation; thus, the household’s net gain as she finds employment outside the home is about 70 percent of what she earns at her new job. The immediate interpretation of θ is as a measure of the cost of women’s participation in the labor market. One must take care in interpreting this parameter. Our estimate says that, for every dollar the wife earns in the market, the *average* cost to the household in terms

⁹In practice, our sample has no observations with male retirement age 56, and we omit one age, namely, age 62, from our list of dummy variables because our regression equation already has a constant.

¹⁰Recall that we model time allocation for dual adult households, where the adults have overlapping life spans.

¹¹We ran OLS regressions imposing values of $\theta = \{0.0, 0.1, 0.2, \dots, 1.0\}$ and $\sigma = \{-0.20, -0.19, \dots, 0.0, 0.01, 0.02\}$ and set our NLLS starting (θ, σ) to minimize the OLS sum of squared residuals.

of lost home production is thirty cents. This is only an average figure; the *marginal* cost of earning an extra dollar in the market should be one dollar in foregone home production.

5.2 The Understatement of National Income

Economists have always known that GDP, as measured, understated the true value of total production. Our focus in this paper, the omission of home production, is a well-known source of mismeasurement. With the exception of imputed rent for owner occupied housing, home production receives no weight at all in the National Income and Product Accounts (NIPA). In 1973, William Nordhaus and James Tobin conjectured that the total value of domestic production and other non-market activity could be as large as measured output.

In this section we construct a measure of the degree to which GDP is understated by the NIPA. To do this, we first spell out the assumptions on household preferences and the production technology for the home good that lead to our assumed cost of woman working in the market of $Al^{1/\theta}$ and reduced form utility function $u(c)$. In particular, suppose that households consume a market good c and a home good h . The home good can be either purchased on the market or produced with woman's labor at home. If the woman works l hours on the market, she produces

$$h = A(1 - l^{1/\theta}) \tag{9}$$

of the home good with the rest of her time. This production function has decreasing returns to scale – the marginal product of the last hour of home production (at $l = 0$) is zero, hence the marginal cost of spending the first hour on the market is zero as well.

Assume that h and c are complements, and a minimum amount of home good $h = A$ is required to enjoy any utility from consumption of the market good. That is, the household utility function is given by

$$v(c, h) = I(h \geq A) \cdot u(c).$$

The motivation for assuming the minimum home production level $h \geq A$ may be that any family requires an equivalent of the services of a full-time housewife for its normal operation. Since producing h is costly, but $\frac{\partial v}{\partial h} = 0$ for all $h > A$, every household will optimally choose to produce $h = A$. Then the reduced form utility function for the household is $v(c, A) \equiv u(c)$. Production function (9) implies that traditional households with $l = 0$ produce all of their home good with their own labor, while working women produce $A(1 - l^{1/\theta})$ at home and buy $Al^{1/\theta}$ on the market. Therefore the cost of working l hours on the market is $Al^{1/\theta}$.

With these assumptions, the amount of home production per family is A . We can estimate A from the optimal time allocation condition (2), given the data on w and l and our estimate of θ . Rearranging equation (2) we can write:

$$A = \theta w l^{\frac{\theta-1}{\theta}}$$

Recall that w is the woman's market wage, l is the fraction of her productive time spent working in the market and A is the real value of home production.

We focus on three distinct types of households: families headed by a single person denoted by S ; households with dual earners denoted by D ; and “traditional” families denoted by T .

Traditional families are couples in which the husband works but the wife does not (i.e. $l = 0$). Let n_S , n_D , and n_T be the numbers of single families, dual-earner, and traditional households respectively.

To derive a coarse measure of “true” GDP, we assume that all married couples have the same home production requirements and we use average wages and hours for working married women to calculate the average annual value of their home production. Pencavel [1998], Table A1 reports the logarithms of annual hours and hourly wages for married women. A married woman who worked full time in Pencavel’s sample earned \$18.07 per hour in 2004 dollars.¹² Married women work roughly 33.5 hours per week (Pencavel, 1998) and employed women spend 25.6 hours per week on housework (Robinson and Godbey (1999, p. 101)). This means that $l = 1$ corresponds to $(33.5 + 25.6) \cdot 50 = 2955$ hours per year of productive time, and a potential annual market income $w = 17.25 \cdot 2955 = 50974$ dollars per year. Then, setting $l = 0.567$ we estimate that, on average, each married couple consumed \$ 60200 worth of home good in 2004. By assumption, a traditional household’s income is mismeasured by this entire amount (and maybe by more, since traditional households might have higher A). An average dual-earner household produces $A - \theta w l = 51200$ worth of home good with their own labor and purchase the remaining \$ 9000 home production substitutes in the market. The mismeasurement of single’s home production is not as obvious. In particular, it is not obvious whether singles have home production needs or not. There are many singles that do not own homes or have children. These households probably produce very little in the form of home goods. However, there are also many families headed by a single parent (especially single women). These families clearly have needs for home goods.

To derive a lower bound on the GDP bias, assume that all single households have $A = 0$. This assumption clearly understates the value of home production done by singles. Under our assumptions, the income of a traditional household would be mismeasured by A and the income of a dual-earner household would be mismeasured by $A - \theta y^F$. The lower bound for the unmeasured value of home production relative to measured GDP:

$$\underline{b} = \frac{n_D (A - \theta y^F) + n_T A}{Y^*} \approx 0.22.$$

We can derive an upper bound on the bias by assuming that non-family single households still have $A = 0$, but a family headed by a single person have to produce the same amount of home goods as a family couple. This leads to an upper bound for GDP bias

$$\bar{b} = \underline{b} + \frac{n_S (A - \theta y^F)}{Y^*} \approx 0.30.$$

Our estimates at the lower end of the 20 to 50 percent range reported in Eisner (1988) and much lower than that of Nordhaus and Tobin (1973). This difference reflects our assumptions on returns to scale in home production. For example, the Nordhaus and Tobin study valued women’s work at their market wage. This essentially said that both the marginal and the average value of home production were equal. If a woman earned \$30,000 more in the market, Nordhaus and Tobin would assume that the value of her home production fell by \$30,000. Our estimate casts doubt on this procedure. Specifically, our estimate of the relative value of a wife’s work suggests that the loss in home production would be only \$10,000.

