Social patience, social credibility and long run inequality.

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Abstract

This paper considers optimal social insurance in a dynamic moral hazard economy. The existing literature has focussed on environments in which a planner and a population agents share the same discount factor. A key finding is that agents are almost surely immiserated: their welfare is driven to its lowest bound. We show that this immiseration result does not hold when the planner's discount factor exceeds the agents'. We give conditions under which optimal allocations in such settings imply a strongly ergodic invariant distribution over endogenously evolving agent Pareto-Negishi weights, agent utilities and consumptions. We consider the implications of this result for inequality and social mobility. Finally, we show that a high social discount factor arises endogenously in environments in which the planner is subject to credibility constraints that require it to maintain a weighted aggregate of agent utilites above a lower bound.

1 Introduction

Many recent papers have analysed optimal dynamic contracts in environments with private information frictions. A key area of application is to the design of optimal social insurance schemes. In these a planner or government[1] insures a population of agents against idiosyncratic, publicly unobserved shocks. In each period, agents report their shocks to the planner and receive a current allocation and a promised future allocation. To induce truthful revelation, it is usually optimal to penalise an agent that reports that he is in need of additional resources today, with a less valuable allocation in the future. Specific applications have considered the optimal design of unemployment and disability insurance, labour and capital taxation. However, a result common to many of these papers is that the optimal social insurance arrangement leads to the eventual immiseration of the agent. Under these arrangements, with probability one, the agent's utility converges to its lowest bound. Discovered initially in a partial equilibrium context by Green (1987) and Thomas and Worrall (1990), this result was extended by Atkeson and Lucas (1992) to general equilibrium settings. In this case, a measure zero set of agents end up with all resources in the economy, while the rest end up with nothing.[2]

The immiseration result has encountered two sorts of criticism. On the one hand some regard it as normatively unappealing, since if an agent is viewed as a dynasty, it implies a very inequitable distribution of consumption across generations. On the other hand, it implies that the optimal social insurance schemes studied in much of the literature exhibit a very severe form of time inconsistency. If a utilitarian planner is unable to commit, she will renege upon such a scheme ex post, undo any immiseration and restart the economy giving all agents the same continuation allocation. This time inconsistency raises questions about the practical implementation and social enforcement of such social insurance. It is difficult to imagine a society commiting to an allocation that

[1] We use the label "planner" throughout. However, we interpret the planner as social device for implementing an allocation, that may itself be subject to incentive problems.

[2] Phelan (1998) links this result to Inada type properties of the agent's utility function.

consigns almost everyone to misery in the long run.

In this paper, we show that the immiseration result does not generally hold when the planner's discount factor exceeds that of the agents. We give conditions under which the optimal allocation in such settings admits a stationary distribution over utilities and consumptions, and conditions under which it admits a strongly ergodic distribution over these variables. In neither case is the minimal bound on an agent's continuation utility an absorbing state. Intuitively, a planner that is relatively more patient than the agents concentrates the future rewards and punishments necessary to induce truthful reporting into the periods immediately after a report. Since agents attach relatively more weight to these periods, such a concentration of rewards and punishments provides the strongest incentives at least cost to the planner. Consequently, (reported) shocks have transitory, rather than permanent effects on the agent's utility. A relatively high planner discount factor becomes a force for mean reversion of an agent's Pareto weight.

The traditional motivation for studying environments in which the planner's discount factor exceeds that of the agents is a normative one. It relies on the interpretation of an agent as a dynasty and the assumption that current generations might discount the well being of future generations at a rate that is too high from a social perspective. In this paper, we also develop a second motivation that stems from our earlier work on credible social insurance (Sleet and Yeltekin (2004)). There we showed that the optimal equilibrium allocation from a game in which the planner and the agents share the same discount factor and the planner cannot commit is essentially the same as the optimal allocation from a planning problem in which the planner can commit, but has an *endogenously higher discount factor* than the agents. Intuitively, the planner with a discount factor equal to that of the agents cannot commit to a policy of immiseration. Since she can only tolerate a moderate level of inequality ex post, she implements an allocation with greater long run equality. Such an allocation coincides with that chosen by a patient planner. Our typical intuition from repeated game-based models of reputation is that social impatience is a source of credibility problems. In the current model, credibility problems are a source of social patience, or, at least patient-like behaviour. Although, we cast these arguments in terms of a planner, we think of our model more generally as capturing the inability of a society, whatever its precise political arrangements, to commit to implementing an immiserating allocation.

The analysis in Sleet and Yeltekin (2004) was in two steps. First, we showed that for an allocation to be credible, it must deliver a sequence of continuation payoffs to the planner above that from the autarkic one in which no insurance is offered. We call these additional constraints "credibility constraints". Since the planner's payoff is a weighted aggregate of agent payoffs, it implies that such a weighted aggregate must exceed the autarky value. The second step entailed constructing an endogenous discounting scheme from the Lagrange multipliers on the credibility constraints. In this paper, we expand upon this idea and consider problems in which the planner and the agents share the same discount factor and the planner must keep some weighted aggregate of agent utilities above a lower bound. We think of this as a generalised credibility constraint capturing the extent to which agents can resist reductions in their continuation utilities even if these reductions are implied by a plan to which they, or their ancestors, previously agreed. Once again, this constrained problem with common discounting is equivalent to one without credibility constraints, but with a social discount factor in excess of the private one.

We conduct most of our analysis in a relatively simple environment close to that considered by Atkeson and Lucas (1992). In this environment agents receive i.i.d taste shocks that affect their desire to consume. These shocks are privately observed by the agent. To induce the agent to reveal that he has a low taste shock and accept low current consumption, the planner must offer the agent more resources and utility in the future. Agents discount the future with discount factor β, while the planner discounts at rate $\lambda \in (\beta, 1)$. We also briefly consider a second environment corresponding to Atkeson and Lucas's (1995) model of unemployment insurance. Here shocks are to job opportunities. Again, we assume a social discount factor greater than the private one.

In these settings we make several technical contributions that are of independent interest. We adopt the recursive saddle point method of Marcet and Marimon (MM) (1999) rather than the more conventional utility promise method of Green, Thomas and Worrall and others.[3] We show how the MM approach can be applied to our private information environment.[4] The approach leads to a recursive reformulation of the Lagrangian from the planner's choice problem. It introduces a "cumulative multiplier", ζ_t, that keeps track of an agent's history by aggregating the Lagrange multipliers on the agent's previous incentive-compatibility conditions. This multiplier can be interpreted as an endogenously evolving Pareto-Negishi weight. We show that in each of our economies this weight evolves according to:

$$\zeta_{t+1} = \frac{1}{1+\psi}\zeta_t + \left(\frac{\psi}{1+\psi}\right) + \varepsilon_t, \qquad \text{with } E_{t-1}\varepsilon_t = 0 \tag{1}$$

Here, $\psi = \frac{\lambda-\beta}{\beta}$ and ε_t is a shock. This shock term is derived endogenously from the Lagrange multipliers on the agent's incentive-compatibility conditions. When $\lambda = \beta$ and $\psi = 0$, the stochastic process in (1) is a martingale. Under appropriate technical conditions, the martingale convergence theorem holds, the process almost surely converges to 0 and the agent is almost surely immiserated. However, when $\lambda > \beta$ and $\psi > 0$, this process tends to mean revert towards 1. It is straightforward to verify that almost sure immiseration cannot happen in this case. We further explore the implications of the law of motion (1) for the cross sectional distribution and long run evolution of the Pareto-Negishi weight process in constant price economies. First, we show that the mean reversion aspect implies social mobility: a given agent is sometimes above and sometimes below the

[3] The MM method provides a convenient and rather natural way to illustrate several features of the planner's problem. In particular, it implies that the planner's policy functions are "affine on average". This connects nicely with some results on Markov process theory that we make use of.

[4] It is well known, that the MM method is applicable to a narrower category of strictly concave problems, than the utility promise approach. The problems in this paper are in this category. The MM method has largely been applied to Ramsey tax problems or full information problems in which agents cannot commit. Especially in the former case this is often done without any formal verification of its applicability. (See Messner and Pavoni (2004) for a discussion).

unconditional long run average Pareto weight. Moreover, appealing to recent results on Markov processes we give conditions under which the Pareto-Negishi weight process admits a stationary distribution and conditions under which it is strongly ergodic. In the latter case, we can rule out, in the language of Phelan (2003), multiple "caste systems", regions of the Pareto weight space in which agents become trapped.

The plan for the remainder of the paper is as follows. In Section 2, we describe the basic environment and planner's problem. The planner is assumed to have an exogenously higher discount factor than the agents and to face an exogenous sequence of resource prices. In Section 3, we break this problem into a family of component planner problems. In each of these a component planner chooses an incentive-compatible allocation to maximise the utility (net of resource cost) of a specific Pareto weighted agent. We show how these component planner problems can be formulated recursively using the agent's Pareto weight. Specifically, we show that the component planner value functions satisfy a sequence of Bellman equations. We give sufficient conditions for the operator associated with these Bellman equations to be a contraction. Since the component planner's value function is typically unbounded, this requires identifying an appropriate weighted norm space on which to search for this value function. In Section 4, we observe that the policy functions from these planner problems are inconsistent with the immiseration of agents. We then turn to a detailed consideration of the planner's problem in constant price economies. We establish conditions under which the Markov process for Pareto-Negishi weights implied by the component planner problems admits a stationary distribution and is strongly ergodic. We interpret these results in terms of both the cross sectional distribution of Pareto-Negishi weights and the social mobility of an individual agent. In Section 5, we endogenise the higher planner discount factor by imposing a sequence of credibility constraints on the planner. In Section 6, we endogenise the resource prices by imposing a period-by-period resource constraint on the planner. Section 7 contains numerical results, while Section 8 concludes.

1.1 Related literature

The literature on optimal dynamic contracting, and specifically the immiseration result, includes Green (1987), Thomas and Worrall (1990) and Atkeson and Lucas (1992). It has elicited various responses. Atkeson and Lucas (1995) consider a model in which the planner must respect an exogenous lower bound on each agent's utility. They motivate this by appealing to considerations of equality. Phelan (1995) imposes a lack of commitment friction on agents. They can exit a social insurance arrangement at a cost and enter into another arrangement. His model can be interpreted as one in which the planner is constrained in its ability to enforce social arrangements.

Phelan (2003) and Farhi and Werning (2005) also consider environments in which the planner's discount factor exceeds the agents. Phelan analyses the case in which the planner's discount factor equals 1. He shows that the immiseration result is overturned in this case. Farhi and Werning is most related to us. Like us they consider environments in which the planner's discount factor is between that of the agent and one. They find conditions under which a stationary distribution over agent utilities exists. Their formulation of the problem is different from ours, it relies upon the utility promise keeping rather than the MM approach, as is their method of proof. Moreover, they take their problem to a different (and very interesting) set of applications; they consider how their optimal allocations could be implemented with estate taxation, while we consider the connection between high planner discount factors and credibility.

Finally, this paper is related to Sleet and Yeltekin (2004) in which we first established a link between optimal credible social insurance and optimal social insurance with a planner discount factor in excess of the private one.

2 Environment

In this section we describe the environment and introduce notation.

Our economy is inhabited by a continuum of infinitely-lived agents and a planner. Each agent has preferences over stochastic sequences for consumption $\{c_t\}_{t=0}^{\infty}$ of the form:

$$E\left[\sum_{t=0}^{\infty}\beta^t\theta_t u(c_t)\right]. \tag{2}$$

Here $\beta \in (0,1)$ denotes the agent's discount factor, while $\theta_t \in \Theta$ is a period t idiosyncratic taste shock that is privately observed by the agent. Shocks are assumed to satisfy:

Assumption 1 *1)* $\Theta = \{\widehat{\theta}_k\}_{k=1}^{K}$, $\widehat{\theta}_{k+1} > \widehat{\theta}_k > 0$*; 2)* $\{\theta_t\}$ *is independently and identically distributed over time and agents with distribution* π.[5]

Define $p_k := \pi(\widehat{\theta}_k)/\pi(\widehat{\theta}_{k-1})$ and let $\theta^t := \{\theta_0, \theta_1, ..., \theta_t\} \in \Theta^{t+1}$ denote a t-period history of shocks. Let π^t denote the corresponding probability distribution. The utility function u is assumed to satisfy:

Assumption 2 *1)* $u : \mathbb{R}_+ \to D \subseteq \mathbb{R} \cup \{-\infty\}$ *is a strictly increasing and strictly concave function. It is continuously differentiable on* $(0,\infty)$*; 2)* $\lim_{c\to\infty} u'(c) = 0$*; 3) Either* $\sup D < \infty$ *or* $\inf D > -\infty$.

It is convenient to work with allocations of utility rather than consumption. Formally, we define a *utility allocation* to be a sequence of functions $\alpha = \{u_t\}_{t=0}^{\infty}$, where $u_t : \Theta^{t+1} \to D$ gives the utility from consumption obtained by an agent at date t. The associated consumption allocation can then be recovered using the inverse of u, $C : D \to \mathbb{R}_+$. For $t > 0$, let $\alpha(\theta^{t-1}) := \{u_{t+s}(\theta^{t-1}, \cdot)\}_{s=0}^{\infty}$ denote the continuation of α after the history θ^t. For $t = 0$, set $\alpha(\theta^{-1}) := \alpha$. Let A denote the set of utility allocations.

