Exchange and Specialization as a Discovery Process

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Abstract

In this paper we seek to identify some minimal structure necessary to support exchange and specialization. We develop a closed economy in which participants must even discover the ability to exchange, in addition to ascertaining what they are comparatively good at producing. As it turns out, many people demonstrate the ability to intuit comparative advantage, exhaust gains from trade, and effectively choose production that is consistent with the choices of others. However, not everyone becomes a specialist, even though full efficiency can only be achieved if everyone does so. Near-full efficiency does frequently occur bilaterally in these economies. That is, many subjects do fully specialize in the productive activity in which they have a comparative advantage, and when they do so it is typically because they have entered a stable trading relationship with a specialist in the other good. Such pairing does not necessarily give impetus to the formation of other pairs, nor are pairs typically open to the inclusion of a third party.

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1. Introduction

Adam Smith is credited with clearly articulating the proposition that specialization creates wealth and specialization is in turn supported by exchange. At the mid-twentieth century, theorists had elegant multiple commodity general equilibrium models of competitive markets based on preference/utility demand theory and resource production supply theory. This solved a long-standing problem: what are the forces behind resource prices and allocations that explain their diversity and responsiveness to external circumstances.

But central new questions were not addressed. In particular, what does the individual need to know beyond his personal circumstances for specialization and exchange to be efficacious? Does each person need information on the circumstances of others? What are the dynamics of that discovery process? How do agents discover comparative advantage in an unstructured, informationally decentralized economy and spontaneously learn to order their production and exchange decisions?

These are important questions that involve the dynamics of specialization and exchange. Of course, if competitive prices are assumed given, then comparative advantage falls out of individual optimization. But this begs the question of how the prices might be formed out of the decentralized private information of the participants. Specialization, exchange and prices have to emerge simultaneously from the same complex discovery process. How might potential trading partners in an unstructured incomplete information environment find their trading partners and arrange their productive activities by comparative advantage so that price coordination can occur?

Previous experimental work on general equilibrium economies has focused on the double auction institution and its remarkable power to draw markets to the competitive equilibrium (Williams and Smith, 1986; Noussair et al., 1995, 1997; Lian and Plott, 1998; Williams et al., 2000; and Gjerstad, 2004). In these studies people had to discover prices and specialization, but an exchange and propertry right institution was defined for them exogenously, and the research program demonstrated how effective and sufficient that institution was for solving the price-allocation discovery problem—prices did not have to be given. How might any exchange institution emerge in an environment in which specialization pays but there are no exogenous institutional guidelines and no third party enforcement of property rights? In this paper we seek to identify some minimal structure necessary to support specialization and exchange. We develop a closed economy in which participants must even discover the ability to exchange, in addition to ascertaining what they are comparatively advantaged at producing. Hence, it is not obvious how or if our economies will reach the competitive equilibrium. Our subjects must discover exchange, specialization, competitive prices, and competitive quantities in less than an hour and a half—without instructional guidance.

We observe that people are quite adept at determining comparative advantage. With no explicit trading rules, our laboratory economies exhibit a significant degree of specialization and become almost fully Pareto efficient in two limited senses. First, holding production decisions constant, few gains from trade remain unexploited. Second, fixing exchange and treating production as unilaterally variable, few agents could substantively gain by changing their output decisions. However, the ability to intuit comparative advantage, produce accordingly, and exhaust gains from trade does not necessarily translate into fully efficient specialization and exchange. Not everyone specializes to the full extent of their comparative advantage, even though full efficiency can be achieved in the economy only if everyone does so. In our experiment, preferences are induced which favor the safety of home production relative to specialization risk if one fails to find trading partners. In this respect we capture the following cultural change hypothesis: there are natural barriers to the emergence of trust in personal exchange where failure is costly, and success through remunerative trade and specialization can only be discovered via experience and by incurring risk. Small economies may inhibit advantageous specialization and exchange as there are fewer opportunities for likeminded matches to explore potential gains from specialization and trade. Several factors may inhibit trade in large economies, including increased noise, longer searches for potential partners with compatible technology, and diminished trust in cooperation given the improved outside option of each subject.[1]

Nevertheless, full specialization and efficient exchange frequently does occur bilaterally in these economies. That is, many subjects do fully specialize in the productive activity in which they have a comparative advantage, and when they do so it is typically because they have entered a stable trading relationship with a specialist in the other good. However, such pairing does not necessarily give impetus to the formation of other pairs, nor are pairs typically open to the

[1] On these points, see Surowiecki (2004) for a discussion on the effect of group size in coordinating economic activities.

inclusion of a third party, even though pairs are never coalition-proof when at least one person in the economy is not paired with anyone else. These findings underscore the emergence of natural human experiential bias in favor of personal exchange, and the need to understand the processes through which exchange becomes less localized and bilateralized (for discussions on personal versus impersonal exchange, see North, 1990, 1991; Smith, 2003).

The paper is organized as follows. In Section 2, we described the economic environment for our experimental economies. Section 3 outlines our design and procedures including the unstructured institution and analytical benchmarks. Our results are discussed in Section 4, and we briefly offer our concluding remarks with Section 5.

2. The Economic Environment

Our economy consists of equal numbers of two types of agents, *odd* and *even*, who produce and consume two types of goods: *red* and *blue*.[2] Both agent types have Leontief preferences over r units of *red* and b units of *blue*:

$$U_{odd} = \min\{r, 3b\}$$

$$U_{even} = \min\{2r, b\}$$

We chose Leontief preferences in part for the relative ease in inducing the preferences for subjects. Such preferences are "easy" for subjects to understand because they do not have to reference tables, functions, calculators, or indifference maps. This then facilitates their focus on the task of discovering how to maximize earnings.

Agents are endowed with T units of time t to produce R units of *red* and B units of *blue*. *Odd* and *even* types also differed in their production functions. For t bounded between 0 and 10, *Odd* agents produce according to the following process:

$$R_{\text{odd}} = \frac{13}{10\sqrt{10}} t^{\frac{5}{2}} \approx 0.41 t^{\frac{5}{2}}$$

$$B_{\text{odd}} = \frac{10}{10-\left(\frac{300\sqrt{10}}{13}\right)^{\frac{2}{5}}} (10-t) \approx 2.25(10-t)\,.$$

For the *even* subjects, we have:

$$R_{\text{even}} = \frac{13}{10-\sqrt{\frac{260}{11}}} t \approx 2.53t$$

$$B_{\text{even}} = \frac{11}{10}(10-t)^2\,.$$

[2] These are two-agent replicator economies in the nomenclature of cooperative game theory.

As will be discussed in more detail in the following section, subjects are not presented with these functions explicitly, but rather will be presented with (red, blue) output pairs for a given set of *t*. The production possibilities frontier for each type of agent and the 2-agent autarky and competitive equilibrium benchmarks are pictured in Figure 1.[3] In autarky, it is optimal for *odd* subjects to spend 56% of their time producing red, thus producing and consuming 30 reds and 10 blues, and earning 30 cents each period. *Even* agents optimally spend 51% of their time producing red, producing and consuming 13 reds and 26 blues, and earning 26 cents per period. In competitive equilibrium, the *odd* subjects fully specialize in the red good, producing 130 units, while the *even* subjects fully specialize in blue, producing 110 units. The competitive price of a blue unit is 4/3 of a red unit, and at this price the *odd* subject will consume 90 red and 30 blue, while the *even* subject will consume 40 red and 80 blue. Note that competitive profits (90 cents for odd, 80 cents for even) are roughly three times greater than autarky profits.

3. Experimental Design and Procedures

3.1 Institution

Subjects are members of a *virtual village*. Each subject owns a *house* and a *field*, which he can monitor on his computer screen. He may also view the houses and fields of other members of the village.[4] Neither the type of the other subjects nor the distribution of types was known to the subjects. Preferences are induced in terms of US cents for units consumed. Depending on the treatment, the subjects are given (1) no ex ante production information (in each period they simply commit to an allocation of time, and then learn the resulting output pair), (2) an output table for a given set of proportional time allocations, or (3) a blank output table that fills in as production choices are made by themselves and by the agents of the same type (i.e., in this case, perfect recall is enforced plus subjects may learn from their counterpart's experience, as well). At the end of the instructions, the subjects were given as much time as they wished to experiment with their production functions before the session began.

Each session lasts for 40 periods, and each period is subdivided into phases *A* and *B* with *t* specified in seconds. Phase *A* is a 10-second production phase during which each subject

[3] Of course, since there are always an equal number of the two types of agents in our economies, per capita autarky and competitive consumption are identical for all economy sizes. The uniqueness of the competitive equilibrium is proven in section 3.3.

[4] The purpose of the italicized words is to motivate our experimental design. Such labels were not presented to subjects to mitigate any unintended influence that these words might have on their decisions.

produces red and blue goods in his field by allocating a proportion of this time to producing each good. Phase *B* is a 90-second consumption phase. Subjects consume goods (thus earning cash paid at the end of the session) by dragging and dropping the red and blue icons into their respective houses. Subjects are not informed they may drag and drop icons into the houses of others, as well. Subjects may communicate with each other in the village chat room. Figure 2 displays a screenshot of a subject's interface. Every 7th period is a 100-second rest period, where no production or consumption takes place, but subjects may still chat with each other.

At any time the subject can adjust the slider in the top center of his screen to determine the proportion of phase *A* to spend producing blue and red goods. The total quantity of each good produced in the phase will appear in his field and icons for each good produced will appear in the upper-left corner of his screen. These icons can be dragged and dropped into any of the houses, which constitutes consumption when the Phase *B* time expires (the subject's own house and field appear in green, while those of others are gray). In the example given in Figure 2, no units can yet be consumed, since no house registers quantities of either good. In all treatments, production and consumption quantities for each villager are public information allowing, without prompting, each to potentially learn from the (production, consumption and any trade decision) experiments of others.