¹²We use the NIPA PCE deflator.

5.3 Growth in Labor Supply of Married Women

Equation (2) and our estimate $\theta = 0.30$ imply that for every one percent increase in w_t^F relative to A_t , l_t^F should increase by 43 percent:

$$\frac{d \ln l}{d \ln w^F} = \frac{\theta}{1 - \theta} \approx 0.43.$$

Our estimated labor supply elasticity for married women is almost exactly the same as Pencavel's [1998, p. 798 table 19] estimate of wage elasticity of hours for employed married women. Hence, women's labor hours should have grown slowly over time. That has not been the case in practice: the observed labor supply of married women grew at a rate of 2.5% per year from 1950 to 1990 (Jones *et al.*, 2003). If we were to explain this change by the rise in women's wages alone, the labor supply elasticity would have to be about 1.25. This suggests that changes in participation are important in understanding the dynamics of hours.

Other recent estimates for the labor supply elasticity of married women are significantly higher than Pencavel's and ours (see Blundell and Macurdy, 1999 for a survey of these findings), partly because they also include the participation margin. However, reconciling the estimates of labor supply elasticity with and without participation margin requires a labor supply curve with a high elasticity at $l = 0$ and a significantly lower elasticity at $l > 0$.

An alternative scenario for reconciling a dramatic change in participation with a low elasticity of labor supply is a change in social and cultural norms and non-wage discrimination blocking women's progress in the labor market. In that scenario, slow but persistent advances in technology would lead to a build up of potential gains to female labor force participation, but the gains at first could not be realized because of social and attitudinal impediments. For instance, a social norm that stigmatizes work or a preference on the part of a husband for a stay-at-home wife would make it costly for a married woman to work. Once impediments dissolved, the pent up economic force was released, and women rapidly entered the labor force. Since change was in a sense overdue, net gains could be large. Our estimate of labor supply elasticity seem, at this point, most consistent with this type of story for increased participation.

If changing social norms are the cause of increased participation, there also has to be an explanation for why these norms changed rapidly after World War II. Fernandez *et al.* (2004) offer a theory based on an externality in men's preferences over the type of wife they seek. They show that men who grew up in families with working mothers were relatively more likely to have working wives. The fact that more women had to work during World War II might have initially fueled the externality, which explains the timing of the change in social norms.

If the social norm explanation for the increase in labor force participation of married women is true, we can interpret the growth in the labor supply of married women in excess of $\frac{\theta}{1-\theta} d \ln w^F$ as a measure of reduction in non-wage discrimination.

Atanasio *et al.* (2004) analyze different alternative sources of increased female labor force participation, including the a reduction in costs of having children, accelerated depreciation of human capital of non-employed women and increased uncertainty about husband's earnings. They stress the importance of reduced cost of child care relative to earnings. In our model, the cost of having children is measured by A , the amount of consumption good

that is required to replace a full-time housewife. In our model, the ratio of A/w^F would have to fall by $2.5/0.43 = 5.8$ percent a year (or by more than 10 times within 40 years) to match the observed change in labor supply.

5.4 The Impact of Changes in Household Composition on the Measured National Saving Rate

There may be important interactions between the change in female labor force participation and the mismeasurement of home production. In particular, as more and more women shift from non-market to market work, production will shift from a unmeasured sectors to measured sectors. To the extent that this is true, some of the growth in GDP and some of the decline in the savings rate (measured as net savings relative to GDP) can be attributed to simple measurement error.

We can use our estimate of θ to calculate the impact of changes in the composition of households on the national savings rate. The ratio of gross private saving in the U.S. to GDP, what we refer to as the average propensity to save, or the APS, rose from 0.17 in 1959 to 0.19 in 1979, but it fell to 0.14 in 1999. The introduction to this paper suggests that increases in the labor force participation rate for women conceivably played a role as follows: a household's saving behavior should be related to its earnings net of expenses — including expenses for replacing lost home production as women choose to work outside their home — whereas measured GDP is proportional to earnings alone; hence, the APS, which equals saving divided by GDP, may have fallen recently as growing replacement expenses caused the numerator to expand more slowly than the denominator. This section provides a quantitative assessment of this hypothesis.

With homothetic preferences, if the economy, for the sake of argument, is always in a long-run steady-state equilibrium, a household's saving at age s should be proportional to the present value of its lifetime resources.¹³ This paper shows that an accurate measure of a household's lifetime resources is

$$Y^M(t) + (1 - \theta) \cdot Y^F(t),$$

where $Y^M(t)$ and $Y^F(t)$ are the present value at fixed household age, say, 50, for a man and his wife if the household begins at time t . The household's saving at age s would be, say,

$$\sigma(s) \cdot [Y^M(t) + (1 - \theta) \cdot Y^F(t)],$$

with $\sigma(s)$ depending only on age. Assuming that earnings grow with technological progress at rate g and that population grows at rate n , that $Y^M(t)$ and $Y^F(s)$ reflect average earnings within a given birth cohort, and that each household's life span is T years, we have

$$APS(t) = \frac{\int_0^T \sigma(s) \cdot [Y^M(t-s) + (1 - \theta) \cdot Y^F(t-s)] e^{n(t-s)} ds}{GDP(t)} =$$