The planner assigns a utility allocation to each agent in period 0. Since she cannot observe an agent's shock history, the planner cannot condition the agent's utility award directly upon this. Instead, she conditions it upon histories of shock reports given by the agent. The reporting behaviour of agents is described by a *reporting policy* $\delta = \{\delta_t\}_{t=0}^{\infty}$, where $\delta_t : \Theta^t \to \Theta$ gives the t-th period report of an agent conditional on the agent's past history of shocks. For each t and all θ^t, let $\delta^t(\theta^t) = (\delta^{t-1}(\theta^{t-1}), \delta_t(\theta^t))$ denote the corresponding history of reports implied

[5] We also interpret $\pi(\widehat{\theta}_k)$ as the fraction of agents receiving the shock $\widehat{\theta}_k$. In doing so we rely on the argument of Judd (1985).

by δ. Agents choose a reporting policy in period 0. If the planner assigns a (report-contingent) utility allocation $\alpha = \{u_t\}_{t=0}^{\infty}$ to an agent and the agent chooses reporting policy δ, then the realised allocation of the agent is $\alpha \circ \delta = \{u_t(\delta^t(\cdot))\}_{t=0}^{\infty}$. Invoking the revelation principle, we can restrict attention to incentive-compatible utility allocations that induce the agent to be truthful. Formally, α is said to be *incentive-compatible* if:

$$\forall \delta : U_0(\alpha) \geq U_0(\alpha \circ \delta), \tag{3}$$

where for $t \geq 0$, $U_t(\alpha(\theta^{t-1})) := E_{\theta^{t-1}} \sum_{s=0}^{\infty} \beta^s \theta_{t+s} u_{t+s}(\theta^{t+s})$ is the payoff to the agent from the continuation allocation $\alpha(\theta^{t-1}) = \{u_{t+s}(\theta^{t-1}, \cdot)\}_{s=0}^{\infty}$.

The planner has a discount factor $\lambda \in [\beta, 1)$. Each agent in the population is indexed by a pair of numbers $(\zeta_0, \gamma) \in Z \times Z$, where $Z = \mathbb{R}_+$ if the agent's utility function is unbounded below and $Z = \mathbb{R}$ otherwise. The (ζ_0, γ)-th agent's utility allocation is denoted $\alpha^{(\zeta_0,\gamma)} = \{u_t^{(\zeta_0,\gamma)}\}_{t=0}^{\infty}$. We assume that each $\alpha^{(\cdot,\cdot)}$ is Borel measurable. The planner weights the per period utility of the (ζ_0, γ)-th agent using the generalised Pareto-Negishi weighting scheme $\{\gamma_t(\zeta_0, \gamma)\}_{t=0}^{\infty}$ where

$$\gamma_t(\zeta_0, \gamma) = \gamma + \left(\frac{1}{1+\psi}\right)^t (\zeta_0 - \gamma) \tag{4}$$

and $\psi = \frac{\lambda - \beta}{\beta}$. Since $\gamma_0(\zeta_0, \gamma) = \zeta_0$ and $\lim_{t\to\infty} \gamma_t(\zeta_0, \gamma) = \gamma$, the parameters (ζ_0, γ) may be interpreted as the agent's initial and long run Pareto weight. Intuitively, we may imagine that events prior to period 0 have caused the agent to be temporarily rewarded (or punished) with an initial Pareto-Negishi weight above (or below) its long run level. If $\zeta_0 = \gamma$ (or $\lambda = \beta$), then $\gamma_t(\zeta_0, \gamma)$ is constant for all t. Thus, our formulation accommodates the more standard constant Pareto weighting scheme as a special case. Generalising the scheme in this way anticipates our later recursive formulation and allows us to be more flexible in our treatment of the initial period. We assume a cross sectional distribution over γ values given by Ψ_0 and a distribution over ζ_0 values given by Φ_0.

The planner faces a sequence of intertemporal resource prices $\{\lambda^t q_t\}_{t=0}^{\infty}$, where each $q_t \in [\underline{q}, \overline{q}]$, $0 < \underline{q} < \overline{q} < \infty$, so that her net-of-cost objective is given by:

$$\begin{aligned} W(\{\alpha^{(\zeta_0,\gamma)}\}; \Psi_0, \Phi_0, \{q_t\}_{t=0}^{\infty}) \quad = \quad & \int_Z \int_Z \sum_{t=0}^{\infty} \lambda^t \gamma_t(\zeta_0, \gamma) \sum_{\theta^t \in \Theta^{t+1}} \theta_t u_t^{(\zeta_0,\gamma)}(\theta^t) \pi^t(\theta^t) \Phi_0(d\zeta_0) \Psi_0(d\gamma) \\ & - \int_Z \int_Z \sum_{t=0}^{\infty} \lambda^t q_t \sum_{\theta^t \in \Theta^{t+1}} C(u_t^{(\zeta_0,\gamma)}(\theta^t)) \pi^t(\theta^t) \Phi_0(d\zeta_0) \Psi_0(d\gamma). \end{aligned} \tag{5}$$

For the moment, we take the price sequence as a parameter. However, this formulation of preferences accommodates a variety of resource constraints upon the planner. For example, the sequence $\{\lambda^t q_t\}$ may represent a sequence of optimal shadow prices from a problem in which the planner faces a per period aggregate resource constraint. We elaborate on these possibilities in Section 7. The *planner's problem* is then given by:

$$\begin{aligned} & \sup_{\{\alpha^{(\zeta_0,\gamma)} \in A\}} W(\{\alpha^{(\zeta_0,\gamma)}\}; \Psi_0, \Phi_0, \{q_t\}_{t=0}^{\infty}) \\ & \text{s.t. } \forall \delta : U_0(\alpha) \geq U_0(\alpha \circ \delta). \end{aligned} \tag{6}$$

3 The component planner's problem

We now turn to the analysis of the planner's problem (6). We proceed in the following steps. First, we define a relaxed planner's problem that incorporates a less restrictive set of "temporary" incentive constraints. Then we disaggregate this problem into a family of component planner problems in which a single planner deals with a single agent. Finally, we obtain a recursive formulation of these component planner's problems using the techniques of Marcet and Marimon. The first two steps are fairly standard in the dynamic contracting literature. The third is less so in models with private information. To date application of the Marcet and Marimon approach has mainly been confined to Ramsey tax problems or to problems without agent commitment. We extend the approach to accommodate our private information environment.

Given an allocation α, we define the *temporary incentive constraints* by

$$\forall t, \theta^{t-1}, k, j : \Delta U_t(\alpha(\theta^{t-1}), \widehat{\theta}_k, \widehat{\theta}_j) \geq 0, \tag{7}$$

where $\Delta U_t(\alpha(\theta^{t-1}), \widehat{\theta}_k, \widehat{\theta}_j) := \widehat{\theta}_k u_t(\theta^{t-1}, \widehat{\theta}_k) + \beta U_{t+1}(\alpha(\theta^{t-1}, \widehat{\theta}_k)) - \widehat{\theta}_k u_t(\theta^{t-1}, \widehat{\theta}_j) - \beta U_{t+1}(\alpha(\theta^{t-1}, \widehat{\theta}_j))$ gives the difference in payoffs obtained by an agent with current shock $\widehat{\theta}_k$ from the allocations $\{u_t(\theta^{t-1}, \widehat{\theta}_k), \alpha(\theta^{t-1}, \widehat{\theta}_k)\}$ and $\{u_t(\theta^{t-1}, \widehat{\theta}_j), \alpha(\theta^{t-1}, \widehat{\theta}_j)\}$.[6] The constraints (7) require that after each history of shocks, the agent is better off truthfully reporting her state, rather than lying and being truthful thereafter. Our earlier incentive constraint (3) clearly implies (7). Conversely, by a well known result (e.g. Atkeson and Lucas (1992)), an allocation satisfying (7), and the limiting conditions

$$\forall \theta^{\infty}, \lim_{t \to \infty} \beta^t \sum_{s=0}^{\infty} \beta^s \theta_{t+s} u_{t+s}(\theta^{t+s}) = 0 \tag{8}$$

satisfies (3) as well. In the sequel we will consider planner's problems subject only to temporary incentive constraints (7). If u is bounded it is immediate that all allocations satisfy (8) and so allocations satisfying (7) are incentive compatible. Similarly, if a solution to the planner's problem with temporary incentive constraints exists and if it ensures that the agent's continuation payoffs are uniformly bounded then it satisfies (8) and is incentive compatible.[7] We further relax the planner's problem by imposing only the *local upwards* temporary incentive constraints:

$$\forall t, \theta^{t-1}, k \in \{0, \ldots, K-1\} : \Delta U_t(\alpha(\theta^{t-1}), \widehat{\theta}_k, \widehat{\theta}_{k+1}) \geq 0. \tag{9}$$

In the appendix we verify that a solution to the relaxed planner's problem with only local upwards constraints

[6] We use the convention $\infty - \infty = -\infty + \infty = 0$ to ensure that $\Delta U_t(\alpha(\theta^{t-1}), \widehat{\theta}_k, \widehat{\theta}_j)$ is always well defined.

[7] The constraints (8) are demanding and it is difficult to check directly that the set of allocations satisfying these and the temporary incentive compatibility constraints is compact. The strategy of relaxing the problem is standard (Atkeson and Lucas (1992)).

satisfies all temporary incentive compatibility constraints. Formally, our *relaxed problem* is:

$$\sup_{\{\alpha^{(\zeta_0,\gamma)}\in A\}} W(\{\alpha^{(\zeta_0,\gamma)}\};\Psi_0,\Phi_0,\{q_t\}_{t=0}^{\infty}) \tag{10}$$

$$\text{s.t. } \forall\gamma,\zeta_0,t,\theta^{t-1},k<K \quad \lambda^t\Delta U_t(\alpha^{(\zeta_0,\gamma)}(\theta^{t-1}),\widehat{\theta}_k,\widehat{\theta}_{k+1})\geq 0.$$

To solve (10), we disaggregate it into a collection of *component planner problems*. In each of these a component planner maximises the net-of-cost utility for a specific (ζ_0,γ)-weighted agent. To economise on notation, we restrict attention to the population of agents sharing a common γ and drop the explicit indexing of objects by γ. The r-th period *continuation component planner problem* is then:

$$V_r^*(\zeta_r) = \sup_{\alpha\in A}\sum_{t=0}^{\infty}\lambda^t\gamma_t(\zeta_r)\sum_{\theta^t\in\Theta^{t+1}}\theta_t u_t(\theta^t)\pi^t(\theta^t)-\sum_{t=0}^{\infty}\lambda^t q_{r+t}\sum_{\theta^t\in\Theta^{t+1}}C(u_t(\theta^t))\pi^t(\theta^t) \tag{11}$$

$$\text{s.t. } \forall t,\theta^{t-1},k<K \quad \lambda^t\Delta U_t(\alpha(\theta^{t-1}),\widehat{\theta}_k,\widehat{\theta}_{k+1})\geq 0.$$

In this continuation problem, the component planner enters period r with current Pareto-Negishi weight ζ_r. The planner faces the price sequence $\{q_{r+t}\}_{t=0}^{\infty}$ and chooses a utility allocation $\{u_t\}_{t=0}^{\infty}$ to maximise her payoff. We reformulate this problem recursively using the method of Marcet and Marimon (1999). Before giving the details, we present a heuristic overview of our formulation.

3.1 Applying the Marcet-Marimon approach: A heuristic overview

In this subsection, we write down and manipulate a Lagrangian for the component planner problem (11). We use the Lagrangian to convert this problem into one without incentive constraints, but with an endogenously evolving Pareto-Negishi weight. After low shock reports this weight is increased, after high shock reports it is reduced, capturing the need to offer future rewards for the first type of report and future punishments for the second. We proceed informally and relegate detailed proofs to later in the section.