In the upper right-hand corner of Figure 2 is the payoff schedule. In the top half of this schedule is a reminder of the relative value to the subject of the goods; in this case we have an *odd* subject, so he requires 1 blue for every 3 reds. In the bottom half of the schedule is a payoff calculator. In this space is calculated the number of "wasted" goods in one's house, and the number of goods of the opposite type necessary so that no quantity of either good is wasted. This calculation aid serves as an "expert" support system to facilitate a focus on thinking about the actions that might be taken.

Our parameterization was chosen to make specialization risky but potentially quite profitable. The Leontief preferences imply that specialization coupled with no trade will result in a payoff of 0¢ per period for the subjects, versus 26¢ and 30¢ in autarky. This circumstance provides some robustness of our results if specialization occurs, because it is clearly not unambiguously profitable to specialize. Furthermore, the competitive equilibrium earnings of 80¢ and 90¢ per period provide a salient incentive to overcome the risk, specialize and trade. We chose these specific parameters for the utility and production functions so that the comparative advantage of each type was different and so that at the competitive equilibrium each type would earn similar but not identical profits. We also attempted to characterize the Leontief preferences as intuitively as possible, so that a subject either needs "1 blue for each 3 red to earn 3 cents" or "1 red for each 2 blue to earn 2 cents". We reiterate that the subjects were not in any way informed that they could exchange items with other subjects. The instructions (intentionally in the passive voice) simply stated that "When the [Phase *B*] clock expires, you earn cash based upon the number of red and blue items that have been moved to your house. To select items to be moved, *left* click on an item or click on the red or blue buttons at the top of the screen. The yellow highlighted items can be moved by dragging with the *right* mouse button."

Why not inform subjects they can trade? Adam Smith writes: "As it is the power of exchanging which gives occasion to the division of labour, so the extent of this division will always be in proportion to the extent of that power."[5] Our goal is to identify some minimal structure necessary to support exchange, which, in turn, supports specialization. Therefore, we do not inform subjects they can trade because we do not want to imply they should be trading and hence specializing; subjects may not believe that we, the experimenters, would design a market in which it is generally better not to participate. In other words, if informed they may trade, subjects may limit themselves to strategies that anticipate exchange. We want subjects to choose from a broader class of strategies. We stack the deck against specialization by not even mentioning its necessary precursor, exchange, and then observe its frequent occurrence despite this disadvantage. In fact, not explicitly revealing opportunities for exchange favors the establishment of an autarky baseline, one that perhaps subjects may not want to deviate from. Then, once exchange is discovered, we can observe whether subjects learn to specialize, despite the risks, by coordinating with others. In fact, this is what we observe explicitly: Prior to exchange, near-autarky prevails, and once the "power of exchanging" is discovered, specialization gradually evolves, despite any framing effects from beginning in autarky.

This approach of discovering the opportunity of exchange may also be of broader methodological import. Researchers have asked whether the strong "fairness" results observed in certain contexts would obtain if a more market-oriented frame were created (see Hoffman, et al., 1994, who make such comparisons in ultimatum and dictator games). Here we develop a way in which the importance of market framing can presumably be tested. That is, we observe

[5] See his Lectures On Jurisprudence, p. 582.

that there are numerous ways subjects can be introduced to a market (gradually-immediately, endogenously-exogenously), not only the standard immediate exogenous approach, and one could presumably test the impact of such frames on the dynamic paths adopted in such markets. While this is not the focus of our present research, future experiments along these lines may help bridge the gap between various literatures.[6]

3.2 Treatments

We expect that the outcomes of these economies may be influenced by variation in two dimensions: The number of subjects in a session and the extent to which each subject knows his own production function. We adopt two 3×1 treatment designs with one shared cell and replicate each treatment with 6 sessions; see Table 1.[7] The number of subjects in each session was 2, 4, or 8, divided into an equal number of even and odd types. We first conducted the treatments with ex ante unknown production, and anticipated two effects of increasing economy size, both improving the rate of specialization and efficiency. First, we hypothesized that larger economies would more quickly discover the possibility of exchange. This turned out to be true, and in fact the powerful effect was fully realized in moving from 2- to 4-subject economies. Second, we hypothesized that specialists would emerge more quickly in larger economies, and that specialization cascades would frequently occur. This was not the case. Much attention will be paid to these issues in Section 4. We note here that in 8-subject economies that the probability a random subject has a complementary technology to one's own is decreasing in population size, and the presence of more potential partners increases one's outside option relative to a particular exchange relationship, potentially inhibiting trust.

[6] To anticipate a result, 2-subject economies that fully specialize are more likely to implement the socially optimal allocation (where utilities are lent equal weight) than pairs of complementary specialists in larger economies. It is also the case that 2-subject economies discovered the opportunity of exchange much more slowly than larger economies. It is possible that chatting in autarky over an extended period of time creates a cooperative frame that carries over into exchange, whereas such framing is weaker in larger economies.

[7] We also conducted three pilot sessions with classroom participants and without a rest period every 7th period. It was evident that the students could use an intermittent break to relax and reflect upon the experiment.

Table 1: Experimental Design

	Production		
	Ex Ante Unknown	Perfect Recall	Ex Ante Known
2 Subjects	*Unknown2*		
4 Subjects	*Unknown4*	*Recall4*	*Known4*
8 Subjects	*Unknown8*		

After running the three size treatments for ex ante unknown production, we speculated that one inhibitor of specialization may have been that subjects were unwilling to explore their productive capabilities. Recall that specialization is costly ex post if a subject cannot find a trading partner or specializes in his comparatively disadvantageous good. If output (particularly the endpoints) were known ex ante, we thought it would facilitate the coordination of specialization.[8] Since two of the six 2-subject sessions did not discover the ability to exchange (and one of the others did so only very late in the experiment), and the 8-subject sessions exhibited great difficulty in specializing at all, we decided the most interesting case in which to apply ex ante known production capabilities was in 4-subject sessions. If we did not observe a significant effect here, it is unlikely we would observe one in 2 or 8-subject sessions. See Figure 4 for the output tables given to subjects in the *Known4* treatment.

Anticipating the results, although the *Known4* treatment reduced the incidence of specializing in a good in which one is comparatively disadvantaged, there was not a significant increase in the rate of specialization. It is plausible that ex ante known production had a mixed effect on specialization: It may have facilitated coordination, but also made explicit the potential cost of experimenting with different output levels, since subjects could now directly compute optimal autarky production. Therefore, we also chose to run a treatment with ex ante unknown production with perfect recall, in an effort to mitigate playing it safe early in the session while facilitating the coordination of production levels between subjects in the chat room. In the *Recall4* treatment, a table similar to Figure 4 was presented to subjects, but the third and fourth columns were blank. After each production phase, the appropriate line in the table was completed and maintained for the duration of the session.

[8] That is, if an odd-numbered subject knows he can produce 130 reds, and learns in the chat room that someone else can produce 110 blues, we conjectured it should be apparent that gains from trade exist, and this realization would prompt more specialization than when neither party necessarily knows their capabilities.

Subjects interacted via visually-isolated computer terminals and read self-paced instructions prior to the first period of the session. The subjects were George Mason University undergraduates randomly recruited from the university at large during the 2004-05 academic year. In addition to $5 for showing up on time, the average earnings in the 2-, 4- and 8-agent sessions are $13.93, $13.56, and $10.58, respectively for a 90-minute session in this experiment.[9] The competitive equilibrium earnings for the *even* and *odd* agent types are $28.00 and $31.50, respectively, and the sum of these amounts, $61.50, represents the maximum potential profit per pair of subjects (the competitive equilibrium coincides with the solution to the egalitarian social planner's problem).

3.3 Analytical Benchmarks

We have already described the competitive equilibrium (CE), but should verify its uniqueness, and also discuss other analytical benchmarks, like Pareto optima, the contract set, and the core. In consideration of CE, it suffices to consider a two-subject economy. Let *p* be the price of one blue unit in terms of red units. In CE, a subject in production must maximize wealth conditional on this price. Then, the intersection of his budget line and offer curve will determine his unique optimal consumption bundle. The resulting allocation is CE if aggregate excess demand is zero; that is, the offer curves in the Edgeworth box intersect at *p*.

Consider the wealth function for subject $i \in \{odd, even\}$, which is equal to $R_i + pB_i$. The second derivative of this function with respect to *t* is strictly positive for $p > 0$ and $t \in [0,10]$; thus, the function is strictly convex on the relevant support, and its maximum is a corner. When price is sufficiently high, both subjects produce only blue, and when price is sufficiently low, both produce only red. The price at which an individual switches is equal to the ratio of his corner output. For the odd subject, this price is roughly 5.77, and for the even subject, 0.23. Thus, for a price greater than 5.77 or less than 0.23, only one type of good is produced, and since demand for both goods is strictly positive, such a price cannot be competitive. For a price between 0.23 and 5.77, the economy engages in fully specialized production by comparative

[9] After observing the results, one might ask why the experiment only lasted 90 minutes when more time might have substantially changed the convergence properties of the economies. Observations of these sessions qualitatively indicate that the experiment, the chat room in particular, is cognitively taxing. A longer session would have required a significantly higher earning potential, at least a two-fold increase (based upon past experience), to achieve dominance in the salient payoffs (Smith, 1982).

advantage, and the only price that supports the intersection of the offer curves is 4/3. Of course, in the experiment such a price is not exogenously given.[10]

There are two notions of Pareto optimality we consider, ex ante and ex post production, which are formally defined and characterized in Appendix I. In the ex post case, gains from trade are exhausted given fixed production; thus, an allocation is ex post Pareto optimal if and only if there are no wasted units of red and blue.[11] Figure 3 displays the ex post Pareto set for a fully specialized 2-subject economy, which is approximately equal to the shaded region lying between the two offer curves. At any allocation in the red region only red units are wasted, and in the blue region only blue units are wasted. Outside of the shaded region both red and blue units are wasted. The bit of waste outside of the offer curves is a consequence of the discreteness of these economies, and reflects the odd agent partially matching a blue unit (with one or two reds) or the even subject partially matching a red unit (with one blue). As is apparent from the figure, the competitive equilibrium allocation is unique in fully utilizing all goods. Of course, the ex post Pareto set is quite large as we may consider any level of production.