¹³Our discussion here assumes that technological progress would raise the value of time in home production, A_t , in the same proportion as it raises market wage rates over time.

$$= \frac{[Y^M(t) + (1 - \theta)Y^F(t)] \cdot e^{nt}}{GDP(t)} \int_0^T \sigma(s) e^{-(n+g)s} ds$$

Thus,

$$\frac{APS(t_1)}{APS(t_0)} = \frac{\frac{[Y^M(t_1) + (1 - \theta)Y^F(t_1)] \cdot e^{n \cdot t_1}}{GDP(t_1)}}{\frac{[Y^M(t_0) + (1 - \theta)Y^F(t_0)] \cdot e^{n \cdot t_0}}{GDP(t_0)}}$$

If labor earnings in total are a constant fraction of GDP over time, we have

$$\frac{GDP(t_0)}{GDP(t_1)} = \frac{[Y^M(t_0) + Y^F(t_0)] \cdot e^{n \cdot t_0}}{[Y^M(t_1) + Y^F(t_1)] \cdot e^{n \cdot t_1}}.$$

Call women's share of total earnings

$$f(t) \equiv \frac{Y^F(t)}{Y^M(t) + Y^F(t)}.$$

Together the above expressions imply

$$\frac{APS(t_1)}{APS(t_0)} = \frac{\frac{Y^M(t_1) + (1 - \theta) \cdot Y^F(t_1)}{Y^M(t_1) + Y^F(t_1)}}{\frac{Y^M(t_0) + (1 - \theta) \cdot Y^F(t_0)}{Y^M(t_0) + Y^F(t_0)}} = \frac{1 - \theta \cdot f(t_1)}{1 - \theta \cdot f(t_0)}. \quad (10)$$

To determine the quantitative impact of changes in $f(t)$, Table 5 compares male and female labor earnings in 1959, 1979, and 1999. We use Census micro-data from the Integrated Public Use Microdata Series (IPUMS).¹⁴ For each year, we have a one percent sample of the U.S. population. We compute the average wage income for men and for women who are 22-62 years old — using all men and women, including those who work and those who do not (entered with zero earnings).

Census year ^b	1960	1980	2000
Male average pre-tax wage earnings	\$ 4,111	\$ 14,300	\$ 36,253
Female average pre-tax wage earnings	\$ 989	\$ 5,160	\$ 19,554
Ratio of male to female earnings	3.9664	2.6307	1.7995
Ratio male to total earnings	0.7986	0.7246	0.6428
Ratio female to total earnings	0.2014	0.2754	0.3572
Number of males	425,022	564,381	748,358
Number of females	445,426	594,511	771,042

a. IPUMS (see text) are random samples of one percent of U.S. population. From the samples, we use all men and women age 22-62, including top-coded observations and zeros.

b. Data from each Census refer to earnings the years before.

Table 5. IPUMS Census male and female earnings over time: current dollars^a

¹⁴See Ruggles *et al.* [2004]. Note that data from the Census refers to earnings the previous year.

As one might expect, the change in female earnings relative to males is stark. In the 1960 Census, a man earned on average about four times (3.97) as much as a woman. By 2000, however, the ratio had fallen to less than two (1.80). As a share of total labor income earnings by the adult population, women account for only 20 percent in the 1960 Census, while they made up more than 35 percent in the 2000 Census. Comparing 1959 and 1999, equation (10) then implies

$$\frac{APS(1999)}{APS(1959)} = \frac{1 - \theta \cdot (.36)}{1 - \theta \cdot (.20)}.$$

If $\theta = 0$, changes in female labor force participation would have had no impact upon national saving — the ratio in (39) would be 1.0. If $\theta = 1.0$ — so that each dollar of a woman’s market earnings lead to a dollar less in home production — the saving rate would have fallen 19 percent because of the change in female participation. According to Section 4, $\theta \approx 0.3$; thus, the APS should have fallen about 5 percent due to participation changes.

Using $\theta = 0.3$, increasing female labor force participation 1959-99 should, according to our illustrative calculations, have caused the APS to decline 2.5 percent; from 1979-99, the decline should have been an additional 2.5 percent. Data at the beginning of this section shows an actual 11 percent increase in the APS 1959-79 and a 30 percent decline 1979-99. Evidently the actual changes are very large relative to the model’s predictions. Although the dynamics of the adjustment following changes in $f(t)$ could lead to greater short-run changes in the $APS(t)$ than our simplified calculations imply, our low estimate of θ in Section 4 precludes a large ultimate effect. The message of this rough calculation is clear. While the transition of women from non-market production to market production has been dramatic, the effect that it has on the mismeasurement of the savings rate is quite modest.

Appendix I: Lifetime Earnings

As stated in the text, we assume that males work full time until they retire. We first estimate a standard earnings dynamics regression model.

For male l of age s , let y_{hs} be “real” earnings (nominal earnings deflated with the GDP consumption deflator, normalized to 1 in 1984), and let X_{hs} be a vector including a constant, a quartic polynomial in years of work experience, and time dummies. We think of the polynomial as capturing the accumulation of human capital through work experience, and we think of the time dummies as registering the impact of macroeconomic forces of technological progress. We have one additional constant, a dummy for SSA observations. To economize on parameters for the computations, we employ a linear spline for our time dummies — with constant rates of growth for 1951–60, 1961–65, 1966–70, 1971–75, etc. Our regression equation is

$$\ln(y_{hs}) = X_{hs} \cdot \beta + u_h + e_{hs}.$$

The regression error term has two components: an individual specific random effect u_h , and an independent, non-specific random error e_{hs} . In fact, we have one random error, e_{hs} , for observations prior to 1991 and another, \bar{e}_{hs} , for observations after 1991.