To keep matters as simple as possible suppose that the number of shocks, K, equals 2. In this case, there is only one upwards incentive constraint after each history θ^{t-1}. We denote its multiplier by $\eta_t(\theta^{t-1}, \widehat{\theta}_1)$. The Lagrangian for problem (11) with $r = 0$, can then be written as:

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{t=0}^{\infty} \lambda^t \gamma_t(\zeta_0) \sum_{\theta^t \in \Theta^{t+1}} \theta_t u_t(\theta^t) \pi^t(\theta^t) - \sum_{t=0}^{\infty} \lambda^t q_t \sum_{\theta^t \in \Theta^{t+1}} C(u_t(\theta^t)) \pi^t(\theta^t) \\ & + \sum_{t=0}^{\infty} \lambda^t \sum_{\theta^{t-1} \in \Theta^t} \eta_t(\theta^{t-1}, \widehat{\theta}_1) \Delta U_t(\alpha(\theta^{t-1}), \widehat{\theta}_1, \widehat{\theta}_2) \pi^{t-1}(\theta^{t-1}) \pi(\widehat{\theta}_1).\end{aligned} \tag{12}$$

Suppose that a solution to the component planner's problem and a corresponding optimal multiplier sequence $\{\eta_t^*(\theta^{t-1}, \widehat{\theta}_1)\}_{t=0}^{\infty}$ attain the saddle point of this Lagrangian. For notational convenience, define for each θ^{t-1}, $\eta_t^*(\theta^{t-1}, \widehat{\theta}_2) = \eta_t^*(\theta^{t-1}, \widehat{\theta}_0) = 0$. Also, define $\varsigma : \Theta \to \Theta$ pointwise by $\varsigma(\widehat{\theta}_1) = \widehat{\theta}_2$ and $\varsigma(\widehat{\theta}_2) = \widehat{\theta}_1$. Rearranging (12) evaluated at the optimal multiplier sequence, and using this new notation, we obtain:

$$\sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \lambda^t \pi(\theta^t) \left\{ \left[\zeta_t^*(\theta^{t-1}) + \left(\eta_t^*(\theta^t) - \eta_t^*(\theta^{t-1}, \varsigma(\theta_t)) p_1 \frac{\varsigma(\theta_t)}{\theta_t} \right) \right] \theta_t u_t(\theta^t) - q_t C(u_t(\theta^t)) \right\}, \tag{13}$$

where

$$\zeta_t^*(\theta^{t-1}) = \gamma_t(\zeta_0, \gamma) + \sum_{s=0}^{t} \left(\frac{\beta}{\lambda} \right)^{t-s} \{\eta_s^*(\theta^s) - \eta_s^*(\theta^{s-1}, \varsigma(\theta_s)) p_1\}. \tag{14}$$

It follows that the solution to the component planner's problem maximises (13). Such a maximisation corresponds to a planner's problem without incentive constraints, but with an endogenously evolving Pareto-Negishi weight $\{\zeta_t^*\}$. As (14) indicates this weight augments the agent's original Pareto weight with a term that depends on lagged incentive multipliers. Inspection of (14) reveals that $\{\zeta_t^*\}$ evolves in a recursive fashion according to:

$$\zeta_{t+1}^*(\theta^t) = \frac{\psi}{1+\psi} \gamma + \frac{1}{1+\psi} \zeta_t^*(\theta^{t-1}) + \frac{1}{1+\psi} \varepsilon_t^*(\theta^t), \tag{15}$$

where $\psi = \frac{\lambda - \beta}{\beta} > 0$ and $\varepsilon_t^*(\theta^t) = \eta_t^*(\theta^t) - \eta_t^*(\theta^{t-1}, \varsigma(\theta_t)) p_1$. Now, $\varepsilon_t^*(\theta^{t-1}, \widehat{\theta}_1) = \eta_t^*(\theta^{t-1}, \widehat{\theta}_1)$, while $\varepsilon_t^*(\theta^{t-1}, \widehat{\theta}_2) = -\eta_t^*(\theta^{t-1}, \widehat{\theta}_1)$, so that $\sum_{k=1}^{2} \varepsilon_t^*(\theta^{t-1}, \widehat{\theta}_k) \pi(\widehat{\theta}_k) = 0$. Consequently, we may think of each ε_t^* as an "incentive shock" that perturbs the endogenous Pareto-Negishi weight. If the agent receives the low shock at t, then $\varepsilon_t^*(\theta^{t-1}, \widehat{\theta}_1) = \eta_t^*(\theta^{t-1}, \widehat{\theta}_1) > 0$ and the Pareto-Negishi weight $\zeta_{t+1}^*(\theta^t)$ is relatively increased. Intuitively, to induce an agent to truthfully report the low shock value $\widehat{\theta}_1$ in period t (and receive low consumption in that period), the planner must reward the agent with higher utility in the future. She does this by raising the agent's Pareto-Negishi weight at $t+1$. Conversely, if the agent receives the high shock, then $\varepsilon_t^*(\theta^{t-1}, \widehat{\theta}_2) = -\eta_t^*(\theta^{t-1}, \widehat{\theta}_1)p_1 < 0$ and the agent's Pareto-Negishi weight at $t+1$ falls.

When $\lambda = \beta$, then $\psi = 0$, (15) reduces to:

$$\zeta_{t+1}^*(\theta^t) = \zeta_t^*(\theta^{t-1}) + \varepsilon_t^*(\theta^t) \tag{16}$$

and $\{\zeta_t^*\}$ is a martingale. In Sleet and Yeltekin (2004), we use this observation and the martingale convergence theorem to derive an immiseration result for economies in which the planner discounts at the same rate as the agents. More generally, when $\lambda > \beta$, $\psi > 0$ and the coefficient in front of ζ_t^* in (15) is less than one in absolute value. Thus, the process for Pareto-Negishi weights has a tendency to mean revert. Intuitively, when $\lambda > \beta$, the agent attaches greater relative value to utility received in the shorter run than the planner. Consequently, the planner concentrates rewards and punishments for a report in the periods immediately after the report.

3.2 Analysis of the Component planner's problem

Our goal in this section is to obtain a recursive formulation of the component planner's problem that uses endogenously evolving Pareto-Negishi weights to keep track of histories. We will now assume $\lambda > \beta$.[8] As a preliminary, we first confirm that a solution exists to the component planner's problem (11) and then establish several useful facts about the value function V_r^*. The proof of existence is complicated by the fact that we only impose boundedness above or below (but not both) on the agent's utility function.

Proposition 1 *There exists a solution $\{u_{r+t}^*(\zeta_r)\}_{t=0}^{\infty}$ to the r-th period component planner's problem (11).*

PROOF: See Appendix A. ■

[8] For analysis of the $\lambda = \beta$ case, see Sleet and Yeltekin (2004).

For $\zeta \in Z$, define the allocation $\underline{\alpha}(\zeta)$ by setting, $\forall t$, θ^t, $u_t(\theta^t) = \arg\sup_{u \in D} \gamma_t(\zeta)E[\theta]u - \overline{q}C(u)$. Denote its payoff to the component planner at the constant price sequence $\overline{q}$ by $\underline{V}(\zeta) > -\infty$.[9] $\underline{\alpha}(\zeta)$ offers the agent no insurance. It is clearly incentive compatible and feasible for problem (11) since it is independent of any reports given by the agent. This coupled with the fact that for all t, $q_t \leq \overline{q}$ implies, for all r and ζ, $V_r(\zeta) \geq \underline{V}(\zeta)$. Similarly, define $\overline{\alpha}(\zeta)$ by setting, $\forall t$, θ^t, $u_t(\theta^t) = \arg\sup_{u \in D} \gamma_t(\zeta)\theta_t u - \underline{q}C(u)$. Denote its payoff evaluated at the constant price sequence $\underline{q}$ by $\overline{V}(\zeta)$. Since $\overline{\alpha}(\zeta)$ offers the agent full insurance and for all t, $\underline{q} \leq q_t$, then for all r and ζ, $\overline{V}(\zeta) \geq V_r^*(\zeta)$. We collect this and other facts about V_r^* below.

Lemma 1 *V_r^* is convex, continuous and differentiable. It is bounded above by $\overline{V}$ and below by $\underline{V}$. If* $\inf D \geq 0$*, V_r^* is increasing; if* $\sup D \leq 0$*, V_r^* is decreasing.*

PROOF: See Appendix B. ■

In the proof of Proposition 1, we establish that any candidate optimal plan satisfies $\sum_{t=0}^{\infty} \lambda^t \sum_{\theta^t} \theta_t |u_t(\theta^t)| \pi^t(\theta^t) < \infty$ (which, of course, implies that $\sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \theta_t |u_t(\theta^t)| \pi^t(\theta^t) < \infty$). Moreover, given these inequalities and the bounds in Lemma 1, we have that $\sum_{t=0}^{\infty} \lambda^t |C(u_t(\theta^t))| \pi^t(\theta^t) < \infty$. Let $\Omega = \{\alpha : \sum_{t=0}^{\infty} \lambda^t \sum_{\theta^t} \theta_t |u_t(\theta^t)| \pi^t(\theta^t) < \infty \text{ and } \sum_{t=0}^{\infty} \lambda^t |C(u_t(\theta^t))| \pi^t(\theta^t) < \infty\}$. Associated with each problem (11) is the Lagrangian $\mathcal{L}_r(\cdot; \zeta_r) : \mathbb{R}_+^{K-1} \times \Omega \to \mathbb{R}$:

$$\begin{aligned} \mathcal{L}_r(\eta, \alpha; \zeta_r) \quad = \quad & \sum_{t=0}^{\infty} \lambda^t \gamma_t(\zeta_r) \sum_{\theta^t \in \Theta^{t+1}} \theta_t u_t(\theta^t) \pi^t(\theta^t) - \sum_{t=0}^{\infty} \lambda^t q_{r+t} \sum_{\theta^t \in \Theta^{t+1}} C(u_t(\theta^t)) \pi^t(\theta^t) \\ & + \sum_{k=1}^{K-1} \eta(\widehat{\theta}_k) \Delta U_0(\alpha, \widehat{\theta}_k, \widehat{\theta}_{k+1}) \pi(\widehat{\theta}_k). \end{aligned} \tag{17}$$

This Lagrangian incorporates *only* the initial period incentive constraints, assigning them the Lagrange multiplier $\eta \in \mathbb{R}_+^{K-1}$. It is well defined given the restriction of allocations to Ω. Let $\Omega_j \subset \Omega$ denote the set of utility

[9] If $\zeta = 0$ and $\inf D = -\infty$, then $u_0^* = -\infty$. In this case, we assign ζu_0^* the value of 0.

allocations in Ω that satisfy the local upward incentive compatibility constraints from period j onwards. The following argument relates the Lagrangian $\mathcal{L}_r$ to V_r^* and establishes the existence of an optimising multiplier.

Proposition 2 *V_r^* and $\mathcal{L}_r$ satisfy for each $\zeta \in Z$,*

$$V_r^*(\zeta) = \inf_{\eta \in \mathbb{R}_+^{K-1}} \sup_{\alpha \in \Omega_1} \mathcal{L}_r(\eta, \alpha; \zeta); \tag{18}$$

Additionally, there exists an $\eta_r^*(\zeta) \in \mathbb{R}_+^{K-1}$ such that $(\eta_r^*(\zeta), \{u_{r+t}^*(\zeta)\}_{t=0}^{\infty})$ attains the saddle point in (18).

PROOF: The result follows from Luenberger, Theorem 1 (p. 217) and Corollary 1 (p. 219) and Proposition 1 of this paper. ■

Gathering together common u_t terms in (17) and using the definition of γ_t, we obtain:[10]

$$V_r^*(\zeta_r) = \sup_{\alpha \in \Omega_1} \inf_{\substack{\eta \in \mathbb{R}_+^{K+1}, \\ \eta(\widehat{\theta}_0) = \eta(\widehat{\theta}_K) = 0}} \sum_{k=1}^{K} \Bigg[\left(\zeta_r + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) \frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} \right) \widehat{\theta}_k u_0(\widehat{\theta}_k) - q_r C(u_0(\widehat{\theta}_k)) \tag{19}$$
$$+ \lambda \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \lambda^t \left[\gamma_t \left(\zeta_{r+1}(\zeta_r, \widehat{\theta}_k) \right) \theta_t u_{t+1}(\widehat{\theta}_k, \theta^t) - q_{r+t+1} C(u_{t+1}(\widehat{\theta}_k, \theta^t)) \right] \pi^t(\theta^t) \Bigg] \pi(\widehat{\theta}_k),$$

where

$$\zeta_{r+1}(\zeta_r, \widehat{\theta}_k) := \frac{1}{1+\psi} \zeta_r + \left(\frac{\psi}{1+\psi} \right) \gamma + \frac{1}{1+\psi} [\eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) p_{k-1}].$$

We now turn to the main result of this section: we establish that the functions $\{V_r^*\}_{r=0}^{\infty}$ satisfy a sequence of Bellman equations. In these equations, the Lagrange multipliers from the current incentive constraints η appear as choice variables, while the endogenous Pareto-Negishi weight ζ acts as a state variable. The utility variables are completely eliminated.

We sketch the derivation of the Bellman equations next. First, by the Lagrange duality theorem (Luenberger (1969), Theorem 1, p. 224), we can break the saddle point problem in (19) into two sequential problems. In the

[10] Such rearrangements are possible given the restriction to allocations in Ω.

second of these, we maximise over allocations given a multiplier choice, in the first we minimise over multipliers. Since the initial period incentive constraint has been brought into the Lagrangian, the maximisation step involves choosing a family of allocations, $\forall k$, $(u_0(\widehat{\theta}_k), \alpha(\widehat{\theta}_k)) \in D \times \Omega_0$ to solve:

$$\sum_{k=1}^{K} \Bigg[\sup_{u_0(\widehat{\theta}_k)\in D} \left[\zeta_r + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) \frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} \right] \widehat{\theta}_k u_0(\widehat{\theta}_k) - q_r C(u_0(\widehat{\theta}_k)) \tag{20}$$
$$+\lambda \sup_{\alpha(\widehat{\theta}_k)\in\Omega_0} \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \lambda^t \left[\gamma_t \left(\zeta_{r+1}(\zeta_r, \widehat{\theta}_k) \right) \theta_t u_{t+1}(\widehat{\theta}_k, \theta^t) - q_{r+t+1} C(u_{t+1}(\widehat{\theta}_k, \theta^t)) \right] \pi^t(\theta^t) \Bigg] \pi(\widehat{\theta}_k).$$

Define the indirect cost function W by $W(\rho; q_r) = \sup_{u\in D} \rho u - q_r C(u)$. Using (20), the definition of W and that of the value function V^*_{r+1}, the minimisation step may then be written as:

$$V_r^*(\zeta_r) = \inf_{\substack{\eta \in \mathbb{R}_+^{K+1}, \\ \eta(\widehat{\theta}_0)=\eta(\widehat{\theta}_K)=0}} \sum_{k=1}^{K} \left[W\left(\left[\zeta_r + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) \frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} \right] \widehat{\theta}_k; q_r \right) + \lambda V^*_{r+1}(\zeta_{r+1}(\zeta_r, \widehat{\theta}_k)) \right] \pi(\widehat{\theta}_k). \tag{21}$$

It follows that the functions $\{V_r^*\}$ satisfy a sequence of Bellman equations.