In characterizing the ex ante Pareto set, it will be convenient to assume goods and time spent in production are perfectly divisible; the discrete case is similar but more tedious to describe. With divisibility, no goods can be wasted at all, because the producer, at no cost to anyone, could have converted a portion of waste into the other good. In addition, it must also be the case that no alternative production and exchange decisions would have made any subject worse off, and at least one subject strictly better off. For example, autarky is ex post Pareto optimal and in fact wastes no goods, but it is not ex ante Pareto optimal because it is Pareto-dominated by the CE allocation. The ex ante contract set, the set of ex ante Pareto optimal allocations in which each subject earns at least autarky utility, is fully characterized in Appendix I. We show that for 2- and 4-subject economies, at most one subject may not fully specialize in his comparative advantage. For 8-subject economies, four subjects of the same type must be fully specialized in their comparative advantage, at least two subjects of the other type must be fully specialized in their comparative advantage, and at least one of the other two subjects must be specialized, although it may be in his comparatively disadvantaged good.

[10] Decentralized coordination on competitive prices remains an important open question in general equilibrium theory; see Crockett, Spear, and Sunder (2005) for a new approach to this issue.
[11] A wasted unit of a good is a unit which does not contribute to anyone's utility. For example, if an odd subject consumes 35 reds and 10 blues, he wastes 5 red units.

Finally, the ex ante core is a subset of the ex ante contract set. It will prove useful later to note that in a core allocation, both an odd and an even subject may not receive less than CE utility, or they could form a blocking coalition. Thus, in the core it must be the case that each member of one type of subject receives at least competitive utility, specifically, a subject type in which each member is fully specialized.

4. Results

The chief scientific objective of this paper is to collect some basic stylized observations on Adam Smith's first principles of specialization and exchange. As we discuss in this section, each treatment, save the one with eight subjects, contains at least one economy (session) that achieves or nearly achieves the competitive equilibrium, and at least one economy that does not ever leave autarky. We have replicated each treatment 6 times, but no quantitative analysis, parametric or nonparametric, would indicate any statistical difference among treatments. This is not all too surprising given the unstructured nature of this experiment. Each economy has its own unique properties. Probing below the surface we do find peculiar differences among the treatments that generate rich directions for future research that can build upon the foundations presented here.

4.1 Discovering the Opportunity for Exchange

Recall that subjects were not informed they could drag and drop the goods they produce into the houses of others. Nevertheless, 29 of 31 economies discovered the opportunity for exchange, 14 in the first period and 21 by the second.[12] The rate of discovery was not constant across treatments however; 2-subject sessions were much slower to engage in exchange. The only economies that failed to discover the possibility of exchange were 2-subject economies. Two of the six sessions never exchanged goods, and, incidentally, became near-optimally autarkic. Of the four economies that did engage in trade, one began trading in period 2, another in period 6, and the other two in periods 18 and 19. The speed in which larger economies learned to trade was strikingly fast, by comparison. A total of 12 out of 19 four-subject

[12] In one session of *Known4*, a power cord was tripped in the lab, resulting in some data loss. We lost all chat and exchange data, and some production and consumption data. In particular, we collected consumption data through period 29, and production data through period 33. Therefore, we ran an additional session so that we could include 6 complete sessions in the treatment. We will include data from the session with incomplete data by default, and note, where appropriate, results that do not include data from this session due to incompleteness.

economies traded goods *in the first period*, and 15 did so by the second period. There does not appear to be much of a difference between 4- and 8-subject economies. Combined, 14 of 25 larger economies traded in the first period, 20 of 25 by the second (the other five sessions first traded in periods 3, 4, 9 (2), and 17). Thus, 80% of the large economies traded in the first or second period, relative to 17% of the 2-subject sessions.

As an aid in understanding this effect across treatments, consider a simple model in which the probability that an individual will discover the opportunity for exchange in a given period is a constant ρ, invariant across individuals, session, and time. Then the cumulative density function to discover exchange in period t is given by $1-(1-\rho)^{Nt}$ (here t excludes rest periods). While it is a stretch to invoke a law of large numbers for 18 observations, it is instructive to consider matching the proportion of 4-subject economies that discover exchange in the first period (about 63%) to this cdf, which occurs when $\rho = 0.22$. For this value of ρ, there is a 39% chance that a 2-subject economy discovers exchange in the first period (0 of 6 actually did), a 99% chance by the 9th period (2 of 6 actually did), and the probability of doing so by the end of the session is unity to many significant digits. Even for $\rho = 0.09$ (where it is half as likely for 4-subject economies to discover exchange in the first period as the observed frequency), the probability that a 2-subject economy will discover exchange by the final period is equal to one to three significant digits, and yet two of six sessions never discovered exchange. Similarly, for $\rho = 0.22$, the likelihood that an 8-subject session discovers exchange in the first period is 86%, which is much greater than the proportion in the data (2 of 6).

Two natural ways to extend the model are to let ρ be a function of session size and/or vary by individual. It seems as though the latter alone is not sufficient. If there are some people who are much more likely than others to quickly discover the possibility of exchange, then to support the 63% frequency of first period success in the 4-subject economies, some 2-subject sessions should have discovered exchange in the initial period (none did). Also, most of the 8-subject sessions also should have discovered exchange in the first period, whereas only 2 of 6 actually did so.

However, to explain the variation within session size, it appears necessary to include subject heterogeneity. For example, to obtain a probability of 0.5 that a 2-subject session trades by the 4th period (matching the realized frequency), we have $\rho_2 = 0.09$. In this case, however, the probability that a session trades by the end of the session is 1 to three significant digits. To put things a different way, to have even a 5% chance that two of six sessions would not trade at all, the probability of observing 3 of 6 sessions trading by period 4 would be close to zero.

Therefore, it appears to be the case that some subjects are more likely than others to consider the possibility of exchange, which is to be expected. However, the strong possibility that the likelihood a given individual will consider exchange for the first time is conditional on the size of a session (peaking somewhere near four then declining) was a surprise. Being in an environment with more than one other person may naturally inspire increased consideration of entrepreneurial activity; the presence of too many people may cause confusion and trepidation. However, we have an insufficient number of sessions to consider estimating a model where ρ varies by individual and session size, so we offer this possibility only as a conjecture.

4.2 Specialization and Comparative Advantage

Across treatments, most subjects begin the first several periods of a session producing both goods near the autarkic quantities. By the end of the session, however, specialization increases substantially except in the 8-subject economies. We define a specialist as a subject who spends at least 90% of his time producing the good in which he has a comparative advantage, which corresponds to an output (100 red, 2 blue) for odd subjects and (3 red, 89 blue) for even subjects. Figure 5 plots the mean number of specialists in each treatment across periods. Consider the ex ante unknown treatments. An average of 1-3% of the subjects in *Unknown2* and *Unknown4* specialized during the first six periods. The 8-subject treatment began with considerably more specialists, an average of 16% during the first six periods. However, after the final rest period, the average proportion of specialists had barely increased, to 22%, whereas it had increased to 50% in *Unknown2* and 77% in *Unknown4*. In fact, three-quarters of the *Unknown2* subjects in sessions where the possibility of exchange had been discovered became specialists, which is comparable to Unknown4.

Why do the 8-subject sessions produce significantly fewer specialists? Research on auditory conversations indicates that our natural ability to hear places an upper limit of four on the number of participants in naturally forming conversations (Dunbar et al., 1995). As the background noise changes, this number can move up or down slightly. The conversations in our experiment are text-based, which would only slow down the ability of the subjects to process the content of discussions. With 8 subjects, we observe that subjects give up on attempts at breaking through the disjointed chatter. The inability to effectively communicate may hamper exchange and eventually extinguish any nascent specialization. Additional explanations will be offered in section 4.3.

From the figure it is apparent that the proportion of *Recall4* and *Known4* specialists was quite similar to the *Unknown4* treatment through the fifth rest period. For example, the average proportion of specialists between the fourth and fifth rest periods was roughly 60% in *Unknown4*, 58% in *Recall4*, and 56% in *Known4*. However, there was small drop in specialization after the final rest period in *Recall4* to 55%, whereas *Unknown4* and *Known4* continued to become increasingly specialized, to 77% and 69%, respectively. *Recall4* was slower to discover exchange than the other 4-subject treatments, as well; in the six sessions exchange was first discovered in periods 1, 2 (2), 3, 4, and 17, whereas exchange was discovered in the first period in 5 of 6 sessions in both *Unknown4* and *Known4*. In section 4.3 we discuss how subjects in *Recall4* were less likely to trade in the later periods than subjects in the other 4-subject treatments, as well. Since *Recall4* begins as *Unknown4* and then becomes more like *Known4* as output data is recorded, we conjecture these differences between the treatments are most likely idiosyncratic rather than a reflection of some treatment effect.