Let the term of the overall likelihood function for equation (1) that applies to individual h be L_h . Each individual has three types of observations: $i \in I_h$ earnings figures from the SSA

history that are not right-censored; $j \in J_h$ from the SSA history that are right-censored; and $k \in K_h$ from the HRS public datasets 1992–2002 (never subject to right-censoring). Assume that u_h , e_{hs} , and \bar{e}_{hs} are independent normal random variables with precisions h_u , h_e , and $h_{\bar{e}}$. Let the normal density, say, for u , be $\phi(u, h_u)$, and let the corresponding normal cumulative distribution function be $\Phi(u, h_u)$. Define

$$c_{hs} = \ln(y_{hs}) - X_{hs} \cdot \beta.$$

Then

$$L_h = \int_{-\infty}^{\infty} \phi(u, h_u) \prod_{i \in I_h} \phi(z_i - u, h_e) \prod_{i \in J_h} [1 - \Phi(z_j - u, h_e)] \prod_{k \in K_h} \phi(z_k - u, h_{\bar{e}}) du.$$

Our estimates of $(\beta, h_e, h_{\bar{e}}, h_u)$, which we call $(\hat{\beta}, \hat{h}_e, \hat{h}_{\bar{e}}, \hat{h}_u)$, come from

$$\min_{\beta, h_e, h_{\bar{e}}, h_u} - \sum_h \ln(L_h).$$

For individuals with no right-censored observations, one can evaluate L_h in closed form; otherwise, we use numerical integration. We minimize the log likelihood function with Newton's method.¹⁵

EARNINGS DATA. To minimize complications from mixing full-time and part-time work, we omit observations from ages above 60 or past the man's reported retirement age. Similarly, we drop annual earnings amounts below 1500 hours \times statutory minimum wage, and we drop SSA observations from years with less than four quarters of work. We assume that earnings begin at age $S = \max\{\text{years of education}+6, 16\}$. We drop observations from ages prior to $S + 3$. Technically the HRS asks earnings and hours for the preceding year, but we find it plausible that respondents report amounts for the current calendar year; thus, we assume survey earnings refer to 1992, 1994, etc., rather than 1991, 1993,.... As protection against coding errors, we drop earnings observations above \$1 million. As our focus is couples, we omit single men.

MALE PARAMETER ESTIMATES. Table A1 presents our parameter estimates. As indicated in the text, we provide separate estimates for four education groups.

RETIREMENT. Construct the retirement age of male h , $R = R_h$, as follows: if he retires in 2002 or before and supplies the date, use it to set R ; if he is retired in one wave but not retired in the preceding wave, set R equal from the date of the second wave; if he has not retired by 2002,

set R from his expected retirement age; and, if dies without retiring, set R from his previously expected retirement age.¹⁶

¹⁵This paper's calculations use Compaq Visual Fortran 6.6. The minimization employs Newton's method (IMSL routine DUMIAH); we evaluate $\Phi(\cdot)$ with IMSL function DNORDF, and we evaluate the integral for u with a 21-point Gauss-Kronrod rule (IMSL routine DQ2AGS) — truncating the bounds of integration at plus and minus six standard deviations from 0. Our version of Newton's method employs user-specified first and second derivatives. For individuals with right-censored observations, the derivatives require numerical integration.

¹⁶Actual analysis in Section 4 uses only surviving men who report they have retired.

LIFETIME EARNINGS. We predict annual earnings from our regression equation, generating $(y_S, y_{S+1}, \dots, y_R)$. For ages x with SSA earnings that are not right-censored and that pass our data filters above, we substitute the actual SSA earnings figure for the imputed value of y_x . We compute annual earnings from 1992 onward by linearly interpolating and extrapolating from y_{1991} and survey data. The last step is meant to capture possible part-time work in years immediately before retirement.

We predict as follows. From our regression equation, we have

$$E[y_s | data] = e^{X_s \cdot \beta} \cdot E[e^u | data] \cdot E[e^{e_t} | data].$$

Given the large sample size in our regression, we simply set $\beta = \hat{\beta}$. Since e_t is an independent random variable, assumed normally distributed, in the SSA sample we use $\eta_e \equiv 1/\hat{h}_e$, etc., to set

$$E[e^{e_t} | data] = \begin{cases} e^{\eta_e^2/2}, & \text{if year} \leq 1991, \\ e^{\eta_e^2/2}, & \text{otherwise.} \end{cases}$$

The middle term in the prediction formula is the most complicated. The data provide a vector of points \vec{z}_i , a vector of intervals \vec{Z}_j with $Z_j = [z_j, \infty)$, and a vector of points \vec{z}_k . Letting $f(\cdot)$ be a density function, we have

$$f(u | \vec{z}_i, \vec{Z}_j, \vec{z}_k) = \frac{f(u, \vec{z}_i, \vec{Z}_j, \vec{z}_k)}{f(\vec{z}_i, \vec{Z}_j, \vec{z}_k)}$$

The statistical model implies

$$f(u, \vec{z}_i, \vec{Z}_j, \vec{z}_k) = \phi(u, h_u) \prod_{i \in I} \phi(z_i - u, h_e) \prod_{i \in J} [1 - \Phi(z_j - u, h_e)] \prod_{k \in K} \phi(z_k - u, h_{\bar{e}}) du.$$

Integrating with respect to u generates the marginal density $f(\vec{z}_i, \vec{Z}_j, \vec{z}_k)$; hence, we have

$$E[e^u | data] = \frac{\int_{-\infty}^{\infty} e^u \cdot \phi(u, h_u) \prod_{i \in I} \phi(z_i - u, h_e) \prod_{i \in J} [1 - \Phi(z_j - u, h_e)] \prod_{k \in K} \phi(z_k - u, h_{\bar{e}}) du}{\int_{-\infty}^{\infty} \phi(u, h_u) \prod_{i \in I} \phi(z_i - u, h_e) \prod_{i \in J} [1 - \Phi(z_j - u, h_e)] \prod_{k \in K} \phi(z_k - u, h_{\bar{e}}) du}.$$

Table 1 in the text summarizes the distribution of male lifetime earnings, providing present values at age 50 and in 1984 dollars.