In the subsequent analysis, it is convenient to find a compact constraint correspondence for the multiplier variables η in the minimisation step. This leads us to slightly modify the Bellman equation in (19). In Lemma B2 in the appendix, we find a collection of non-binding constraints on the optimal multiplier choices from (18). We use these to construct a compact-valued, non-binding constraint correspondence which we incorporate into the minimisation step. Specifically, we define $\Lambda : Z \twoheadrightarrow \mathbb{R}_+^{K+1}$ pointwise as follows. If $\zeta < 0$, $\Lambda(\zeta) = \{0\}$, otherwise:

$$\Lambda(\zeta) = \left\{ \eta \in \mathbb{R}_+^{K+1} : \begin{array}{ll} 1 & \eta(\widehat{\theta}_0) = 0,\ \eta(\widehat{\theta}_K) = 0, \\ 2 & \theta_{k+1} \left\{ \zeta + \eta(\widehat{\theta}_{k+1}) - \eta(\widehat{\theta}_k) \frac{\widehat{\theta}_k}{\widehat{\theta}_{k+1}} p_k \right\} \geq \theta_k \left\{ \zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) \frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} \right\} \\ 3 \text{ (if } Z = \mathbb{R}_+) & \frac{1}{1+\psi} [\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) p_{k-1}] + \left(\frac{\psi}{1+\psi} \right) \gamma \geq 0 \end{array} \right\} \tag{22}$$

The second constraint in (22) implies that the weight on current utility is increasing in k, the third implies that the continuation Pareto-Negishi weight remains in Z if $Z = \mathbb{R}_+$. Collectively, these conditions imply that Λ is continuous, compact and convex-valued. The definition of our Bellman operator T_r incorporates Λ and is intuitive given (19):

$$T_r f(\zeta) \quad = \quad \inf_{\eta \in \Lambda(\zeta)} \sum_{\widehat{\theta}_k \in \Theta} \left\{ W\left(\left[\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) \frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} \right] \widehat{\theta}_k ; q_r \right) \right. \qquad (23)$$

$$\left. + \lambda f \left(\frac{1}{1+\psi} [\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) p_{k-1}] + \left(\frac{\psi}{1+\psi} \right) \gamma \right) \right\} \pi(\widehat{\theta}_k),$$

With the definition of T_r in place, we now formally establish that the value functions $\{V_r^*\}_{r=0}^{\infty}$ satisfy a sequence of Bellman equations.

Proposition 3 *For all r, $V_r^* = T_r V_{r+1}^*$.*

PROOF: See Appendix B. ■

We complete this section by showing that the policy functions that solve the problems $V_r^* = T_r V_{r+1}^*$ induce an allocation that solves the component planner's problem (11). Let $\{\eta_t\}_{t=0}^{\infty}$ be a sequence of policy functions with each $\eta_t : Z \times (\{\widehat{\theta}_0\} \times \Theta) \to \mathbb{R}_+$. We say the utility allocation $\{u_t\}_{t=0}^{\infty}$ is *induced by* $\{\eta_t\}_{t=0}^{\infty}$ *from* ζ if $\zeta_0(\theta^{-1}) = \zeta$ and for each $(\theta^{t-1}, \widehat{\theta}_k)$,

$$u_t(\theta^{t-1}, \widehat{\theta}_k) = \arg\sup_{u \in D} W\left(\left[\zeta_t(\theta^{t-1}) + \eta_t(\zeta_t(\theta^{t-1}), \widehat{\theta}_k) - \eta_t(\zeta_t(\theta^{t-1}), \widehat{\theta}_{k-1}) \frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} \right] \widehat{\theta}_k ; q_{r+t} \right)$$

and

$$\zeta_{t+1}(\theta^{t-1}, \widehat{\theta}_k) = \frac{1}{1+\psi} [\zeta_t(\theta^{t-1}) + \eta_t(\zeta_t(\theta^{t-1}), \widehat{\theta}_k) - \eta_t(\zeta_t(\theta^{t-1}), \widehat{\theta}_{k-1}) p_{k-1}] + \left(\frac{\psi}{1+\psi} \right) \gamma.$$

Lemma 2 *Let $\{\eta_r^*\}_{r=0}^{\infty}$ denote a sequence of policy functions with $\eta_r^* : Z \times (\{\theta_0\} \cup \Theta) \to \mathbb{R}_+$ such that for each r and ζ, $\eta_r^*(\zeta, \widehat{\theta}_0) = \eta_r^*(\zeta, \widehat{\theta}_K) = 0$ while $\{\eta_r^*(\zeta, \widehat{\theta}_k)\}_{k=1}^{K-1}$ attains the infimum in*

$$V_r^*(\zeta) = \inf_{\eta \in \mathbb{R}_+^{K-1}} \sup_{\alpha \in \Omega_1} \mathcal{L}_r(\eta, \alpha; \zeta). \qquad (24)$$

Then for each r, 1) η_r^* is the unique policy function that attains the infima in $V_r^* = T_r V_{r+1}^*$; 2) η_r^* is continuous in ζ, 3) if $\{u_{r+t}^*(\zeta)\}_{t=0}^{\infty}$ is a solution to (11) at ζ it is unique and is induced by $\{\eta_{r+t}^*\}_{t=0}^{\infty}$ from ζ.

Proof: See Appendix B. ■

The optimal multiplier policy functions $\{\eta_r^*\}_{r=0}^{\infty}$ may be used to construct a corresponding sequence of *optimal Pareto-Negishi policy functions* $\zeta^* = \{\zeta_{r+1}^*\}_{r=0}^{\infty}$:

$$\zeta_{r+1}^*(\zeta, \widehat{\theta}_{k+1}) = \frac{\psi}{1+\psi}\gamma + \frac{1}{1+\psi}\zeta + \frac{1}{1+\psi}\{\eta_r^*(\zeta, \widehat{\theta}_{k+1}) - \eta_r^*(\zeta, \widehat{\theta}_k)p_k\}. \tag{25}$$

These are central to our analysis and quite intuitive. As noted in the heuristic discussion, they incorporate a shock term $\eta_r^*(\zeta, \widehat{\theta}_{k+1}) - \eta_r^*(\zeta, \widehat{\theta}_k)p_k$ with zero conditional expectation. Additionally, since $\lambda > \beta$, $\frac{1}{1+\psi} \in (0,1)$ and the $\{\zeta_{r+1}^*\}_{r=0}^{\infty}$ incorporate a force for mean reversion that draws the agent's Pareto-Negishi weight back towards γ.

3.3 The component planner's dynamic programming problem

So far we have shown that the optimal value functions $\{V_r^*\}_{r=0}^{\infty}$ solve the functional equations $V_r^* = T_r V_{r+1}^*$. However, we have not shown that they are the *unique* sequence of functions satisfying these equations, nor have we established a method for computing them. We address these important practical issues in this section. Specifically, we find sufficient conditions for T_r to be a contraction. We then use the contractive property of the $\{T_r\}_{r=0}^{\infty}$ operators to establish a computational procedure for obtaining $\{V_r^*\}_{r=0}^{\infty}$. Since the functions $\{V_r^*\}_{r=0}^{\infty}$ are unbounded (in the sup norm), our arguments rely on finding a subset $\mathcal{V}$ of a weighted norm space such that each $V_r^* \in \mathcal{V}$ and $T_r : \mathcal{V} \to \mathcal{V}$. Before stating these arguments, we introduce some preliminary material on weighted norm spaces.

Let $w : Z \to [1, \infty)$ be a Borel-measurable function. For $f : Z \to \mathbb{R}$, define its w-norm by $\|f\|_w = \sup_{\zeta \in Z} \frac{|f(\zeta)|}{w(\zeta)}$. We say that a function f is w-bounded if $\|f\|_w < \infty$. Let $\mathbb{B}_w(Z)$ be the normed linear space of measurable, w-bounded functions with domain Z. $\mathbb{B}_w(Z)$ is a Banach space (see Hernandez-Lerma and Laserre (1999)).

Define the set of candidate value functions:

$$\mathcal{V} = \left\{ f : Z \to \mathbb{R} : f \text{ is continuous and } \underline{V}(\zeta) \leq f(\zeta) \leq \overline{V}(\zeta) \right\}.$$

and suppose the existence of a weighting function satisfying the following assumption.

Assumption 3 *There exists a weighting function $w : Z \to [1, \infty)$ such that 1) $\sup_{\zeta \in Z} \left| \frac{\underline{V}(\zeta)}{w(\zeta)} \right| < \infty$ and $\sup_{\zeta \in Z} \left| \frac{\overline{V}(\zeta)}{w(\zeta)} \right| < \infty$, and 2) for some $\delta < 1$ and all $\zeta \in Z$ and $\eta \in \Lambda(\zeta)$,*

$$\sup_{\zeta \in Z} \lambda \frac{\sum_{k=1}^{K} w\left(\frac{\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1}) p_{k-1} + \psi}{1+\psi} \right)}{w(\zeta)} \in (0, \delta). \tag{26}$$

The first part of the assumption ensures that $\mathcal{V} \subset \mathbb{B}_w(Z)$. The second is a bound on the feasible growth of the Pareto-Negishi weights, we use it to show that T_r satisfies a discounting property and is, hence, a contraction on $\mathcal{V}$. In Appendix C, we find conditions on u that are sufficient for a weight function satisfying Assumption 3 to exist. These conditions encompass many cases including CRRA utility with $\sigma > 1$ and any bounded utility function.

Lemma 3 *1) $T_r : \mathcal{V} \to \mathcal{V}$. 2) Under Assumption 3, T_r is a contraction on $\mathcal{V}$*

Proof: Appendix C. ■

Let $T_r^s = T_r \circ T_{r+1} \circ \cdots \circ T_{r+s}$, $s = 0, 1, \ldots$. Lemma 4 uses the contractive property of each T_r to show that there is a unique sequence of functions in $\mathcal{V}$ satisfying the recursion $V_r^\infty = T_r V_{r+1}^\infty$ and that each element of this sequence can be obtained by iterating on the operators $\{T_r^s\}_{s=0}^\infty$.

Lemma 4 *Let $V \in \mathcal{V}$ and let $V_r^s = T_r^s V$. Then 1) for each r, the sequence $\{V_r^s\}_{s=0}^\infty$ converges uniformly (in the w-norm) to a limiting function $V_r^\infty \in \mathcal{V}$; 2) for each r, $V_r^\infty = T_r V_r^\infty$; 3) $\{V_r^\infty\}_{r=0}^\infty$ is the unique sequence of functions in $\mathcal{V}$ satisfying the recursion for each r $V_r^\infty = T_r V_{r+1}^\infty$.*

PROOF: Appendix C. ■

Finally, we confirm that the functions $\{V_r^\infty\}$ equal the true value functions $\{V_r^*\}$.

Lemma 5 *For all r, $V_r^\infty = V_r^*$.*

PROOF: Appendix C. ■

4 The absence of immiseration and the existence of steady state distributions

4.1 No immiseration

We say that an agent is *immiserated* during shock history θ^∞ if its corresponding Pareto-Negishi weight sequence $\{\zeta_t(\theta^t)\}_{t=0}^\infty$ is absorbed by the non-positive real line, i.e. if $\limsup_{t\to\infty} \zeta_t(\theta^t) \leq 0$. It is readily verified that the continuation utility of an immiserated agent converges to its lowest bound. When $\beta = \lambda$, it is known that for a range of assumptions on u, agents are *almost surely* immiserated. As we have previously argued, this is an unappealing result from both a normative and a practical perspective. In stark contrast, when $\lambda > \beta$, it is immediate from the form of the optimal policy functions that immiseration cannot occur along *any* shock history. Observe that for $\zeta \leq 0$ and each $\widehat{\theta}_k$, $\eta_t^*(\zeta, \widehat{\theta}_k) = 0$ and so

$$\zeta_{t+1}^*(\zeta, \widehat{\theta}_k) = \frac{\psi}{1+\psi}\gamma + \frac{1}{1+\psi}\zeta \geq \zeta + \frac{\psi}{1+\psi}\gamma. \tag{27}$$

Consequently, if an agent's Pareto-Negishi weight enters the non-positive interval, it leaves with probability one. We have proved the following proposition.