As one would expect, increased levels of specialization are generally identified with comparatively advantaged exchange; typically, the odd subjects are specializing in red, the evens in blue, and they are trading with each other. Let V_{tk1} be the sum of red and blue goods exchanged in period t of treatment k between odd and even subjects, and V_{tk2} be the corresponding sum of goods exchanged between subjects of the same type. We define $\lambda_{tk} = V_{tk1} / (V_{tk1} + V_{tk2})$ to be the proportion of trading volume in a period that occurred between subjects of opposite type. Since there are more potential odd-even pairs than same-type pairs, we also consider lending more weight to V_{tk2}. Let σ_k be the proportion of potential opposite-type pairings in treatment k (1 for two subjects, 2/3 for four subjects, and 4/7 for eight subjects). We denote weighted proportional trading volume in comparatively advantaged exchange as $\lambda'_{tk} = V_{tk1} / (V_{tk1} + \phi_k V_{tk2})$, where $\phi_k = \sigma_k / (1 - \sigma_k) > 1$.

In Figure 6, λ_{tk} and λ'_{tk} are plotted for the 4- and 8-subject treatments. The weighted statistic has a reasonably stationary mean between 72% and 88% in the four treatments after the third rest period. A stable odd-odd pair emerged in *Unknown4*, which explains why this treatment is lower than the others in the final 12 periods or so. It turns out that an odd-odd pair can earn a total of 69¢ by specializing (one in red, one in blue); this is slightly better than autarky, so such a pairing is reasonable if arrangements cannot be coordinated between opposite-type pairs. The other three treatments average 79% to 88% over the second half of the experiment. In fact, in *Recall4* and *Unknown8*, where some progress in specialization deteriorated over time, the weighted proportion of trade in comparative advantage over the final few periods was in the mid-90s, suggesting that some failed specialists gave up and returned to near-autarkic production. Therefore, most trade aligns according to comparative advantage, even after accounting for the fact that there are fewer same-type pairs in the economies. Those who specialize generally do so in their comparative advantage and trade with subjects with complementary technologies, and those who do not settle into autarky. Further support for this statement will be presented in the following section.

4.3 Ex-ante Efficiency

We next address realized efficiency relative to the competitive equilibrium. In our chosen parameterization the CE allocation doubles as the optimal allocation in the egalitarian social planner's problem, so relating realized total profits to autarky and CE profits can provide a measure of social efficiency. The particular measure we adopt for a given treatment is:

$$\Pi_{st} = \frac{\sum_{s=1}^{S}\sum_{n=1}^{N}\pi_{snt} - \frac{SN(30+26)}{2}}{\frac{SN(90+80-30-26)}{2}},$$

where S is the number of sessions in the treatment, N is the number of subjects per session, and π_{snt} is realized profit of subject n in period t of session s. Π_{st} is thus the ratio of realized to competitive profit, normalized so that autarky profits equal zero (recall odd subjects make 30¢ in autarky and 90¢ in CE, while even subjects make 26 and 80 cents, respectively).

Figure 7 displays the mean ex ante efficiency for each of the five treatments. Prior to the first rest period, the economies are 10-20% *less efficient* than autarky on average, even in the *Known4* treatment where autarky production was relatively easy to determine from the subjects' production tables. As will be detailed in Section 4.4, many goods are wasted in these early periods, as subjects attempt to discover their comparative advantage through trial and error, fail to coordinate production decisions, and over-consume goods they end up not matching (for example, an even subject might produce 10 reds and 40 blues and immediately place them in his house, potentially wasting 20 blues in the process by not keeping them available for trade).

After the second rest period, mean efficiency exceeds autarky in all treatments, as many subjects who desire to trade have determined what they are comparatively good at producing (recall Figure 6), reducing the number of unmatchable goods. It is typically after the second rest period that opposite-type pairs begin to coordinate their specialization activities, although a couple did so beforehand. Mean efficiency generally increases in all sessions throughout the remaining the periods, to 40-50% in 2- and 4-subject sessions and 15% in 8-subject sessions.

The treatment mean masks much individual profit variation. Consider Figure 8, which presents two individual profit distributions for each treatment, one for mean profits between periods 1-6, the other for mean profits between periods 36-40. The former is clustered a bit to the left of autarky. The second distribution, for all but the 8-subject treatment, has two roughly equal peaks, one near autarky and one near the competitive equilibrium, with a small bit of activity between. Thus, it is not the case that each subject eventually earns half of the difference between autarky and CE profits; in fact, such subjects are uncommon. Rather, most subjects earn near-competitive or near-autarkic profits. Figures 9a and 9b present mean ex ante efficiency by session. From these figures we learn that not only are there clusters of individuals near autarky and CE by the end of the experiment, but that these individuals are not distributed uniformly across sessions. We observe some sessions that converge near the CE allocation, some that remain near the autarky allocation, and some where there is great variance in profits among individuals, with individual profits typically pooling near autarky and CE.

An earlier question bears repeating, "What happened in *Unknown8*?" In session 2, six specialists emerged and pushed mean profits to about 65%. In both sessions 1 and 5, two specialists emerged and pushed mean profits to 20%. These ten specialists earned profits much closer to CE than autarky, while the other subjects (almost 80% of the total) remained near autarky. We have already suggested that the 8-member chat rooms exceed the size of naturally occurring conversations, and thus may introduce excessive noise, confusion, and trepidation. This may not be the whole story, however.

Figure 10 depicts the proportion of subjects who send goods to other subjects, by treatment and period. The contribution of each session to this proportion begins with its first period of exchange. Thus, if trade in sessions 1-6 of *Recall4* began in periods 2, 1 2, 4, 3, and

17, respectively, then the first data point for the treatment is equal to ($a_{1,2} + a_{2,1} + a_{3,2} + a_{4,4} + a_{5,3} + a_{6,17}$)/24, where a_{st} is equal to the number of agents in session *s* who sent goods to other agents in period *t*. The measure is computed only up to the number of periods in which final session to discover exchange traded, so that the full complement of sessions contributes to each observation. The exception is *Unknown2*; since two sessions never traded goods, they have been discarded from the analysis. The important thing to note in the figure is that through 19 periods (the number of periods for which we have observations from each treatment), the proportion of traders in *Unknown8* is comparable to the 4-subject sessions. Seventy-three percent of the subjects in *Unknown8* sent goods to other subjects in the 19th period of exchange, relative to 79% in *Unknown4*, 71% in *Recall4*, and 54% in *Unknown4*. Even afterward, this proportion hovers in the high-50s to mid-60s through the 28th period (the final observation), similar to *Known4*, though less than *Unknown4*. So it is not the case that 8-subject sessions trade much less frequently than 4-subject sessions. They clearly understand the technology of exchange, and are presumably sending goods for some purpose rather than at random. The key is that the volume of trade per capita is much lower. These subjects are generally not specializing, and only send small quantities of goods back and forth.

The problem appears to be about coordination and less about noise. There are at least two plausible reasons why coordination could be more difficult in larger economies. First, the proportion of subjects with compatible technologies gets smaller as *N* gets larger; this proportion is equal to $(N/2)/(N–1)$. In a 2-subject economy, one's (potential) partner is always technologically compatible. (Each person still has to discover the good in which each has an comparative advantage.) In a 4-subject economy, 2/3 of the other subjects are compatible, while only 4/7 are compatible in the 8-subject economies. It may be the case that some same-type subjects attempted to specialize and trade with each other, and then decided to give up on specialization entirely, not realizing there were more compatible partners in the village. A second inhibitor of exchange may be that the outside option of each subject increases in the number of subjects. A partner may be implicitly more trustworthy in a 2-subject setting, because a violation of trust could likely result in a severance of all future exchange. In a larger economy, a subject may have some expectation that he can find another "partner" in later periods. Isolating the effect of noise, search, and trust in the evolution of exchange and specialization and is something we are developing in future research.

4.4 Ex-post Efficiency

So far we have discussed efficiency relative to benchmark markets in competitive equilibrium. However, other notions of efficiency may help us address two more questions: (1) To what extent are goods wasted, either consumed unmatched or not consumed at all; (2) Is such waste due to poor coordination in exchange or production? We develop three measures of efficiency that address these questions, which together imply these economies became almost fully Pareto efficient in the ex post sense already described and in a limited ex ante sense, as well. We conclude that few subjects would have significant incentive to deviate from their output or exchange decisions by the end of the session, fixing such decisions of others.

For a given period and session, we first determine the number of red and blue *contributions* to a subject's utility. For odd subjects, this is *c* reds and ⅓*c* blues, and for evens this is ½*c* reds and *c* blues, where *c* is the number of cents earned by the subject, with fractions rounded up. For example, if an even subject earns 13 cents in a period, the contribution is 7 reds (rounded up from 6.5) and 13 blues. A *wasted* good is one that does not contribute to the utility of any subject in the economy. The number of wasted goods of a certain color in a period is equal to its total production minus the sum of contributions of that good across subjects.

We first consider to what extent waste is ex post Pareto inefficient. Let r_w and b_w be the total amount of wasted red and blue goods, respectively, in a given period and session. We define exchange waste *W* as the optimized value of the egalitarian social planner's problem that reassigns wasted goods to subjects. That is, we act as if we could assign wasted goods to the subjects who would receive the greatest benefit from receiving them. Weighting utilities equally makes *W* as large as possible, providing an upper bound on inefficiency. An allocation is in the ex post Pareto set if and only if $W = 0$. See Appendix II for a formal definition of *W*.

Now let *P* equal total realized profit in the period. Maximum potential profit in the session, conditional on realized production and exchange, is $W+P$, and *ex post Pareto efficiency* as a percentage of maximum potential profit is $P/(W+P)$. Across treatments, these economies became almost fully ex post-efficient. In Figure 11, the blue line segments represent mean ex post efficiency by treatment and period. Across treatments, the mean economy is 65-75% efficient in the first period, and 95-100% efficient in the final period. While there is some variation in efficiency across individual sessions, most almost completely exhaust gains from trade. Nearly 70% of the final five periods across sessions are at least 95% Pareto efficient (85% of the sessions are at least this efficient in the final period), and almost 85% of the final five periods are at least 90% efficient (95% in the final period).