FEMALE EARNINGS We are unwilling to assume that women necessarily work in the labor market full time; thus, our imputation procedure is more selective than the one that we use for men.

We use the same earnings dynamics equation as for men, though for each education group we estimate the equation twice for women. First, we employ the same filters as in the case of males — calling this our “exclusive sample.” For each earnings figure at the censoring limit, we use the maximum of the limit and the prediction from the first earnings dynamics equation. Table A2 presents our earnings dynamics equation coefficient estimates.¹⁷

¹⁷Comparing Tables A1-A2, in terms of coefficient signs and magnitudes one can see that the earnings dynamics model is much more successful for men — undoubtedly due to the heterogeneity of women’s labor force hours in the birth cohort of our sample. This paper relies primarily on actual earnings from the data. Beyond that, it uses Table A1 more extensively than Tables A2-A3.

Second, we enlarge our data set to what we call our “inclusive sample.” This includes ages from S up to retirement, earnings below 1500 hours \times statutory minimum wage, and those with less than four SSA quarters of work. In 1996 the HRS inquired about number of years of non-FICA employment and the corresponding dates. We predict non-FICA earnings for jobs prior to 1980 (after 1980, our data includes non-FICA earnings) with the second earnings dynamics equation (and the prediction steps described above). If the survey reports, for instance, two years of non-FICA work 1955-58, we impute an annual amount for each of the four years but multiply predicted values by one half. If a non-FICA prediction overlaps an observation in the data, we sum the two. Table A3 presents our second earnings dynamics equation coefficient estimates.¹⁸

Table 2 in the text summarizes the distribution of female lifetime earnings.

¹⁸See the preceding footnote.

Table A1. Male Earnings Regression Coefficients (Standard Error), by Education; Weighted Regressions; 1984 Prices with NIPA PCE Deflator^a

Description	Less Than High School	High School	Some College	College or more
DUM SSA	-0.0384 (0.0386)	-0.0243 (0.0250)	-0.0182 (0.0327)	0.0031 (0.0329)
CONSTANT	8.7160 (0.0578)	8.8087 (0.0418)	8.9343 (0.0642)	9.0599 (0.0835)
EXP	0.1305 (0.0101)	0.1419 (0.0073)	0.1397 (0.0102)	0.1109 (0.0135)
EXP**2/100	-0.5894 (0.0758)	-0.7606 (0.0586)	-0.7629 (0.0868)	-0.6963 (0.1188)
EXP**3/1000	0.1322 (0.0232)	0.1822 (0.0189)	0.1932 (0.0293)	0.2062 (0.0419)
EXP**4/10000	-0.0115 (0.0024)	-0.0162 (0.0021)	-0.0184 (0.0034)	-0.0233 (0.0050)
DUM 51-60	-0.0095 (0.0042)	-0.0023 (0.0032)	0.0020 (0.0055)	0.0243 (0.0080)
DUM 61-65	0.0228 (0.0046)	0.0379 (0.0034)	0.0191 (0.0053)	0.0619 (0.0068)
DUM 66-70	0.0299 (0.0048)	0.0413 (0.0035)	0.0490 (0.0054)	0.0386 (0.0069)
DUM 71-75	-0.0064 (0.0045)	-0.0037 (0.0033)	-0.0058 (0.0050)	0.0179 (0.0060)
DUM 76-80	-0.0218 (0.0042)	-0.0133 (0.0030)	-0.0245 (0.0044)	-0.0366 (0.0049)
DUM 81-85	-0.0144 (0.0041)	-0.0110 (0.0028)	-0.0029 (0.0041)	0.0169 (0.0042)
DUM 86-90	-0.0164 (0.0043)	-0.0107 (0.0029)	-0.0043 (0.0042)	0.0026 (0.0043)
DUM 91-95	-0.0037 (0.0119)	0.0103 (0.0077)	0.0130 (0.0101)	0.0343 (0.0103)
DUM 96-00	0.0372 (0.0121)	0.0324 (0.0078)	0.0338 (0.0097)	0.0249 (0.0101)
h_e	2.9986 (0.0209)	2.7966 (0.0137)	2.6460 (0.0187)	2.7476 (0.0205)
h_u	2.7660 (0.0833)	2.6664 (0.0531)	2.4399 (0.0649)	1.9540 (0.0484)
$h_{\bar{e}}$	2.4560 (0.0637)	2.3261 (0.0364)	2.3627 (0.0475)	2.1224 (0.0402)
Addendum: Summary Statistics				
Observations	15,628	35,481	17,641	18,834
Households	675	1,568	877	1,032
$-\ln(\text{likelihood})$	7201.3026	17616.3343	9321.1486	9106.7372

a. See text.