Proposition 4 ***(No immiseration)*** *For all histories θ^∞, $\limsup_{t\to\infty} \zeta_{t+1}(\theta^t) > 0$.*

4.2 Stationary distributions in constant price economies

When the sequence of resource prices are constant, $q_t = q$ for all t, we can obtain a sharper characterisation of the agent's Pareto-Negishi weight process $\{\zeta_{t+1}\}$. In this case, a specialisation of our earlier arguments can be used to show that the component planner's value and policy functions are time invariant. We denote them V_q^*, η_q^* and ζ_q^* respectively. We have previously observed that the form of the optimal policy function ζ_q^* incorporates a force for mean reversion to γ. More formally, we now show that the Markov process for Pareto-Negishi weights induced by ζ_q^* satisfies a type of mixing. If the agent's Pareto-Negishi weight is less than γ today, then it will almost surely be above γ at some point in the future and vice versa. This sort of mixing can be interpreted as social mobility around the average Pareto-Negishi weight γ.[11] Consequently, agents are socially mobile in the sense that their Pareto-Negishi weight is sometimes above and sometimes below this average value.

Proposition 5 *Let $\{\zeta_t\}$ denote the sequence of Pareto-Negishi weights induced by ζ_q^* from some ζ_0. If $\zeta_0 < \gamma$, then $\{\zeta_t\}$ reaches the set $\{\zeta_t > \gamma\}$ with probability one. If $\underline{\zeta} = \min\{0, \inf_{\mathbb{R}_+\times\Theta} \zeta_q^*(\zeta, \widehat{\theta}_k)\} > -\infty$, and $\zeta_0 > \gamma$, then $\{\zeta_t\}$ reaches the set $\{\zeta_t < \gamma\}$ with probability one.*

PROOF: Suppose $\zeta_0 < \gamma$. Let $\widetilde{\zeta} = \min\{\zeta_0, \min_{[0,\gamma]\times\Theta} \zeta_q^*(\zeta, \widehat{\theta}_k)\}$ and $\widehat{\zeta} = \max_{[0,\gamma]\times\Theta} \zeta_q^*(\zeta, \widehat{\theta}_k)$. Let $\{\zeta_t\}$ denote the process for Pareto-Negishi weights induced by ζ_q^* from ζ_0. Define the random variable $T = \inf\{t : \zeta_t > \gamma\}$ and the stopped process ζ^T by $\zeta_t^T = \zeta_{\min\{t,T\}}$. Now, if $t \geq T$, $\zeta_{t+1}^T = \zeta_t^T$, otherwise, using the optimal policy function ζ_q^*, $E_t[\zeta_{t+1}^T] = \frac{1}{1+\psi}\zeta_t + \left(\frac{\psi}{1+\psi}\right)\gamma \geq \zeta_t = \zeta_t^T$. Hence, $\{\zeta_t^T\}$ is a submartingale bounded on $[\widetilde{\zeta}, \widehat{\zeta}]$. Thus, $\{\zeta_t^T\}$ almost surely converges. By (27), it cannot converge to a point on $[\widetilde{\zeta}, 0]$. Since the optimal multiplier functions $\eta_q^*(\zeta, \widehat{\theta}_k)$ are strictly positive on $(0, \infty)$, $\{\zeta_t^T\}$ cannot converge to a point on $(0, \gamma)$. We deduce that T is almost surely finite and that the process $\{\zeta_t\}$ reaches the set $\{\zeta_t > \gamma\}$ with probability one.

[11] It is easy to check that ζ_q^* implies $\lim_{t\to\infty} E[\zeta_t] = \gamma$ for any initial ζ_0.

Now assume that $\zeta_0 > \gamma$. Note that for all $(\zeta, \widehat{\theta}_k) \in [\underline{\zeta}, \infty) \times \Theta$, $\zeta_q^*(\zeta, \theta_k) \in [\underline{\zeta}, \infty)$, so if $\underline{\zeta} > -\infty$, then the process $\{\xi_t\}$ with $\xi_t = \zeta_t - \underline{\zeta}$ takes its values in $[0, \infty)$. Define the stopping time $T = \inf\{t : \xi_t < \gamma - \underline{\zeta}\}$ and the stopped process $\xi^T = \{\xi_t^T\}_{t=0}^\infty$ by $\xi_t^T = \xi_{\min\{t,T\}}$. If $t \geq T$, $\xi_{t+1}^T = \xi_t^T$, otherwise, $E_t[\xi_{t+1}^T] = \frac{1}{1+\psi}(\xi_t - \underline{\zeta}) + \left(\frac{\psi}{1+\psi}\right)(\gamma - \underline{\zeta}) \leq \xi_t - \underline{\zeta} = \xi_t^T$. Hence, ξ^T is a non-negative supermartingale and it almost surely converges. By the same logic given above, it cannot converge to a point in $[\gamma - \underline{\zeta}, \infty)$. We deduce that T is almost surely finite and that the process $\{\zeta_t\}$ reaches the set $\{\zeta_t < \gamma\}$ with probability one. ■

Proposition 5 assumes that the optimal policy function is bounded below on $\mathbb{R}_+ \times \Theta$. Lemma B5 in the appendix establishes such boundedness for the case in which u is bounded above (but not necessarily bounded below). Lemma B6, gives a condition for each $\zeta_q^*(\cdot, \widehat{\theta}_k)$ to be monotone. In this case $\inf_{\mathbb{R}_+ \times \Theta} \zeta_q^*(\zeta, \widehat{\theta}_k) \geq \zeta_q^*(0, \widehat{\theta}_k) = \frac{\psi}{1+\psi}$.

Let P_q denote the Markov operator induced by ζ_q^*:

$$P_q \Phi(H) = \int_Z \sum_{\widehat{\theta}_k \in \Theta} 1_H(\zeta_q^*(\zeta, \widehat{\theta}_k))\pi(\widehat{\theta}_k)\Phi(d\zeta), \quad \Phi \in \mathcal{M}(Z),\ H \in \mathcal{B}(Z),$$

where $\mathcal{M}(Z)$ is the space of probability measures on Z, $\mathcal{B}(Z)$ is the Borel sigma algebra on Z and $1_H(\cdot)$ is the indicator function for the set H. Given an initial cross sectional distribution of Pareto-Negishi weights Φ_0, repeated application of P_q induces a sequence of cross sectional distributions $\{\Phi_t\}_{t=1}^\infty$ for subsequent periods. A basic question concerns the existence of an invariant probability measure for P_q. Such an invariant measure has an interpretation as a stationary cross sectional Pareto weight distribution for our environment. Since, an agent's current consumption and utility is a function of its Pareto-Negishi weight and shock, it implies a stationary cross sectional distribution for these variables as well. We now show that such an invariant measure exists when ζ_q^* is bounded below on $\mathbb{R}_+ \times \Theta$.

Our proof of the existence of an invariant measure relies on an argument from the theory of iterated function systems, see Lasota and Mackay (1994). In order to state the result, we give a preliminary definition.

Definition 1 *A* ***stochastic system*** *is a pair of sets X and Y, a function $R : X \times Y \to X$ and a probability distribution υ on Y satisfying the following. 1) X is a closed subset of $\mathbb{R}^d$. Y is a Borel subset of $\mathbb{R}^k$, 2) $R(\cdot, y)$ is continuous, each $y \in Y$; $R(x, \cdot)$ is measurable, each $x \in X$, 3) the system evolves according to: $x_{t+1} = R(x_t, y_t)$, with each y_t distributed iid according to υ and x_0 given.*

A stochastic system induces a Markov operator P according to $P\mu(H) = \int_X \int_Y 1_H(R(x, y))\ \upsilon(dy)\ \mu(dx)$, where $\mu \in \mathcal{M}(X)$, and $H \in \mathcal{B}(X)$. The following result provides a sufficient condition for a stochastic system to admit an invariant measure.

Proposition 6 *Let P be the Markov operator corresponding to the stochastic system (X, Y, R, υ). Assume that there exists non-negative constants α_0 and α_1, $\alpha_1 < 1$ such that:*

$$\int_Y |R(x, y)| \upsilon(dy) \leq \alpha_1 |x| + \alpha_0 \quad \text{for all } x \in X. \tag{28}$$

Then P has an invariant distribution.

Proof: Lasota and Mackey (1994), Theorem 12.5.2, p. 420. ■

Remark: The analogue of the function R in our analysis is the policy function $\zeta_q^*(\zeta, \widehat{\theta}_k) = \frac{\psi}{1+\psi}\gamma + \frac{1}{1+\psi}\zeta + \eta_q^*(\zeta, \widehat{\theta}_k) - \eta_q^*(\zeta, \widehat{\theta}_{k-1})p_{k-1}$ less a non-positive constant. Proposition 6 is useful because its key requirement is the moment condition (28). We know relatively little about ζ_q^* except that $\sum_{\widehat{\theta}_k} \zeta_q^*(\zeta, \widehat{\theta}_k)\pi(\widehat{\theta}_k) = \frac{\psi}{1+\psi}\gamma + \frac{1}{1+\psi}\zeta$. We translate this fact into one concerning absolute values, as in (28), by a simple renormalisation. Notice that Proposition 6 does not require compactness of the state space. ■

Our next proposition applies this result to our economy.

Proposition 7 *Assume $\underline{\zeta} = \min\{\inf_{\mathbb{R}_+ \times \Theta} \zeta_q^*(\zeta, \widehat{\theta}_k), 0\} > -\infty$. Then, P_q admits an invariant distribution Φ_q.*

PROOF: For all $(\zeta,\widehat{\theta}_k) \in [\underline{\zeta},\infty) \times \Theta$, $\zeta_q^*(\zeta,\theta_k) \in [\underline{\zeta},\infty)$. Since $\underline{\zeta} > -\infty$, $\xi_q^*(\xi,\theta) := \zeta_q^*(\xi+\underline{\zeta},\theta) - \underline{\zeta}$ maps $[0,\infty)$ into itself. Set $X = [0,\infty)$, $Y = \Theta$, $v = \pi$, and $R = \xi_q^*$. Note that for $\xi \geq 0$

$$\begin{aligned}
\sum_k \left|\xi_q^*(\xi,\widehat{\theta}_k)\right| \pi(\widehat{\theta}_k) &= \sum_k \xi_q^*(\xi,\widehat{\theta}_k)\pi(\widehat{\theta}_k) = \sum_k \{\zeta_q^*(\xi+\underline{\zeta},\widehat{\theta}_k) - \underline{\zeta}\}\pi(\widehat{\theta}_k) \\
&= \sum_k \left\{ \frac{\xi + \underline{\zeta} + \psi + \eta_q^*(\xi+\underline{\zeta},\widehat{\theta}_{k+1}) - \eta_q^*(\xi+\underline{\zeta},\widehat{\theta}_k)p_k}{1+\psi} - \underline{\zeta} \right\} \pi(\widehat{\theta}_k) \\
&= \frac{\xi}{1+\psi} + \frac{\psi}{1+\psi}\{1-\underline{\zeta}\}.
\end{aligned}$$

Where the first equality uses $\xi_q^*(\xi,\widehat{\theta}_k) \geq 0$, the second and third the definitions of ξ_q^* and ζ_q^* and the fourth $\sum_k \{\eta_q^*(\xi+\underline{\zeta},\widehat{\theta}_{k+1}) - \eta_q^*(\xi+\underline{\zeta},\widehat{\theta}_k)p_k\}\pi(\widehat{\theta}_k) = 0$. Setting $\alpha_1 = \frac{1}{1+\psi} \in (0,1)$ and $\alpha_0 = \frac{\psi}{1+\psi}\{1-\underline{\zeta}\} \geq 0$ verifies (28). $R(\cdot,\widehat{\theta}_k) = \xi_q^*(\cdot,\widehat{\theta}_k)$ is continuous since $\zeta_q^*(\cdot,\widehat{\theta}_k)$ is. It then follows from Proposition 6 that the process for ξ admits an invariant distribution. Consequently so too does the process for ζ. ■

It is easy to see that Φ_q is non-degenerate. In particular, it follows from our discussion of immiseration that $\Phi_q(0,\infty) > 0$. This is in contrast to the equal discounting case $\lambda = \beta$, when the immiseration result implies that Φ_q puts all mass on the non-positive Pareto-Negishi weights. We now show that when ζ_q^* is monotone in ζ, Φ_q is unique and any sequence of measures induced by P_q converges strongly and at a geometric rate to Φ_q. This ergodic result strengthens our earlier conclusions about mixing and social mobility. Following Phelan (2003), multiple ergodic sets can be interpreted as caste systems: sets of Pareto-Negishi weights from which an agent cannot escape. Our results imply a unique ergodic set and, hence, the absence of (multiple) caste systems.