There is waste that is not a result of inefficient exchange, however. Suppose two odd and one even subject fully specialize in a 4-subject economy, while the remaining even subject produces and consumes optimally in autarky. If an odd subject and the even specialist implement the pair-wise competitive allocation, the 'odd man out' will waste 130 units of red, although none of this waste is ex post Pareto-inefficient. We provide a broader characterization of inefficiency that includes such production coordination failures. Ex ante Pareto optimality is too strong for this purpose, so we consider a limited notion of ex ante Pareto efficiency, one in which unilateral changes in production determine "wasted profit" rather than multilateral ones. We begin by attributing wasted goods, defined as above, to individual subjects. Since we have collected transaction information sequentially, we can determine which particular goods were wasted *a posteriori*. For example, if an even subject consumes 20 red and 13 blue units, then the final 13 red units were wasted by sender (possibly himself). For each subject *i*, we determine individual accounts of wasted goods r_{w_i} and b_{w_i}, noting that $\sum_{\{i\}} r_{w_i} = r_w$ and $\sum_{\{i\}} b_{w_i} = b_w$.

We next determine how much such mistakes implicitly cost each individual. We do so by considering the change in production that would have caused the imbalance to be consumed optimally. For example, suppose an odd subject produced 63 reds and 6 blues, and that 45 of those reds and none of the blues became realized production mistakes. We calculate the change in production that would have put these mistakes "in balance." To produce (63,6), the subject actually spent 75% of his time producing red goods. If he had instead used 56% of his time to produce reds, he would have produced 30 reds and 10 blues, producing 33 less units of red and 4 more units of blue. He then could have consumed the remaining 12 units of wasted reds with the 4 new units of blue to attain an additional utility of 12 cents in the period. This is an implicit cost of his mistakes. We normalize this cost by autarky utility, so his *total realized waste* is 40% of autarky profits. The green line segments in Figure 11 represent mean *total realized waste*.

Note both production and exchange inefficiency are captured by *total realized waste*. Production inefficiency in this sense represents a failure to coordinate one's production decisions with others, whereas exchange inefficiency represents waste that could have been eliminated with better coordination in the exchange process alone. To separate these inefficiencies, we define one final measure, *realized production waste*. When $W > 0$, we reallocate the exchange-inefficient waste as before and reciprocally credit the contributing individual waste accounts. The process is detailed in Appendix II. For each individual with residual production-inefficient waste, we consider how he could have changed output to eliminate it. The (normalized) utility increase associated with this change is *realized production waste*. Such waste is bounded from above by *total realized waste* and below by zero; thus it serves to distinguish how much waste could have been eliminated by trade, and how much waste resulted from poorly coordinated production decisions. The residual waste between total and production waste is an alternative measure of ex post Pareto inefficiency, though normalized by autarky profits rather than potential profits.

Figure 11 represents *realized production waste* as red line segments. From the figure it is clear that little waste of any kind is taking place in these economies by the end of the session. In fact, with the exception of the 2-subject sessions, production inefficiencies are small throughout, as mean *realized production waste* in each treatment is no more than 12% in any period, and no greater than 5% in 85% of the periods. Put a different way, if each session had been able to exhaust gains from trade, only a penny or two more per subject in each period could have been earned by readjusting output decisions and consuming the balance in autarky. To the extent that inefficiency existed, as it did in early periods, much can be attributed to failing to coordinate exchange rather than output decisions. It is intuitive that production inefficiencies are greatest in smaller economies, since in larger economies it is more likely that subjects who have not directly communicated hold unmatched goods.

Exchange inefficiencies were substantial in early periods. In each treatment, mean *total realized waste* in the first period was between 35 and 40%, and generally above 15% until after the second rest period. However, such waste gradually decreased over time, finishing in the 10-12% range for *Recall4* and *Unknown4*, and below 5% in the other three treatments. Note that 5% inefficiency reflects a potential utility loss of less than 1.5 cents per period.

To summarize, we defined wasted goods as output that does not contribute to any subject's utility, and considered three ways to characterize such inefficiency. We first played the role of social planner, matching wasted goods optimally and relating the derived value of this waste to profits actually attained. We next partitioned wasted goods among individuals. We consider how much each subject would have had to change his output in order to "balance" the goods he wasted, relating the value of such a change to autarky profits. Since *total realized waste* captures failures to coordinate both production and exchange, we isolate the effects. We find that while inefficiencies are substantial early on, they eventually become small, and that most early inefficiencies are due to failures in coordinating exchange rather than production. The economies in this experiment become nearly ex post Pareto efficient, and also ex ante efficient in a limited sense. The mean subject would have little incentive to go back in time and change his output late in the session given the realized production and exchange of others.

4.5 Evolution of Bilateral Exchange in 4-subject Markets

As we have already alluded, specialization and exchange evolves bilaterally in most sessions. To see specific evidence in support of this claim, first note that there are six potential bilateral partnerships in a 4-subject session and 28 such pairs in an 8-subject session. We define trading volume as before, and allocate this volume among the partnerships. For a given session and period, we determine the set of mutually exclusive pairs that produced the largest trading volume. For example, there are three such sets in a 4-subject session, (1-2, 3-4), (1-3, 2-4), and (1-4, 2-3); we determine which one accounted for the highest trading volume in each period. Then, for a given period, we take the sum of these high-volume sets across all sessions in a treatment and divide it by the corresponding sum of total trading volume to determine the mean proportion of bilateral trade for the period. This statistic is bounded from above by one, which implies that trade is entirely bilateral. From below, the statistic is bounded by 1/3 and 1/7 in the 4- and 8-subject treatments, respectively. These bounds reflect perfectly multilateral trade; that is, there is an equal volume of trade between all potential partnerships. Since some of these potential partnerships are same-type pairs, who generally do not trade many goods, we also consider a modified lower bound on trade that assumes all trade occurs between opposite-type pairs only. These bounds are 1/2 and 4/15 for 4- and 8-subject treatments, respectively.[13]

Figure 12 plots this bilateralism measure for the 4- and 8-subject treatments, and the two lower bounds discussed above are marked for easy reference. Clearly, trade becomes quite bilateral in nature by the end of the experiment across treatments, as the measure falls between

[13] It is also possible to reach this bound if some subjects do not trade at all. For example, imagine one odd subject with trading volume split equally among the two even subjects in a 4-subject session, and suppose the other odd subject does not trade at all. Then the "bilateralism" measure is 50%, which is the same as if both odd subjects had split their trading volume equally with both even subjects.

the mid-80s and mid-90s in the final several periods. There is also less variance as the sessions progress, as many subjects establish permanent exclusive trading partnerships. The *Unknown4* and *Unknown8* treatments are quite similar, with significant variation between 60% and 90% prior to the first two rest periods, more stable means between 65% and 80% between the second and fourth rest periods, and stabilizing in the mid-80s to low-90s after the final rest period.

Recall4 becomes predominantly bilateral much more quickly, in the low-90s after the second rest period, but actually becomes a bit more variable throughout the remainder of the experiment (although the treatment generally remains quite bilateral). *Known4* sessions begin similarly to those in *Unknown4* and *Unknown8*, but by the second rest period actually *Known4* sessions become less bilateral through the fourth rest period, actually being only 50-60% bilateral for many periods. These periods correspond to a time when subjects in this treatment were more likely to be engaged in same-type exchange and less likely to trade with others, as well. Immediately after the fourth rest period, the treatment becomes quite bilateral for the remainder of the session, according to comparative advantage. It is unclear why subjects in this treatment were less likely early on to trade according to comparative advantage with stable partners than other sessions. Perhaps having full knowledge of productive capabilities made some of these subjects less likely to experiment with exchange than other subjects early on, causing their coordination of such activity to occur later. The most important point to be made here is that all treatments became quite bilateral by the end of the experiment.

4.6 Terms of Trade

Recall that the competitive price is 4/3; that is, each blue being worth roughly 1.33 reds. How near are the prices that emerged in the sessions to this price? Are emergent patterns of price and specialization in some economies consistent with those of core allocations?

To compute the price for a particular pair of subjects, we first calculate the net trade in red and net trade in blue for one subject in the pair during a specified number of periods. Price is defined when the product of net red and net blue are negative; otherwise, one subject is net gift-giving in this time frame. When defined, price equals the absolute value of net red divided by net blue. We focus on the price that emerges over several periods rather than a single period for two reasons. First, some subjects enter partnerships in which each subject consumes all pair-wise output every other period, so price is not defined in any given period but is reasonably stable when averaged over several periods. Second, complementarity implies that a subject may hold goods at the end of a period which are worthless to himself. He may give these goods to a subject who does value them in hope of reciprocation in subsequent periods. Price computed over several periods may reflect such dynamics.

There are several benchmark prices of particular interest for fully specialized bilateral partnerships, displayed in Table 2. For a price between 5 and 10, the odd subject should be just indifferent between trade and autarky. In this range 50 red units will be wasted unless exchanged with a third party (given our characterization of the core, this circumstance should give impetus to the odd subject to find another partner or produce some units of blue, a fact we will return to momentarily). Similarly, if the price is between 0.17 and 0.36, the even subject will be just indifferent between trade and autarky. These individually rational price bounds are exactly 3.75 times greater or less than the competitive equilibrium price, so we also note the midpoint prices, which are 1.875 times removed. Thus, a price between 0.71 and 2.50 is closer to the CE price than the boundary of the individually rational set; we characterize such a price as "central."