Table A2. Female Earnings Regression Coefficients (Standard Error), by Education; Weighted Regressions; 1984 Prices with NIPA PCE Deflator; “Exclusive Sample”^a

Description	Less Than High School	High School	Some College	College or more
DUM SSA	-0.0741 (0.0325)	-0.0311 (0.0197)	-0.0417 (0.0278)	-0.0114 (0.0295)
CONSTANT	8.6949 (0.0547)	9.0010 (0.0361)	8.9508 (0.0706)	9.0521 (0.1564)
EXP	0.0212 (0.0112)	-0.0070 (0.0068)	0.0070 (0.0098)	-0.0762 (0.0134)
EXP**2/100	-0.1383 (0.0839)	0.0491 (0.0547)	-0.0664 (0.0817)	0.6109 (0.1176)
EXP**3/1000	0.0481 (0.0255)	0.0020 (0.0176)	0.0349 (0.0275)	-0.1713 (0.0418)
EXP**4/10000	-0.0058 (0.0027)	-0.0019 (0.0019)	-0.0056 (0.0032)	0.0156 (0.0051)
DUM 51-60	0.0149 (0.0048)	0.0058 (0.0034)	0.0134 (0.0074)	0.0426 (0.0162)
DUM 61-65	0.0136 (0.0054)	0.0134 (0.0036)	0.0292 (0.0059)	0.0389 (0.0097)
DUM 66-70	0.0266 (0.0048)	0.0243 (0.0033)	0.0129 (0.0049)	0.0392 (0.0073)
DUM 71-75	0.0017 (0.0042)	-0.0036 (0.0029)	0.0047 (0.0042)	-0.0035 (0.0058)
DUM 76-80	0.0024 (0.0038)	0.0075 (0.0026)	0.0192 (0.0037)	-0.0001 (0.0047)
DUM 81-85	0.0027 (0.0035)	0.0078 (0.0024)	0.0188 (0.0033)	0.0298 (0.0041)
DUM 86-90	0.0004 (0.0036)	-0.0016 (0.0024)	0.0157 (0.0033)	0.0229 (0.0040)
DUM 91-95	-0.0004 (0.0100)	0.0099 (0.0060)	0.0224 (0.0084)	0.0377 (0.0090)
DUM 96-00	0.0320 (0.0095)	0.0229 (0.0053)	0.0253 (0.0071)	0.0396 (0.0077)
h_e	3.9150 (0.0318)	3.4870 (0.0172)	3.3102 (0.0228)	3.1271 (0.0263)
h_u	3.1901 (0.0934)	2.8766 (0.0526)	2.7064 (0.0697)	2.1436 (0.0599)
$h_{\bar{e}}$	2.6811 (0.0677)	2.7892 (0.0406)	2.5524 (0.0469)	2.6801 (0.0547)
Addendum: Summary Statistics				
Observations	9,223	26,015	13,899	10,609
Households	798	1,946	999	782
$-\ln(\text{likelihood})$	1927.8852	7996.1526	5141.4456	4603.7158

a. See text.

Table A3. Female Earnings Regression Coefficients (Standard Error), by Education; Weighted Regressions; 1984 Prices with NIPA PCE Deflator; “Inclusive Sample”^a

Description	Less Than High School	High School	Some College	College or more
DUM SSA	-0.0814 (0.0591)	-0.0236 (0.0339)	-0.0653 (0.0472)	-0.0415 (0.0503)
CONSTANT	7.5107 (0.0832)	8.2312 (0.0504)	8.2383 (0.0862)	7.8950 (0.1583)
EXP	0.0923 (0.0129)	0.0251 (0.0071)	-0.0350 (0.0098)	-0.0512 (0.0135)
EXP**2/100	-0.3493 (0.0978)	-0.1496 (0.0574)	0.1984 (0.0804)	0.4207 (0.1197)
EXP**3/1000	0.0799 (0.0300)	0.0697 (0.0185)	-0.0389 (0.0266)	-0.0830 (0.0423)
EXP**4/10000	-0.0076 (0.0031)	-0.0097 (0.0020)	0.0018 (0.0029)	0.0024 (0.0050)
DUM 51-60	-0.0317 (0.0083)	-0.0176 (0.0051)	0.0225 (0.0088)	0.0788 (0.0173)
DUM 61-65	0.0232 (0.0104)	0.0082 (0.0065)	0.0218 (0.0097)	0.0121 (0.0145)
DUM 66-70	0.0343 (0.0102)	0.0222 (0.0067)	0.0387 (0.0093)	0.0319 (0.0125)
DUM 71-75	-0.0056 (0.0096)	0.0138 (0.0062)	0.0325 (0.0087)	0.0196 (0.0113)
DUM 76-80	0.0095 (0.0091)	0.0258 (0.0058)	0.0517 (0.0081)	0.0438 (0.0101)
DUM 81-85	-0.0215 (0.0089)	-0.0052 (0.0056)	0.0358 (0.0076)	0.0262 (0.0092)
DUM 86-90	0.0184 (0.0090)	0.0100 (0.0056)	0.0269 (0.0076)	0.0392 (0.0090)
DUM 91-95	0.0145 (0.0175)	0.0213 (0.0098)	0.0373 (0.0135)	0.0308 (0.0144)
DUM 96-00	0.0350 (0.0139)	0.0298 (0.0078)	0.0163 (0.0103)	0.0431 (0.0111)
h_e	1.1161 (0.0065)	1.1375 (0.0043)	1.1160 (0.0061)	1.0546 (0.0072)
h_u	1.3572 (0.0364)	1.4392 (0.0252)	1.4521 (0.0365)	1.2222 (0.0333)
$h_{\bar{e}}$	1.3197 (0.0239)	1.4644 (0.0164)	1.4047 (0.0208)	1.5002 (0.0257)
Addendum: Summary Statistics				
Observations	17,375	43,648	21,685	15,615
Households	981	2,252	1,120	862
$-\ln(\text{likelihood})$	23,826.32	57,277.90	28,878.51	20,661.63

a. See text.