We prove our ergodicity result in two steps. First, we establish that P_q has a unique invariant probability measure and that any sequence of probability measures induced by P_q from some arbitrary $\Phi_0 \in \mathcal{M}(Z)$ converges weakly to this invariant measure. We follow Lasota and Mackey in referring to Markov operators with these properties as *weakly asymptotic*. We then build on this to establish the geometric ergodicity result. Our first step uses the following theorem of Lasota and Mackey

Proposition 8 *Let P be the Markov operator corresponding to (X, Y, R, υ). Assume that:*

$$\int_Y (|R(x,y) - R(z,y)|)\, \upsilon(dy) \leq \alpha_1 |x - z|\,, \qquad x, z \in X, \tag{29}$$

and $\int_Y (|R(0,y)|)\, \upsilon(dy) \leq \alpha_0$, where $\alpha_1 \in (0,1)$ and $\alpha_0 \in (0,\infty)$. Then the system (X, Y, R, υ) is weakly asymptotically stable.

Proof: Lasota and Mackey (1994), Theorem 12.6.1, p. 423. ■

The next proposition applies this result to the stochastic system implied by $(Z, \Theta, \zeta_q^*, \pi)$.

Proposition 9 *Assume that each $\zeta_q^*(\cdot, \widehat{\theta}_k)$ is monotone, then P_q is weakly asymptotically stable.*

Proof: Set $X = Z$, $Y = \Theta$, $R = \zeta_q^*$ and $\upsilon = \pi$. Suppose $\zeta \geq \zeta'$, then

$$\begin{aligned}
\sum_k \left|\zeta_q^*(\zeta, \widehat{\theta}_k) - \zeta_q^*(\zeta', \widehat{\theta}_k)\right| \pi(\theta) &= \sum_k (\zeta_q^*(\zeta, \widehat{\theta}_k) - \zeta_q^*(\zeta', \widehat{\theta}_k))\pi(\widehat{\theta}_k) \\
&= \sum_k \left(\frac{\zeta + \psi + \eta_q^*(\zeta, \widehat{\theta}_{k+1}) - \eta_q^*(\zeta, \widehat{\theta}_k)p_k}{1+\psi} - \frac{\zeta' + \psi + \eta_q^*(\zeta', \widehat{\theta}_{k+1}) - \eta_q^*(\zeta', \widehat{\theta}_k)p_k}{1+\psi} \right) \pi(\widehat{\theta}_k) \\
&= \frac{\zeta - \zeta'}{1+\psi} = \frac{|\zeta - \zeta'|}{1+\psi},
\end{aligned}$$

where we use the monotonicity of each $\zeta_q^*(\cdot, \widehat{\theta}_k)$ in the first line, the definition of ζ_q^* in the second, and $\sum_k \{\eta_q^*(\widehat{\zeta}, \widehat{\theta}_{k+1}) - \eta_q^*(\widehat{\zeta}, \widehat{\theta}_k)p_k\}\pi(\widehat{\theta}_k) = 0$ at $\widehat{\zeta} = \zeta$ and ζ' in the third. The argument for $\zeta \leq \zeta'$ is very similar. This verifies (29) with $\alpha_1 = \frac{1}{1+\psi}$. To verify the second condition in Proposition 8, use the definition of ζ_q^* and, for all k, $\eta_q^*(0, \widehat{\theta}_k) = 0$, to obtain: $\sum_k \left|\zeta_q^*(0, \widehat{\theta}_k)\right| \pi(\widehat{\theta}_k) = \sum_k \left|\frac{\psi + \eta_q^*(0,\widehat{\theta}_{k+1}) - \eta_q^*(0,\widehat{\theta}_k)p_k}{1+\psi}\right| \pi(\widehat{\theta}_k) = \frac{\psi}{1+\psi}$. The result then follows from Proposition 8. ■

Remark: We give conditions for ζ_q^* to be monotone in Appendix B. These require that the shock spread not be too large and that $\partial W/\partial \rho$ be concave, where $W(\rho; q) = \sup_{u \in D} \rho u - qC(u)$. The latter is satisfied if u is CRRA with $\sigma > 1/2$. There is a long tradition of studying Markov processes induced by monotone policy functions in economics, see for example Hopenhayn and Prescott (HP) (1992). For the most part these results rely on a compact state space. In the context of the present model, this is restrictive. Proposition 9 does not require such a condition. Additionally, to obtain uniqueness and weak asymptotic stability, most contributions to the economics literature supplement monotonicity with a mixing condition, see HP, Theorem 2. In the present model, the underlying mean reversion property of the ζ_q^*, coupled with monotonicity, ensures the contractivity condition and no additional mixing conditions are needed. ■

To strengthen Proposition 9 and obtain the geometric ergodicity result, it is necessary to embed $\mathcal{M}(Z)$ into a normed space. Let $w : Z \to [1, \infty)$ be a weight function. If μ is a signed measure on $\mathcal{B}(Z)$ with total variation $|\mu|$, denote its w-norm by $\|\mu\|_w = \int_Z wd|\mu|$. Let $\mathbb{M}_w(Z)$ denote the set of signed measures on $\mathcal{B}(Z)$ with finite w-norm. $P : Z \times \mathcal{B}(Z) \to [0, 1]$ is a signed kernel if for each $\zeta \in Z$, $P(\zeta, \cdot)$ is a signed measure on Z and for all $H \in \mathcal{B}(Z)$, $P(\cdot, H)$ is a measurable function. If P is a signed kernel define its w-norm by: $\|P\|_w = \sup_{\mu \in \{\mathbb{M}_w(Z): \|\mu\|_w \leq 1\}} \|P\mu\|_w$.

Definition 2 *Let $w : Z \to [1, \infty)$ be a weight function and P a Markov operator such that $\|P\|_w < \infty$. Then P is called w-**geometrically ergodic** with convergence rate $\rho \in (0, 1)$ if there is a $\mu_* \in \mathbb{M}_w(Z)$ and $R \geq 0$ such that for all t, and $\mu \in \|P^t - \mu_*\|_w \leq R\rho^t$.*

Thus, if a Markov operator P is w-geometrically ergodic, it has a unique invariant measure and any sequence of measures induced by P converges strongly and at a geometric rate ρ to this measure.

Proposition 10 *Define $w : Z \to [1, \infty)$ pointwise by $w(\zeta) = 1 + |\zeta|$. Assume that each $\zeta_q^*(\cdot, \widehat{\theta}_k)$ is monotone. Then P_q is w-geometrically ergodic with convergence rate $\frac{1}{1+\psi}$.*

Proof: By the previous proposition P_q is weakly asymptotically stable. Hence, it has a unique invariant measure, μ_q. It follows that the Markov process associated with P_q is μ_q-irreducible and aperiodic. By Theorem

7.3.10 p. 12, Hernandez-Lerma and Laserre, P_q is w-geometrically ergodic with convergence rate $\rho \in (0,1)$ if, in addition, for all ζ, and some $b < \infty$, $\sum_k w(\zeta_q^*(\zeta, \widehat{\theta}_k))\pi(\widehat{\theta}_k) \leq \rho w(\zeta) + b$.

Now,

$$\begin{aligned}\sum_k (1 + |\zeta_q^*(\zeta, \widehat{\theta}_k)|)\pi(\widehat{\theta}_k) &= \sum_k \left(1 + \left|\frac{\zeta + \psi + \eta_q^*(\zeta, \widehat{\theta}_{k+1}) - \eta_q^*(\zeta, \widehat{\theta}_k)p_k}{1+\psi}\right|\right)\pi(\widehat{\theta}_k) \\ &\leq \frac{1}{1+\psi}(1 + |\zeta|) + \frac{2\psi}{1+\psi}.\end{aligned}$$

The first line above uses the definition of ζ_q^*. The second line uses the fact that if $\zeta \leq 0$, for all k, $\eta_q^*(\zeta, \widehat{\theta}_k) = 0$, while if $\zeta \geq 0$, by monotonicity, $\zeta_q^*(\zeta, \widehat{\theta}_k) \geq \zeta_q^*(0, \widehat{\theta}_k) = \frac{\psi}{1+\psi} > 0$ and $\sum_k \{\eta_q^*(\zeta, \widehat{\theta}_{k+1}) - \eta_q^*(\zeta, \widehat{\theta}_k)p_k\}\pi(\widehat{\theta}_k) = 0$. This confirms the condition with $\rho = \frac{1}{1+\psi}$ and $b = \frac{2\psi}{1+\psi}$, proving the result. ■

5 The Atkeson-Lucas unemployment insurance problem

The analysis developed in this paper can be applied to variety of other environments. One such is the unemployment insurance model of Atkeson and Lucas (1995). We sketch this application here. An agent receives either a positive or a zero productivity shock in each period. The former is interpreted as a job, the latter as unemployment. Thus, we set $\Theta = \{E, U\}$, for employment and unemployment respectively. These shocks are private information. If the agent has a job, she is able to exert effort in work. Here preferences over consumption and effort are given by:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t(\theta^t)) - 1_E(\theta_t)v(y_t(\theta^t))]\pi(\theta^t),$$

where $u : \mathbb{R}_+ \to D$ is as before, $v : L \to B$ is convex, increasing and differentiable and 1_E is an indicator function for the employment state. Let $Y = v^{-1}$. Atkeson and Lucas assume that $L = [0, \overline{y}]$ and that v is linear. To avoid considering corner solutions we will assume that v is strictly convex, $\lim_{y \downarrow 0} v'(y) = 0$, $\lim_{y \uparrow \overline{y}} v'(y) = \infty$. We will also assume that u is bounded above.

The planner's problem proceeds as in the previous environment, with appropriate respecifications of shocks and preferences. Assuming constant prices, the recursive component planner problem is:

$$V(\zeta) = \left[W^E(\zeta+\eta, q) + \lambda V\left(\frac{\zeta+\eta+\psi}{1+\psi}\right)\right]\pi(E) + \left[W^U(\zeta-\eta p, q) + \lambda V\left(\frac{\zeta-\eta p+\psi}{1+\psi}\right)\right]\pi(U),$$

where $W^E(\rho, q) = \sup_{u\in D, v\in B} \rho\{u - v\} - q\{C(u) - Y(v)\}$ and $W^U(\rho, q) = \sup_{u\in D} \rho u - qC(u)$. Let $\zeta_q^*(\zeta, \theta)$ denote the optimal continuation Pareto-Negishi weight function, and $\eta^*(\zeta)$ the optimal Lagrange multiplier policy function. A convenient feature of this problem is that both $\zeta_q^*(\zeta, E)$ and $\zeta_q^*(\zeta, U)$ are increasing, under no additional assumptions. Thus, the implied Markov operator P_q is w-geometric with unique invariant measure μ_q^*. Since $\eta_q^*(0) = 0$ as in our earlier environment, we have that $\zeta_q^*(\zeta, E) \geq \zeta_q^*(\zeta, U) \geq \zeta_q^*(0, U) = \frac{\psi}{1+\psi}$. Thus, an agent's utility from consumption is bounded below by $u = \arg\sup_{u\in D} \frac{\psi}{1+\psi}u - qC(u)$, her distutility from effort is bounded above by: $v = \arg\sup_{v\in B} -\frac{\psi}{1+\psi}v + qY(v)$.

6 Social credibility and social discounting

Optimal allocations in dynamic mechanism design models are rarely time consistent. The long run immiseration of agents means that this time inconsistency takes an extreme form when the agent and the planner share the same discount factor. This raises natural questions about the practical implementation of such allocations. Motivated by this observation in Sleet and Yeltekin (2004), we consider the subgame perfect equilibria of a policy game in which the planner cannot commit to implementing a particular allocation and has the same discount factor as agents. In other respects the physical environment is the same as that considered in the current paper. We call equilibrium allocations in this game credible. A key finding of that paper is that optimal credible allocations solve the problem of a planner who can commit, but who has an *endogenously higher discount factor than the agent.* The argument is in two steps. First, we show that optimal credible allocations solve a planner's problem that incorporates additional *credibility constraints.* These require that the planner's continuation payoff remains above its autarkic value (i.e. above the value it would take if the planner were to offer no insurance). Since the planner's payoff is a weighted aggregate of agent utilities, this, in turn, requires that this weighted aggregate is maintained above its autarkic value. Second we reformulate the Lagrangian from the problem with credibility constraints, constructing a new discounting scheme from the Lagrange multipliers on these constraints.

We now generalise this logic. When a planner proposes a report-contingent allocation in the initial period, agents, in selecting their reporting strategies, must assess whether the planner will adhere to the allocation. If we assume that the planner is constrained to maintain some weighted aggregate of agent utilities above a lower bound, no allocation will be viable unless it respects these bounds at all future dates. Although the planner might propose an allocation that violates these constraints, in an attempt to reduce the social cost (from her perspective) of truthful revelation, agents will not find the allocation credible. We assume that these bounds stem from the ability of agents to collectively resist or otherwise avoid reductions in their continuation utility, even if they, or their ancestors, previously accepted an allocation that called for such reductions. This resistence might take the form of social disruption or some other sort of political pressure, but our model will not be explicit about this. The logic described above may be used to convert a planner's problem with such generalised credibility constraints into a problem without such constraints, but with high planner discounting.