Table 2: Benchmark Prices for Fully Specialized Economy (Value of Blue per Red)

5 to 10	Price range that gives the odd subject exactly autarky utility
2.50	Midpoint price between CE and odd subject's individually rational set
1.33	Competitive equilibrium
0.71	Midpoint price between CE and even subject's individually rational set
0.17 to 0.36	Price range that gives the even subject exactly autarky utility

We have computed prices over the final six periods of each session, one for each of the possible pairs of subjects. We have focused on the end of the sessions for this analysis because the economies are the most bilateral and ex post efficient in these periods, and also involve the largest trading quantities. We argue that a price between two predominantly specialized agents near the end of the session that involves a large quantity transfer, and which has (typically) been conditioned on some history of trading within the pair, is a more interesting statistic vis-à-vis the benchmark prices than one formed between two casual partners trading small quantities of goods for the first time in an early period.[14]

We begin with a discussion of the prices in 4-subject economies, which are displayed in Table 3. We will focus on the highlighted prices in the table, which reflect prices supported by substantial trading volume. Each highlighted cell corresponds to trading volume in excess of 250 over the 6 periods. By way of comparison, the competitive equilibrium trading volume for 6 periods is 420 units. The cells not highlighted have volume less than 180, and the 6 non-highlighted cells with volume greater than 120 correspond to prices 14.55, 7.27, 4.81, 3.19, 0.07, and one undefined; these partnerships are essentially gift-giving arrangements. Therefore, the highlighted cells reflect the only real, substantial trade.

Table 3: Mean Price in 4-Subject Sessions, by Pair

Final 6 Periods																		
Pair	Unknown4						Recall4						Known4					
	S1	S2	S3	S4	S5	S6	S1	S2	S3	S4	S5	S6	S1	S2	S3	S4	S5	S6
(1,2)	NaN	0.56	1.64	NaN	NaN	NaN	1.33	1.06	14.55	NaN	2.00	NaN	1.13	NaN	0.92	NaN	NaN	3.19
(1,3)	2.81	NaN	NaN	NaN	4.16	NaN	NaN	NaN	4.81	5.00	NaN	NaN	NaN	NaN	NaN	6.03	NaN	2.88
(1,4)	NaN	3.26	NaN	1.40	NaN	0.07	NaN	NaN	2.43	1.03	1.06	1.30	NaN	NaN	NaN	NaN	NaN	0.86
(2,3)	NaN	0.02	NaN	1.46	NaN	2.25	NaN	NaN	NaN	NaN	0.94	NaN	NaN	NaN	0.43	2.58	1.00	0.33
(2,4)	NaN	0.91	NaN	NaN	NaN	0.12	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
(3,4)	NaN	7.27	1.57	NaN	0.34	0.77	1.33	0.29	NaN	0.24	NaN	NaN	0.88	1.06	1.40	NaN	1.00	NaN

We make several observations regarding these prices. Twenty-two pairs (of 36 potential mutually exclusive pairs) meet the volume restriction. One of these pairs was a same-type pair (odd-odd). This pair remarkably traded in the very narrow contract set for an odd-odd pairing. Both fully specialized, the one producing red typically made 3-6 cents more per period than in autarky, the one producing blue typically made 0-3 cents more than in autarky. Of the other 21 pairs, 17 fall into the central range of individually rational prices. This fact was expected; it is risky to specialize, and if one is only averaging slightly more than autarky utility by trading, it may be better to produce and consume (safely) in autarky.

[14] It should be noted that in comparing the price from the final six periods for a "high volume pair" (a pair who traded quantities above a certain threshold) to the quantity-weighted price at which this pair traded over all 40 periods, there is typically not much difference between the two prices at all. In cases where there is a difference, there is no clear pattern regarding the relationship of these prices to the competitive equilibrium price; that is, price can move toward or away from competitive equilibrium over time. We have chosen 6 periods rather than 5 (the latter corresponds to the number of periods following the final rest period) because two pairs of subjects engage in partnerships where each consumes all pair-wise output every other period; thus, an even number of periods will better reflect the true mean price.

We know from Section 3.3 and the predominantly bilateral nature of exchange that candidate economies for the core should have two high-volume pairs with both prices above or below 4/3. Mutually exclusive high-volume pairs form in 7 of the 18 sessions, and in 6 of these cases, prices are either competitive or on the same side of the competitive price. In the 7th case, prices are 0.92 and 1.40, and the latter price actually drops to 1.27 while the former remains the same if we compute price over the final 5 periods. While not all of these economies are actually in the core, the prices are suggestive that the competitive pressures that motivate the theoretical core may be at work in these economies.

Three economies actually are in the core (or extremely close). Session 1 of the *Recall4* treatment implements the competitive equilibrium in the final 11 periods, and Session 4 of the *Unknown4* treatment is quite close. In the fourth and fifth-to-last periods, one odd-even pair earned (90,80) per period and the other earned (84,82) per period. In the final three periods, both pairs earned (84,82) per period. This allocation is in the discrete core of the experiment (as opposed to the continuous one described in 3.3): Each odd subject wastes 5 units of red, but the marginal production cost of one blue unit exceeds 10 reds for a fully specialized odd subject. Session 5 of the *Recall4* treatment is just outside the core. Both odd subjects fully specialize in red, and each even subject produces 101 blues and 1 red. One odd-even pair earned (99,64) in the final period, so they wasted no red and 4 blues. The other pair earned (96,68), so they wasted 1 red and 1 blue. Thus, the outcome is not quite Pareto efficient; either subject in the latter pair could have earned 1 additional penny by consuming their wasted goods, or an even subject could have made 2 extra pennies by consuming all wasted goods. However, this amount is trivial, and no change in production could have better "balanced" the waste. Finally, the even subject who traded 33 blues for 32 reds could not form a coalition with the odd subject who traded 34 reds for 32 blues, because to make that subject better off he would need to send 33 blues, leaving himself with 68.

One final observation is that when only one significant pair forms in a session, the other mutually exclusive (potential) pair is inactive. In 5 of 7 cases (excluding the session with the odd-odd pair, since an even-even pair cannot achieve autarky utility for both subjects), the off-pair did not trade apart from some small gifts, and in the other two cases the prices were 0.02 and 0.33, which are essentially gifts by the even subject. In fact, four of these seven off-pairs do not have a price defined for the session in its entirety, and the other four have "gift prices" of 0.11, 0.07, 0.50, and 13.80 with very low volume. This price and volume data further corroborate our earlier observation that specialization and exchange typically occur in large jumps; stable partnerships where both members partially specialize are rare. In 4 of these 7 one-pair sessions one subject does not trade at all in the final 6 periods, which is also the case in 1 of the 3 sessions where no high-volume pairs form. Some subjects are simply resigned to autarky.

Prices in 2-subject economies for high volume pairs over the final 6 periods were very close to the competitive price. Four of the six sessions engaged in exchange. Three of those four pairs fully specialized, while the fourth produced near autarky and only exchanged small quantities of goods. The prices for the three fully specialized pairs were 1.33, 1.31, and 1.49 on trading volumes in excess of 375. Trading volume for the fourth pair was only 46. In fact, the pair that traded at 1.49 was actually closer to CE than this price suggests. In one period they compared earnings and determined that the odd subject had earned more money, after which the odd subject decided to send his entire output of reds to the even subject. He offered to do the same in the following period, but the even subject declined!

Only 5 partnerships emerged in *Unknown8* that meet the requisite volume threshold of 250. As in the 4-subject case this threshold is quite robust; the highest excluded trading volume is 115, and the lowest included volume is 348. Prices for the high-volume pairs in the final 6 periods were 0.61, 1.48, 2.25, 0.94, and 0.82. The first three pairs were in the same session (ex ante efficiency in the session was 0.6, the other five were below 0.25), and the other two were in two different sessions. Four of these prices are closer to the competitive than autarky price, but only one is anywhere near the competitive equilibrium. There is substantial dispersion of prices in the session with three significant partnerships; we detect no evidence of the co-evolution of these prices, as (casually) appears to be the case in 4-subject sessions. We speculate that there is too much information in the chat room to observe closely what others are doing.

The sample is too small to make strong claims with regards to why the specialized pairs in 2-subject economies typically traded closer to the competitive price than pairs in larger economies. However, it is difficult to ignore the fact that only 5 of 21 odd-even pairs in the 4-subject treatments and 1 of 5 odd-even pairs in the 8-subject treatments establish prices as close to the competitive price as the 2-subject pair *farthest* from this benchmark. Economic intuition suggests the reverse relationship, as we might expect competitive pressure (i.e., the potential ability to enter alternative partnerships) to drive prices towards the competitive equilibrium.

Crockett (2005) observes several 2 × 2 economies operating under an alternative institution where subjects can be extremely demanding in their terms of exchange, exactly the opposite of what we observe here. It seems plausible that the interplay of the chat feature and the "ah-hah" discovery nature of this experiment created a team-oriented approach to decision-making in 2-subject economies that was diminished in larger economies, although this is purely speculative. Also, as discussed above with respect to *Unknown8*, the cognitive tasks of maintaining discourse is the hardest with eight subjects and is clearly the easiest with two. This suggests that formal market institutions, such as the double auction, play a role in reducing conversational noise in coordinating the activities of larger groups attempting to extract gains from exchange and specialization.