Appendix II: Feasible Generalized Least Squares Estimation¹⁹

At time t , let $s = s(i, t)$ be the age of household i . Define

$$e(i, t, \beta) \equiv \ln(NW_{is} \cdot e^{-r(s(i,t)-50)}) - \beta_0 - \ln(Y_{i,50}^M + (1 - \beta_1)Y_{i,50}^F) - \beta_2(s(i, t) - 50) - \ln\left(\frac{3}{2} \cdot \int_0^{\phi(s(i,t),i)} e^{\eta \cdot v} dv + \int_{\phi(s(i,t),i)}^{\psi(s(i,t),i)} e^{\eta \cdot v} dv\right) - \sum_{j=3}^{18} \beta_j X_{i,s(i,t),j}$$

For each household i , we have observations for a set of times $\mathcal{T}(i)$.

Step 1. We obtain a consistent estimate of β from

$$\hat{\beta} = \arg \max_{\beta} \sum_i \sum_{t \in \mathcal{T}(i)} e(i, t, \beta) \cdot e(i, t, \beta).$$

Step 2. We generate a set of residuals $e_{it} = e(i, t, \hat{\beta})$. Equation (8) shows the residuals represent the sum of two errors, μ_i and ϵ_{it} . We obtain a consistent estimate of the ratio of variances

$$\frac{\sigma_{\mu}^2}{\sigma_{\mu}^2 + \sigma_{\epsilon}^2}$$

using

$$\hat{z} = \frac{\sum_i \left(\sum_{\mathcal{T}(i)} e_{it}\right)^2 - \sum_i \sum_{\mathcal{T}(i)} e_{it}^2}{\hat{s}^2 \cdot \sum_i ([t(i)]^2 - t(i))}$$

where

$$t(i) = \#\mathcal{T}(i),$$

$$\hat{s}^2 = \frac{\sum_i \sum_{\mathcal{T}(i)} e_{it}^2}{\sum_i t(i)}.$$

Step 3. For each i , let the $t(i) \times t(i)$ matrix $\Omega(i)$ have \hat{z} off of the main diagonal and 1 on the main diagonal. Let $\vec{e}(i, \beta)$ be the (column) vector $(e(i, 1, \beta), \dots, e(i, t(i), \beta))$. Then our FGLS estimate of β is

$$\bar{\beta} = \arg \max_{\beta} \sum_i \vec{e}(i, \beta)^T \cdot [\Omega(i)]^{-1} \cdot \vec{e}(i, \beta).$$

Forming a new estimate of s^2 from the residuals based on $\bar{\beta}$, the covariance matrix for $\bar{\beta}$ is

$$\hat{s}^2 \cdot \left[\sum_i F_i^T \cdot [\Omega(i)]^{-1} \cdot F_i \right]^{-1},$$

where F_i is the matrix with (18–element) row $\partial e(i, t, \bar{\beta})/\partial \beta$ for each t , $t = 1, \dots, t(i)$.

¹⁹See, for example, Gallant [1987, ch. 2].

Appendix III: Proof of Proposition 1

The current-value Hamiltonian for the household's optimization problem is

$$\mathcal{H} = u(c_s) + \lambda_s \cdot [r \cdot a_s + y_s^M + y_s^F (1 - \theta) - c_s]. \quad (11)$$

Since the Hamiltonian is jointly concave in c_s and a_s , the following conditions are necessary and sufficient for an optimum:

$$\frac{\partial \mathcal{H}}{\partial c_s} = 0 \iff c_s^{\gamma-1} = \lambda_s \quad (12)$$

$$\dot{\lambda}_s = \rho \cdot \lambda_s - \frac{\partial \mathcal{H}}{\partial a_s} \iff \dot{\lambda}_s = \lambda_s \cdot [\rho - r], \quad (13)$$

$$\dot{a}_s = r \cdot a_s + y_s^M + y_s^F (1 - \theta) - c_s \quad (14)$$

$$\lambda_T \geq 0 \text{ and } \lambda_T \cdot a_T = 0 \quad (15)$$

$$a_0 = 0. \quad (16)$$

At an optimum, (12) and (13) show $\lambda_s > 0$ all s ; thus, (15) implies $a_T = 0$. Taking logs and time derivatives of (12), using (13) and the fact that the interest rate is constant implies that the growth rate of net consumption c_s is constant.

$$\dot{c}_s/c_s = (r - \rho)/(1 - \gamma).$$

Integrating both sides from 0 to s ,

$$c_s = c_0 e^{\left[\frac{r-\rho}{1-\gamma}\right]s} \quad (17)$$

Multiplying every term in (14) by $e^{-r \cdot (s-R)}$ we can show:

$$\frac{d}{ds} (e^{-r(s-R)} a_s) = e^{-r(s-R)} [\dot{a}_s - r a_s] = e^{-r(s-R)} [y_s^M + y_s^F (1 - \theta) - c_s]. \quad (18)$$

Integrating both sides from $s = R$ to $s = T$, using the fundamental theorem of calculus, and noting that the first three right-hand side terms are zero for $s > R$, we have

$$a_R = \int_R^T e^{-r \cdot (s-R)} \cdot c_s ds \quad (19)$$

Moreover, integrating both sides of (18) from $s = 0$ to $s = T$, gives:

$$Y^M + Y^F (1 - \theta) = \int_0^T e^{-r(s-R)} c_s ds \quad (20)$$

where Y_R^M and Y_R^F are the capitalized value of total lifetime male and female market earnings at retirement.