More formally suppose that the planner solves the credible social insurance problem:

$$\sup_{\{\alpha^{(\zeta_0,\gamma)}\in A\}} \int_Z \int_Z \zeta_0 \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t\in\Theta^{t+1}} \theta_t u_t^{(\zeta_0,\gamma)}(\theta^t)\pi^t(\theta^t)\Phi_0(d\zeta_0)\Psi_0(d\gamma)$$
$$- \int_Z \int_Z \sum_{t=0}^{\infty} \widehat{q}_t \sum_{\theta^t\in\Theta^{t+1}} C(u_t^{(\zeta_0,\gamma)}(\theta^t))\pi^t(\theta^t)\Phi_0(d\zeta_0)\Psi_0(d\gamma) \tag{30}$$

subject to $\forall \zeta_0, \gamma, t, \theta^{t-1}, k < K$, $\Delta U_t(\alpha^{(\zeta_0,\gamma)}, \gamma(\theta^{t-1}), \widehat{\theta}_k, \widehat{\theta}_{k+1}) \geq 0$ and for all t,

$$\int_Z \int_Z \gamma \sum_{\theta^{t-1}\in\Theta^t} U_t(\alpha^{(\zeta_0,\gamma)}(\theta^{t-1}))\pi^{t-1}(\theta^{t-1})\Phi(d\zeta_0)\Psi(d\gamma) \geq \underline{U}. \tag{31}$$

The new constraints (31) require that the planner maintains a γ-weighted aggregate of continuation agent utilities above a lower bound. Consistent with the discussion above, we interpret the γ-weight as capturing the ability of an agent (or dynasty) to resist or avoid reductions in its continuation utility. In Sleet and Yeltekin (2004), these weights coincided with the ζ_0 weights in the planner's objective, since they stemmed from the planner's inability to commit. Here, however, we allow them to differ.[12] We also treat the lower bound $\underline{U}$ as a parameter (in Sleet and Yeltekin (2004), it is given by the autarkic utility). We call (31) a *social credibility constraint.*

We now construct a Lagrangian for the above planner's problem. Denoting the multiplier on the t-th social credibility constraint (31) by $\beta^t\psi_t\prod_{s=0}^{t-1}(1+\psi_t)$, we construct the following Lagrangian:

$$\sup_{\{\alpha^{(\zeta_0,\gamma)}\in A\}}\int_Z\int_Z \zeta_0 U_0(\alpha^{(\zeta_0,\gamma)})\Phi(d\zeta_0)\Psi(d\gamma)-\sum_{t=0}^{\infty}\widehat{q}_t\int_Z\int_Z\sum_{\theta^t\in\Theta^{t+1}}C(u_t^{(\zeta_0,\gamma)}(\theta^t))\pi^t(\theta^t)\Phi(d\zeta_0)\Psi(d\gamma)$$
$$+\sum_{t=0}^{\infty}\beta^t\psi_t\prod_{s=0}^{t-1}(1+\psi_t)\left[\int_Z\int_Z\gamma\sum_{\theta^{t-1}\in\Theta^t}U_t(\alpha^{(\zeta_0,\gamma)}(\theta^{t-1}))\pi^{t-1}(\theta^{t-1})\Phi(d\zeta_0)\Psi(d\gamma)-\underline{U}\right] \tag{32}$$

subject to $\forall\zeta_0,\gamma,t,\theta^{t-1},k<K$, $\Delta U_t(\alpha^{(\zeta_0,\gamma)}(\theta^{t-1}),\widehat{\theta}_k,\widehat{\theta}_{k+1})\geq 0$. Notice that this Lagrangian absorbs the social credibility (but not the incentive) constraints into the objective. If the utility functions of the agents are bounded either above or below and if $\sum_{t=0}^{\infty}\beta^t\psi_t\prod_{s=0}^{t-1}(1+\psi_t)<0$, the Lagrangian (32) may be rearranged to give the following optimisation:

$$\sup_{\{\alpha^{\zeta_0,\gamma}\in A\}}\int_Z\int_Z\sum_{t=0}^{\infty}B_0^t\gamma_t(\zeta_0,\gamma)\sum_{\theta^t\in\Theta^{t+1}}\theta_t u_t^{(\zeta_0,\gamma)}(\theta^t)\pi^t(\theta^t)\Phi(d\zeta_0)\Psi(d\gamma)$$
$$-\sum_{t=0}^{\infty}q_tB_0^t\int_Z\int_Z\sum_{\theta^t\in\Theta^{t+1}}C(u_t^{(\zeta_0,\gamma)}(\theta^t))\pi^t(\theta^t)\Phi(d\zeta_0)\Psi(d\gamma), \tag{33}$$

subject to $\forall\zeta_0,\gamma,t,\theta^{t-1},k<K$, $\Delta U_t(\alpha^{(\zeta_0,\gamma)}(\theta^{t-1}),\widehat{\theta}_k,\widehat{\theta}_{k+1})\geq 0$. Here $q_t=\widehat{q}_t/B_0^t$, $B_0^t=\beta^t\prod_{s=0}^{t-1}(1+\psi_s)$ and $\gamma_t(\zeta_0,\gamma)=\gamma+\psi_t+(\zeta_0-\gamma)/\prod_{s=0}^{t-1}(1+\psi_t)$. It then follows that the planner's problem (30) can be transformed into one *without* credibility constraints, but *with* a new discounting scheme $\{B_0^t\}_{t=0}^{\infty}$ and a new Pareto weighting

[12] We do assume that both ζ_0 and γ are constants. More generally still, they could be time varying or stochastic.

scheme $\{\gamma_t\}_{t=0}^{\infty}$. Since for all t, $B_0^t \geq \beta^t$, a credibility-constrained planner will behave as if she has a higher discount factor and is more patient than the agents. In this sense credibility is a force for social patience. More formally, the following result links credible social insurance problems to problems with relatively patient planners. The proof is obtained by minor modification of the proof in Sleet and Yeltekin (2004).

Proposition 11 *Fix* $\{\{q_t\}_{t=0}^{\infty}, \underline{U}, \Phi, \Psi\}$ *and suppose there exists an allocation* $\{\alpha^{(\zeta_0,\gamma)}\}$ *and a sequence* $\{\psi_t\} \in \mathbb{R}_+^{\infty}$ *that satisfy the following conditions:*

1. $\sum_{t=0}^{\infty} \beta^t \psi_t \prod_{s=0}^{t-1}(1+\psi_s) < \infty$, $\sum_{t=0}^{\infty} \beta^t q_t \prod_{s=0}^{t-1}(1+\psi_s) < \infty$;
2. $\psi_t \left(\int_Z \int_Z \sum_{\theta^t \in \Theta^{t+1}} \gamma U_t(\alpha^{(\zeta_0,\gamma)}(\theta^t))\pi^t(\theta^t)\Phi(d\zeta_0)\Psi(d\gamma) - \underline{U} \right) = 0$

   and $\int_Z \int_Z \sum_{\theta^t \in \Theta^{t+1}} \gamma U_t(\alpha^{(\zeta_0,\gamma)}(\theta^t))\pi^t(\theta^t)\Phi(d\zeta_0)\Psi(d\gamma) - \underline{U} \geq 0$;
3. $\alpha^{\zeta_0,\gamma}$ *solves* $\sup_{\{u_t\}_{t=0}^{\infty} \in A} \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} B_0^t[\gamma_t(\zeta_0,\gamma)\theta_t u_t(\theta^t) - q_t C(u_t(\theta^t))]\pi^t(\theta^t)$ *subject to* $\forall t, \theta^{t-1}, k < K$, $\Delta U_t(\{u_{t+s}(\theta^{t-1},\cdot)\}_{s=0}^{\infty}, \widehat{\theta}_k, \widehat{\theta}_{k+1}) \geq 0$.

   Then α *solves the planner's problem (30) at* $\{\{q_t\}_{t=0}^{\infty}, \underline{U}, \Phi, \Psi\}$.

To simplify the analysis, we assume that the Ψ distribution is degenerate at one, and drop the explicit indexing of variables by γ. It is natural to focus on stationary allocations that solve the planner's problem (30) for a fixed sequence of prices and an appropriate initial and invariant measure Φ_q. Such allocations are more easily computable and bring out more clearly the link between crediblility and social patience. More explicitly, we define a stationary credible equilibrium as follows.

Definition 3 *A stationary credible equilibrium at* $(\underline{U}, q)$ *is a tuple* $(\Phi_q, \psi_q, V_q^*, \zeta_q^*, \eta_q^*)$ *satisfying*

1.

$$V_q^*(\zeta) = \inf_{\eta\in\Lambda(\zeta)} \sum_{\widehat{\theta}_k\in\Theta} \left\{ W\left(\left[\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1})\frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k} p_{k-1} + \psi_q \right] \widehat{\theta}_k; q \right) + \beta(1+\psi_q)V_q^*\left(\frac{1}{1+\psi_q}[\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1})p_{k-1}] + \left(\frac{\psi_q}{1+\psi_q}\right)\right)\right\} \pi(\widehat{\theta}_k), \quad (34)$$

2. η_q^* *attains the solution to (34);*

3. ζ_q^* *satisfies* $\zeta_q^*(\zeta, \widehat{\theta}_k) = \frac{1}{1+\psi}[\zeta + \eta_q^*(\zeta, \widehat{\theta}_k) - \eta_q^*(\zeta, \widehat{\theta}_{k-1})p_{k-1}] + \left(\frac{\psi_q}{1+\psi_q}\right)$;

4. Φ_q *is an invariant measure of the Markov process induced by* ζ_q^* *and* π*;*

5. $\sum_{t=0}^{\infty} \beta^t (1+\psi_q)^t < \infty$*; and*

6. letting $U(\zeta)$ *denote the lifetime utility to an agent with initial Pareto-Negishi weight* ζ *under the allocation induced by* ζ_q^* *and* η_q^**,* $\int_Z U(\zeta)\Phi_q(d\zeta) = \underline{U}$.

Thus, the allocation induced by ζ_q^* and η_q^* solves the credibility problem (30) at $\{\{q\}_{t=0}^{\infty}, \underline{U}, \Phi_q, 1_{\{1\}}\}$ and induces an average continuation utility to agents of $\underline{U}$. A stationary credible equilibrium is essentially the same as an optimal stationary allocation from a constant price economy with a planner discount factor of $\beta(1+\psi_q)$. In particular, the process for Pareto-Negishi weights has the same form. Consequently, given a pair (q, ψ_q), under the conditions of Proposition 7, optimal value and policy functions $(V_q^*, \zeta_q^*, \eta_q^*)$ exist with the latter inducing a stationary distribution Φ_q. The tuple $(\Phi_q, \psi_q, V_q^*, \zeta_q^*, \eta_q^*)$ is then a stationary credible equilibrium at $(q, \int_Z U(\zeta)\Phi_q(d\zeta))$.

Our definition of a stationary credible equilibrium resembles that of a stationary equilibrium in a Bewley-type economy with ψ_q serving as the analogue of an equilibrium price. Consequently, it is relatively straightforward to compute. We provide such numerical calculations in Section 8. We use them to illustrate the equilibrium policy functions and stationary distributions for some sample economies. Specifically, we show these objects change as $\underline{U}$ and ψ_q vary.

7 Budget constraints

So far we have treated the pricing sequence $\{q_t\}_{t=0}^{\infty}$ as a parameter. However, if the model is augmented with a resource or budget constraint for the planner, then these prices can be derived endogenously as shadow resource prices. We briefly discuss the issue in the context of a model with an exogenous planner discount factor.[13] To simplify the exposition, we assume a distribution over γ values concentrated at 1 and drop explicit reference to the long run weight γ in the notation.

We refer to an initial Pareto-Negishi weight distribution Φ_0 and a family of utility allocations $\{\alpha^{\zeta_0} \in A\}$ as a population allocation. Fixing a sequence of resource quantities $\{R_t\}$, we will say that a population allocation $\{\Phi_0, \{\alpha^{\zeta_0}\}\}$ satisfies the per period resource constraints if for all t,

$$R_t \geq \int_Z \sum_{\theta^t \in \Theta^{t+1}} C(u_t^{\zeta_0}(\theta))\pi^t(\theta^t)\Phi_0(d\zeta). \tag{35}$$

The planner's problem with a budget constraint can then be written as:

$$\sup_{\{\alpha^{\zeta_0} \in A\}} \int_Z \sum_{t=0}^{\infty} \lambda^t \gamma_t(\zeta_0) \sum_{\theta^t \in \Theta^{t+1}} \theta_t u_t^{\zeta_0}(\theta^t)\pi^t(\theta^t)\Phi_0(d\zeta_0) \tag{36}$$

$$\text{s.t. } \forall \zeta_0, t, \theta^{t-1}, k < K, \qquad \lambda^t \Delta U_t(\alpha^{\zeta_0}(\theta^{t-1}), \widehat{\theta}_k, \widehat{\theta}_{k+1}) \geq 0$$

$$\forall t, \qquad R_t \geq \int_Z \sum_{\theta^t \in \Theta^{t+1}} C(u_t^{\zeta_0}(\theta))\pi^t(\theta^t)\Phi_0(d\zeta).$$

We then have by a fairly standard argument (e.g. Luenberger (1969)) that if a family of allocations $\{\alpha^{\zeta_0}\}$ solve the component planner problems (11) at prices $\{q_t\}_{t=0}^{\infty} \in \ell_{1,+}(\lambda)$ and if the population allocation $\{\Phi_0, \{\alpha^{\zeta_0}\}\}$ satisfies the per period constraints with equality at all dates, then $\{\alpha^{\zeta_0}\}$ solves (36) subject to these constraints.