5. Conclusions

In this paper we report an experiment on the minimal structure necessary to support exchange and specialization. Our subjects must first discover that exchange is a feasible activity, enabling value to be created by supporting specialization, and then to develop the limits to that discovery; to achieve this, they must ascertain their particular circumstances and unknown possibilities for production. We find that a small majority of subjects either settle into autarky or specialize in their comparative advantage immediately. The rest typically do so over the course of time, and most trade occurs between appropriately specialized subjects of the opposite type. With ex ante unknown production functions, we observe the following results on the effect of the economy size on exchange and specialization. None of the 2-, 4- and 8-person economies rise above the autarky equilibrium in periods 1-6, and most continue this performance into periods 7-12 (Figure 9). In 2-person economies, three pairs discover and achieve the competitive equilibrium by the 40th period; it took 28 periods for the three pairs to establish their like-minded willingness to explore exchange and specialization, and step out of their autarky homes; and having done so, they almost immediately find the competitive equilibrium. Three pairs discover the autarky equilibrium by the 20th period, but are content to hover near it thereafter (Figure 9a).

With four subjects, only one session discovers and achieves complete specialization; one session fails completely; and the remaining four sessions cluster in the range of 25% to 75% of full efficiency at the competitive equilibrium (Figure 9b). An inference from comparing 4-person with 2-person economies is that the existence of more potential partners is detrimental to the full development of building an exchange and specialization relationship between individual pairs, even though exchange is commonly discovered in economies with more than two people. It also appears that with four people it takes longer to develop and grow a bilateral relationship of exchange and specialization. When again doubling the size of the economy to eight people, no session achieves complete specialization: three sessions lock on to autarky; two achieve 20% of the available surplus; and one about 60%. (Figure 9c) These 8-person sessions are remarkably self-reliant and hence grossly inefficient. These regularities are further explicated by the finding that although the proportion of subjects who engage in some specialization in 4-person economies rises to about 70%, while 2-person economies are stuck at 50% (Figure 5), the proportion of subjects who trade conditional on discovering exchange stabilizes much earlier at about 80%, in 2-person than in 4-person economies (Figure 10). On both of these measures 8-person economies underperform 4 and 2-person economies (Figures 5 and 10). Moreover, the time paths of ex-post efficiency, given production decisions, decrease monotonically with the size of the economy (Figure 11). Providing more ('recall') or complete information on production possibilities does not in any meaningful way alter these observations[15] (Figures 5, 9 and 10).

In this experiment we find that there seems to be three learning steps to achieving the competitive equilibrium in these exchange and production economies: (1) arriving at the idea to trade, which may require "mind reading" (inferring thoughts from words and actions) and imitation (limited to 4- and 8-person economies), (2) finding a suitably endowed trading partner with whom one can exploit the power of exchanging through specialization, and (3) building the relationship by increased specialization over time. In the larger economies imitation can hasten the discovery of exchange, but (2) and (3) can conflict and increase waste: effort spent searching for the "best" partner match reduces effort invested in developing the exchange-specialization relationship with any one partner.

The following excerpt from a transcript demonstrates how quickly, upon achieving step 1, two people alone in Session 6 of the *Unknown2* treatment can complete the next two steps and achieve complete specialization:

***** Period 18-B *****
2: wonder if u can give me objects

[15] If anything can be said, it may appear that complete information creates a mental framework that hinders subjects from leaving autarky.

1: oh yeah..
2: heyyy
2: i make blues faster
2: what color do u make?
2: faster
1: red
2: lol[16]
2: ok
1: LOL
2: so ill make all blues
2: and u make all reds
1: then drop them to each other's houses?
2: yea
2: do it
1: ok
1: 100% red
2: 100% blue
[The subjects achieve complete specialization in the next period.]

The tone in the above excerpt also clearly indicates that the emergence of a natural propensity to exchange is indeed personal and social.[17] As the findings in our larger economies indicate, exchange in this setting is increasingly bilateral over time, i.e., not of the multilateral sort usually associated with markets. Thus, none of our economies discover multilateral 'double auction' trading through, say, an extended order of bilateral negotiation to explore and facilitate pairing. Why? Is it "unnatural" to thereby depersonalize the exchange-specialization relationship, and perhaps disrupt reputation formation, as in finding a way that "maybe we could help each other?" Is the "double auction" an example of a constructivist formal institution, the product of entrepreneurial "invention" which is then selected wherever it has fitness advantages? Is it simply that necessity is the mother of invention and our environment demands only a bilateral structure to solve the optimization problem?

Our experiment is but a first step in understanding the dynamics of human behavior in personal social exchange and its extension to much more abstract institutions of impersonal

[16] The shorthand "lol" stands for "laugh out loud".
[17] And it is frequently conveyed explicitly as such:

Session 4, *Unknown2*
***** Period 17-B *****
Person 2: can i put them things in your house?
Person 1: i dont think so
1: oh thats ur house
***** Period 18-B *****
2: you can put them in each others house
1: oh ok
2: do you know which one you make faster
***** Period 19-B *****
2: im trying to think how maybe *we could help each other* [italics added]

market exchange. This central question is often implicitly posed or thought of as one of cooperative versus non-cooperative behavior or personal *versus* impersonal exchange, when, to the contrary, the point is to understand the terms of the *trade-off* between engaging in personal and impersonal exchange. The discovery-based framework of this experiment provides a foundation on which to vary those terms in an incremental way, thereby shedding new light on the means by which exchange and specialization create wealth.

Figure 1: Production Possibilities Frontier

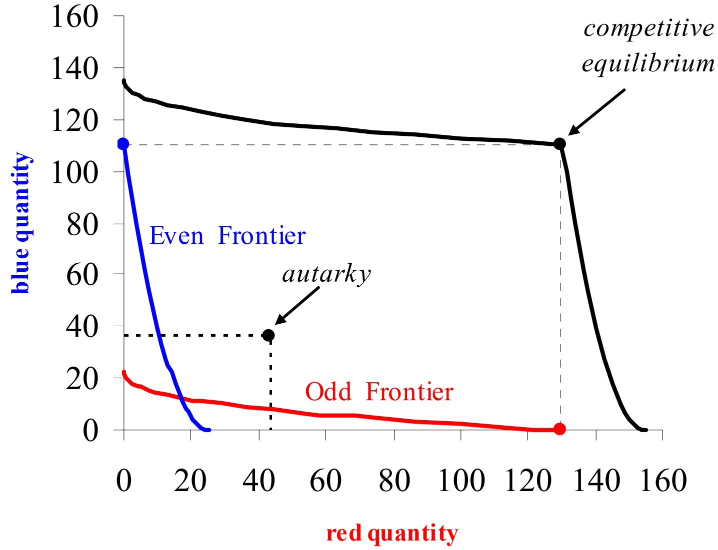


Figure 2: Example of a Subject Interface (*Unknown8* Treatment)

Figure 3: Ex post Pareto Set for Fully Specialized Economy

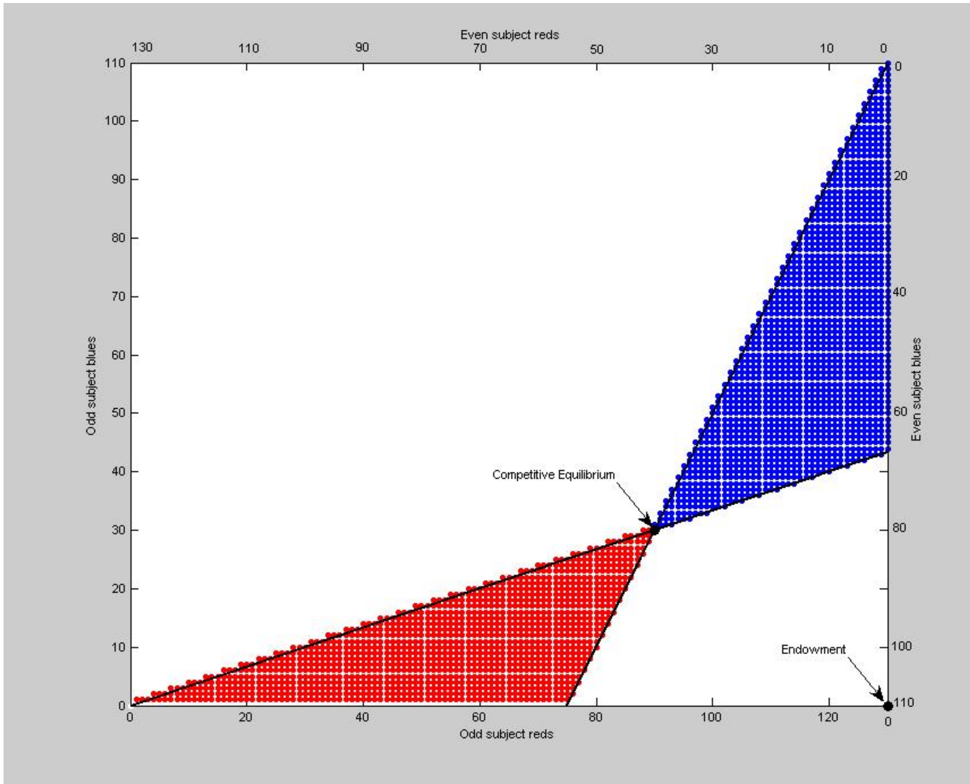


Figure 4: Subject Output Tables (*Odd* subjects first, *Even* second)

Production Table

Production			
Blue %	Red %	Blue	Red
0%	100%	0	130
5%	95%	1	114
10%	90%	2	100
15%	85%	3	87
20%	80%	5	74
25%	75%	6	63
30%	70%	7	53
35%	65%	8	44
40%	60%	9	36
45%	55%	10	29
50%	50%	11	23
55%	45%	12	18
60%	40%	14	13
65%	35%	15	9
70%	30%	16	6
75%	25%	17	4
80%	20%	18	2
85%	15%	19	1
90%	10%	20	0
95%	5%	21	0
100%	0%	23	0