$$Y_R^M = \int_0^T e^{-r \cdot (s-R)} \cdot y_s^M ds \quad \text{and} \quad Y_R^F = \int_0^T e^{-r \cdot (s-R)} \cdot y_s^F ds$$

Since $NW_R = a_R$ we have:

$$\frac{NW_R}{Y^M + Y^F(1 - \theta)} = \frac{\int_R^T e^{-r(s-R)} \cdot c_s ds}{\int_0^T e^{-r(s-R)} \cdot c_s ds}$$

Cancelling the $e^{r \cdot R}$ from the top and bottom and using (17), gives:

$$\frac{NW_R}{Y_R^M + Y_R^F(1 - \theta)} = \kappa$$

where $\kappa \equiv \frac{\int_R^T e^{\eta s} ds}{\int_0^T e^{\eta s} ds}$ and $\eta = -r + \frac{r-\rho}{1-\gamma}$. \square

References

- [1] Attanasio, Orazio; Low, Hamish; and Sanchez-Marcos, Virginia “Changes in Female Labor Supply in a Life-Cycle Model,” CWPE 0451 working paper, October 2004.
- [2] Benhabib, Jess; Rogerson, Richard; and Wright, Randall. “Homework in Macroeconomics: Household Production and Aggregate Fluctuations,” *Journal of Political Economy* 99, no. 6 (December 1991): 1166–1187.
- [3] Blundell, Richard and Thomas MaCurdy, “Labor Supply: A Review of Alternative Approaches,” in A. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Volume 3 (Amsterdam: Elsevier, 1999): 1569–1695.
- [4] *Economic Report of the President 2002*. Washington: U.S. Government Printing Office, 2002.
- [5] Eisner, Robert. “Extended Accounts for National Income and Product,” *Journal of Economic Literature* 26, no. 4 (December 1988): 1611–1684.
- [6] Feldstein, Martin. *Privatizing Social Security*. Chicago: The University of Chicago Press, 1998.
- [7] Fernandez, Raquel; Alessandra Fogli; and Claudia Olivetti, “Mothers and Sons: Preference Formation and Female Labor Force Dynamics”, forthcoming, *Quarterly Journal of Economics*.
- [8] Gallant, A. Ronald. *Nonlinear Statistical Models*. New York: John Wiley & Sons, 1987.
- [9] Goldin, Claudia. *Understanding the Gender Gap*. Oxford: Oxford University Press, 1990.
- [10] Greenwood, Jeremy; Rogerson, Richard; and Wright, Randall. “Household Production in Real Business Cycle Theory,” in Thomas F. Cooley (ed.), *Frontiers of Business Cycle Research*. Princeton, N.J.: Princeton University Press, 1995.

- [11] Jones, Larry E.; Rodolpho E. Manuelli; and Ellen R. McGrattan, “Why Are Married Women Working So Much?”, Federal Reserve Bank of Minneapolis Research Department Staff Report 317, June 2003
- [12] Haider, Steven, and Gary Solon, “Nonrandom Selection in the HRS Social Security Earnings Sample.” Working Paper 00–01, RAND Labor and Population Program, 2000.
- [13] Haider, Steven, and Gary Solon, “Life–Cycle Variation in the Association between Current and Lifetime Earnings.” Mimeo, The University of Michigan, 2004.
- [14] Laitner, John. “The Dynamic Analysis of Continuous Time Life Cycle Saving Growth Models,” *Journal of Economic Dynamics and Control* 11, no. 3 (September 1987): 331–357.
- [15] Laitner, John. “Tax Changes and Phase Diagrams for an Overlapping Generations Model,” *Journal of Political Economy* 98, no. 1 (February 1990): 193–220.
- [16] Laitner, John. “Secular Changes in Wealth Inequality and Inheritance,” *The Economic Journal* 111, no. 474 (October 2001a): 349–371.
- [17] Laitner, John. “Wealth Accumulation in the U.S.: Do Inheritances and Bequests Play a Significant Role?” University of Michigan mimeo, November 2001b.
- [18] Laitner, John. “Wealth Inequality and Altruistic Bequests,” *American Economic Review* 92, no. 2 (May 2002a): 270–273.
- [19] Laitner, John. “Transition Paths and Social Security Reform,” Michigan Retirement Research Center working paper 2002–25, June 2002b.
- [20] Laitner, John. “Labor Supply Responses to Social Security,” University of Michigan mimeo, April 2003.
- [21] Laitner, John, and Juster, F. Thomas. “New Evidence on Altruism: A Study of TIAA–CREF Retirees,” *American Economic Review* 86, no. 4 (September 1996): 893–908.
- [22] Laitner, John, and Stolyarov, Dmitriy. “Technological Change and the Stock Market,” *American Economic Review*, to appear. University of Michigan mimeo, October 2002.
- [23] Laitner, John, and Ohlsson, Henry. “Bequest Motives: A Comparison of Sweden and the United States,” *Journal of Public Economics* 79, no. 1 (January 2001): 205–236.
- [24] Modigliani, Franco. “Life Cycle, Individual Thrift, and the Wealth of Nations,” *American Economic Review* 76, no. 3 (June 1986): 297–313.
- [25] Rupert, Peter; Rogerson, Richard; and, Wright, Randall, “Homework in Labor Economics: Household Production and Intertemporal Substitution.” *Journal of Monetary Economics* 46 (December 2000): 557–579.
- [26] Social Security Administration, *Annual Statistical Supplement, 2002, to the Social Security Bulletin*. Washington, D.C.: Social Security Administration, 2002.