In general, it is difficult to characterise the optimal allocation $\{\alpha^{\zeta_0}\}$ and price sequence at a particular sequence of resource quantities. On the other hand, given a price sequence $\{q_t\}_{t=0}^{\infty}$ and initial distribution Φ_0 one can first solve for the optimal allocation at these parameters (by solving the earlier problems (11)) and

[13] The credibility model could be similarly augmented.

then in a second step, compute the corresponding resource costs. The allocation and prices then solve (36) at these resource costs. In particular, if q is constant, and the conditions of Proposition 7 hold, then the component planner's optimal policy functions admit an invariant distribution Φ_q over Pareto-Negishi weights. The corresponding constant resource cost R_q is given by:

$$R_q = \int_Z \sum_{\widehat{\theta}_k} C(u^*(\widehat{\theta}_k(\zeta + \eta_q^*(\zeta, \widehat{\theta}_k) - \eta_q^*(\zeta, \widehat{\theta}_{k-1})\frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k}); q))\pi(\widehat{\theta}_k)\Phi_q(d\zeta),$$

where $u^*(\rho, q) = \arg\sup_{u \in D} \rho u - qC(u)$. Since $u^*(\rho; q) = \frac{\partial C}{\partial u}^{-1}(\frac{\rho}{q})$, we have that R_q is finite if $q > 0$ and $C \circ C'^{-1}$ is concave. In this case, the allocations $\{\alpha^{\zeta_0}\}$ induced by the component planner's optimal policy functions solve the planner's problem (36) when $\Phi_0 = \Phi_q$ and each $R_t = R_q < \infty$.

Rather more can be said for the CRRA economy. In this case, with constant prices, the component planner's value function satisfies:

$$\begin{aligned} V_q(\zeta) \quad = \quad \inf_{\eta \in \Lambda(\zeta)} \sum_{\widehat{\theta}_k \in \Theta} \Bigg\{ & \frac{\sigma}{1-\sigma}\left(\frac{1}{q}\right)^{\frac{1-\sigma}{\sigma}} \left(\left[\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1})\frac{\widehat{\theta}_{k-1}}{\widehat{\theta}_k}p_{k-1}\right]\widehat{\theta}_k\right)^{\frac{1}{\sigma}} \\ & + \lambda V_q\left(\frac{1}{1+\psi}[\zeta + \eta(\widehat{\theta}_k) - \eta(\widehat{\theta}_{k-1})p_{k-1}] + \left(\frac{\psi}{1+\psi}\right)\right)\Bigg\}\pi(\widehat{\theta}_k). \end{aligned}$$

Dividing through by $\left(\frac{1}{q}\right)^{\frac{1-\sigma}{\sigma}}$, we observe that $\left(\frac{1}{q}\right)^{\frac{\sigma-1}{\sigma}} V_q(\zeta) = T_1 \left(\frac{1}{q}\right)^{\frac{\sigma-1}{\sigma}} V_q(\zeta)$, where T_1 denotes the Bellman operator when $q = 1$. Now for $q > 0$, $\left(\frac{1}{q}\right)^{\frac{\sigma-1}{\sigma}} V_q \in \mathcal{V}$.[14] By arguments similar to those given earlier, when $\sigma > 1$, T_1 is a contraction on $\mathcal{V}$ with unique fixed point V_1 in $\mathcal{V}$. Hence, $V_q = \left(\frac{1}{q}\right)^{\frac{1-\sigma}{\sigma}} V_1$. It follows that the policy functions and stationary distribution over Pareto-Negishi weights are independent of q; q simply scales the utilities and consumptions of agents up or down. In particular, it is easy to check that $R_q = \left(\frac{1}{q}\right)^{\frac{1}{\sigma}} R_1$. Once R_1 is obtained, it is straightforward to compute the price that would support an arbitrary resource level as part of a stationary solution.

[14]The bounding prices $\underline{q}$ and $\overline{q}$ can be set arbitrarily small or large.

8 Numerical computations

In this section, we briefly present some numerical calculations that illustrate and expand upon our previous analysis. In particular, we compute steady state credible equilibria for several economies, focussing on policy functions and steady state distributions. Throughout, CRRA preferences are assumed: $u(c) = c^{1-\sigma}/1-\sigma$. The parameters for our economy are given by $\{\sigma, \beta, \Theta, q, \psi\}$. We set $\sigma = 2$, the agent's discount factor $\beta = 0.9$ and $q = 1$. We assume four shock values uniformly distributed between 0.8 and 1.2. We allow ψ to vary, considering a low value, 0.001, an intermediate value of 0.0024 and a high value of 0.035. Each ψ value corresponds to a different value for $\underline{U}$, the bound from the credbility constraint and the steady state expected utility to agents. The $\underline{U}$ values are increasing in ψ with $\psi = 0.0024$ corresponding to the autarkic value for $\underline{U}$.

Solutions to the component planner's dynamic programming problem are computed using value iteration. We approximate value functions using Schumaker splines on an interval. These are extended off of the interval when necessary using polynomials that remain within the bounds $\underline{V}$ and $\overline{V}$. We use the approximated policy functions to construct a discretised approximation to the Markov operator.

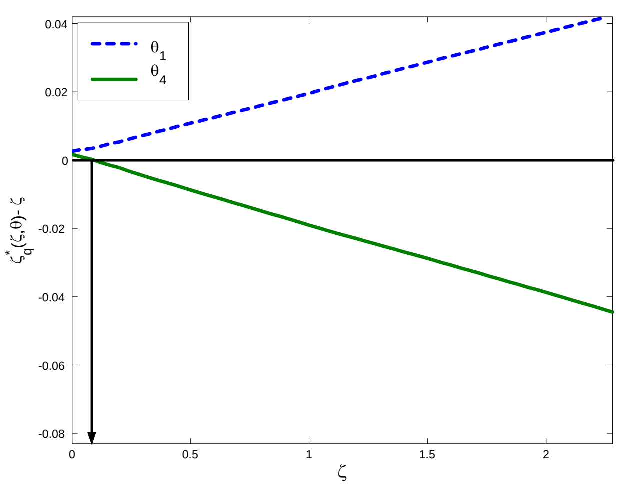

Figure 1A: $\zeta_q^*(\zeta, \theta) - \zeta$, when $\psi = 0.0024$.

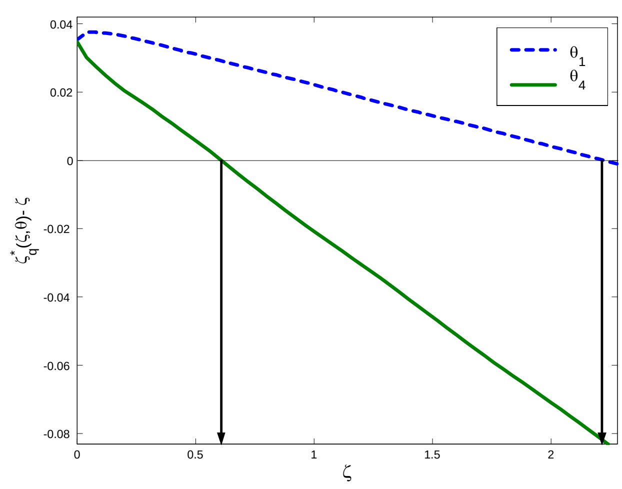

Figure 1B: $\zeta_q^*(\zeta, \theta) - \zeta$, when $\psi = 0.035$.

Figures 1A and 1B show $\Delta\zeta(\zeta, \theta) := \zeta_q^*(\zeta, \theta) - \zeta$, for $\theta = \{\widehat{\theta}_1, \widehat{\theta}_4\}$. Thus, they show the change in an agent's Pareto-Negishi weight contingent on the realisation of the lowest and highest shocks. Figure 1A shows what happens when $\psi = 0.0024$. In this case, the $\Delta\zeta(\zeta, \theta)$ functions diverge from a positive value at $\zeta = 0$. The lower policy function, $\Delta\zeta(\zeta, \widehat{\theta}_4)$, associated with the high shock, crosses the ζ-axis at ζ value, $\underline{\zeta}$, slightly below 0.1, indicating that the interval $[0, \underline{\zeta}]$ is non-absorbing. In fact, the $\zeta_q^*(\cdot, \widehat{\theta}_k)$ policy functions are monotone, so that this set is transient. Figure 1B plots the $\Delta\zeta(\zeta, \theta)$ for the much higher ψ value, 0.035. Now, $\Delta\zeta(\zeta, \widehat{\theta}_4)$ crosses the ζ-axis at a higher value of $\underline{\zeta} = 0.6$. Again, ζ_q^* is monotone and $[0, \overline{\zeta}]$ is transient. Evidently, this lower transient region is larger than before. In addition, the upper function $\Delta\zeta(\zeta, \widehat{\theta}_1)$ crosses zero from above at 2.42, indicating that the region above $\zeta = 2.42$ is transient as well. This has immediate implications for inequality. When $\psi = 0.035$, except temporarily during the initial transition, an agent's Pareto-Negishi wanders in the region $[0.6, 2.42]$, when $\psi = 0.0024$, agent's Pareto-Negishi weights wander over the much larger region $[0.1, \infty)$, modulo the initial transition, indicating a much greater scope for inequality. By way of contrast, for the case $\psi = 0$ (not shown), the functions $\Delta\zeta(\zeta, \theta)$ diverge from 0 as ζ increases above 0. By the immiseration result, agents' Pareto-Negishi weights are almost surely absorbed by 0.

The monotonicity of the computed policy functions indicates that the underlying constant price economies exhibit an ergodic Markov process for ζ weights. Consequently, they admit a unique invariant probability distribution Φ_q. In Figure 2, we illustrate the cumulative distribution functions $F(\zeta) = \Phi_q([0, \zeta])$ implied by Φ_q at three different values of ψ.

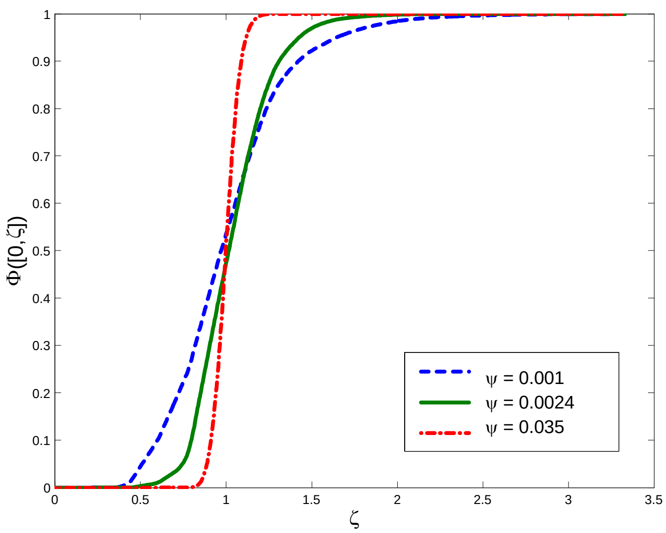

Figure 2: CDFs

The lowest value of ψ implies considerable inequality with non-negligible probability mass across a wide range of ζ values. The high value of ψ implies a distribution concentrated in a narrow interval of 1, the equilibrium value of ψ, though small, indicates an intermediate distribution. This figure complements our existing theoretical results by suggesting that the degree of inequality at the stationary distribution is monotone decreasing in ψ. Since $\underline{U}$ is increasing in ψ, it further indicates that the tighter the credibility constraint the more equal the steady state distribution of Pareto weights and, hence, of consumption and utility.

9 Conclusion

The analysis in this paper implies that the immiseration result is a pathology of environments with equal planner and agent discount factors. In environments in which the planner's discount rate exceeds that of the agents, a stationary distribution over utilities and consumptions often emerges. Relatedly, immiseration assumes that the planner (or government) is free to treat future generations arbitrarily badly in the interests of current generations. As we have argued, at a practical level this seems implausible. Optimal allocations in economies in which such treatment is not possible resemble those in ones with a high planner discount factor. Since allocations are only credible if they are consistent with the feasible long run treatment of agents, credibility restrictions imply social patience, or, at least, patient-like behaviour.

At a technical level this paper, shows the value of the Marcet-Marimon procedure for the analysis of (concave) dynamic moral hazard problems. It also applies new techniques from Markov process theory to the study of such problems. These technical contributions should be useful in settings beyond the one considered in the paper.

Two major areas of extension remain. Our social credibility constraints seek to capture the ability of groups of agents to resist ex post reductions in their continuation utility. This ability is captured by an agent specific weight and by a lower bound on the weighted sum of agent utilities. The weights and the bound are parameters. We think it would be extremely interesting to endogenise these parameters in the context of explicit policy or political economy games. In Sleet and Yeltekin (2004), we consider one such game in which the planner or government cannot commit. But many other possibilities exist. The second area of extension is to more general idiosyncratic shock processes. Although the restriction to taste shocks is not particularly important for the analysis, the results can easily be extended to other sorts of private information, the restriction to i.i.d. shocks is important. We leave both extensions to future work.

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