Production Table

Production			
Blue %	Red %	Blue	Red
0%	100%	0	25
5%	95%	0	24
10%	90%	1	23
15%	85%	2	22
20%	80%	4	20
25%	75%	7	19
30%	70%	10	18
35%	65%	13	16
40%	60%	18	15
45%	55%	22	14
50%	50%	28	13
55%	45%	33	11
60%	40%	40	10
65%	35%	46	9
70%	30%	54	8
75%	25%	62	6
80%	20%	70	5
85%	15%	79	4
90%	10%	89	3
95%	5%	99	1
100%	0%	110	0

Figure 5: Proportion of Specialists

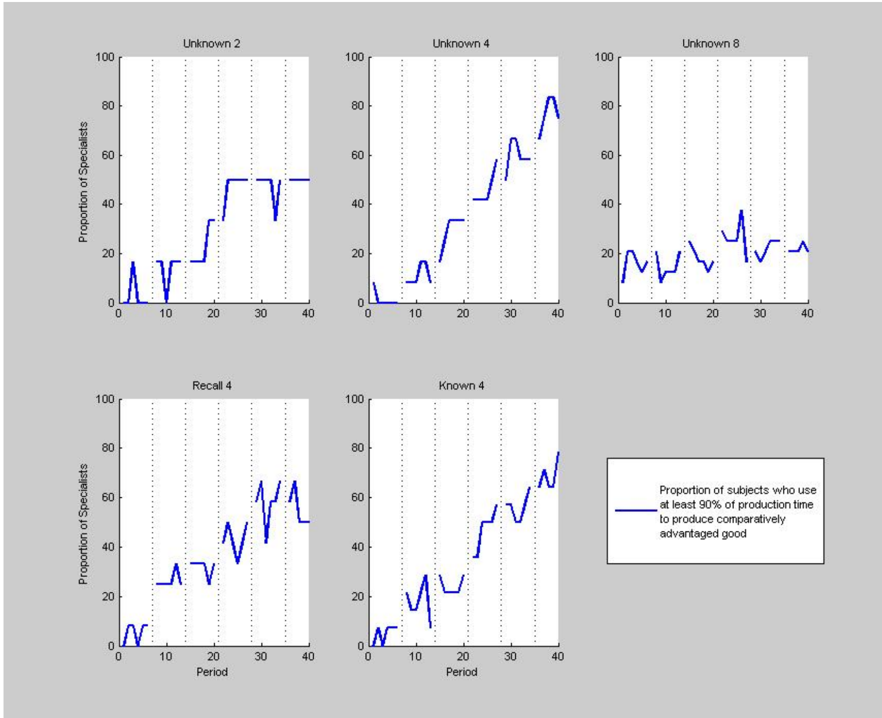


Figure 6: Trade in Comparative Advantage

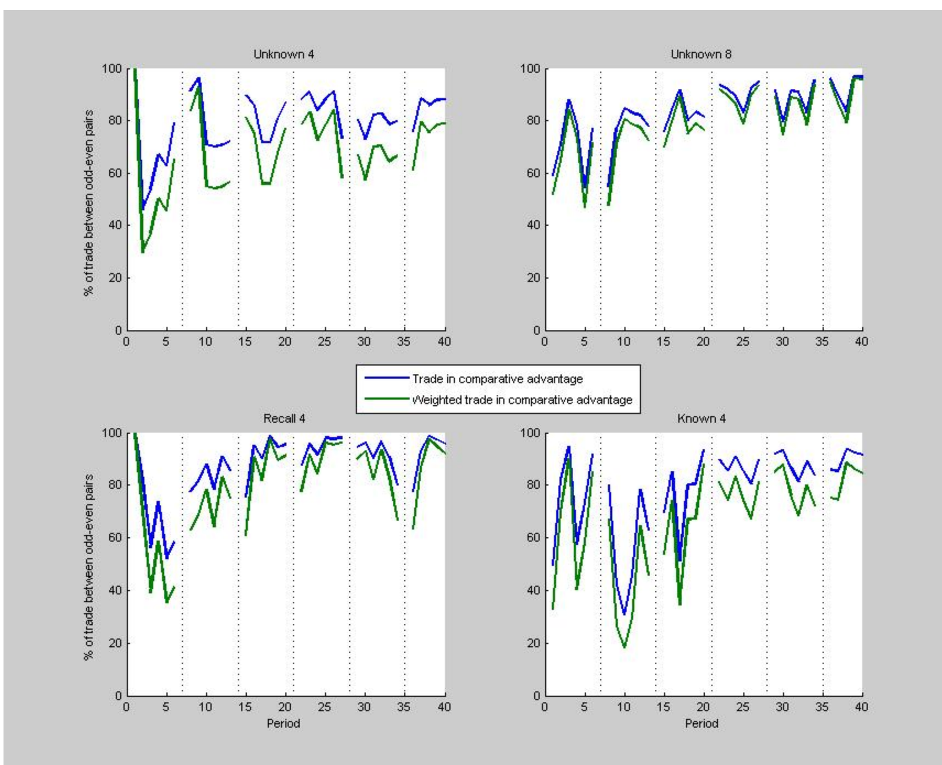


Figure 7: Mean Efficiency of Competitive Equilibrium Profits Relative to Autarky

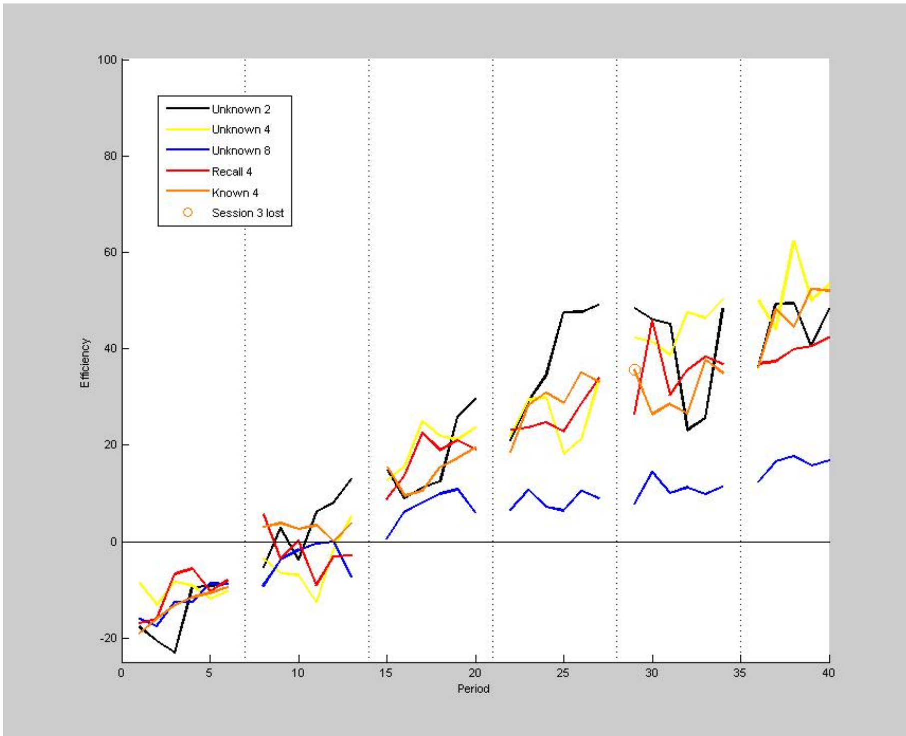


Figure 8: Distribution of Profits, Beginning and End of Session

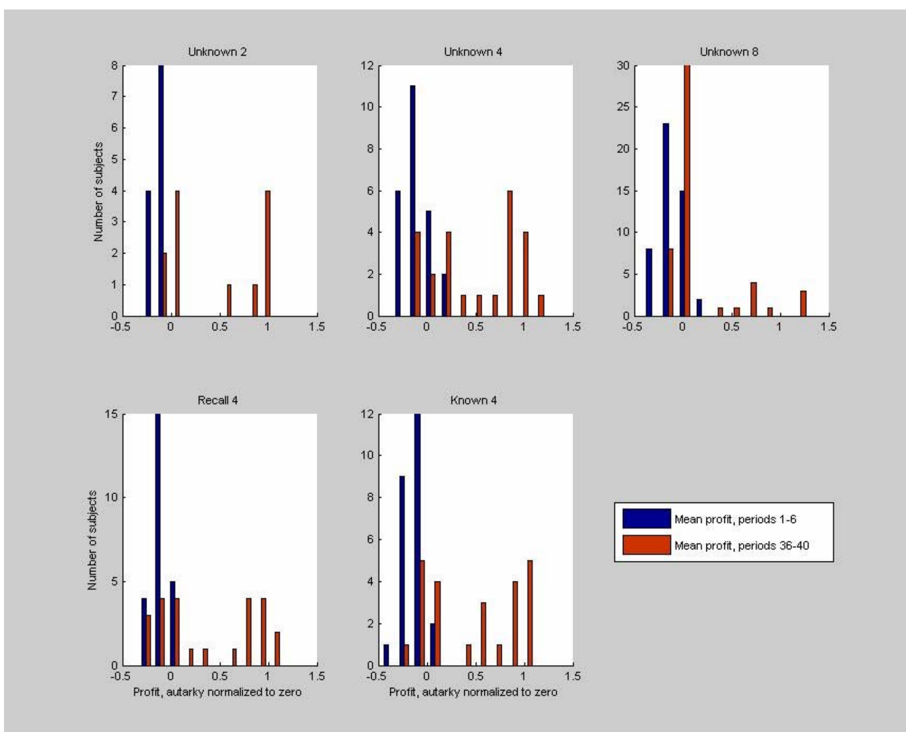
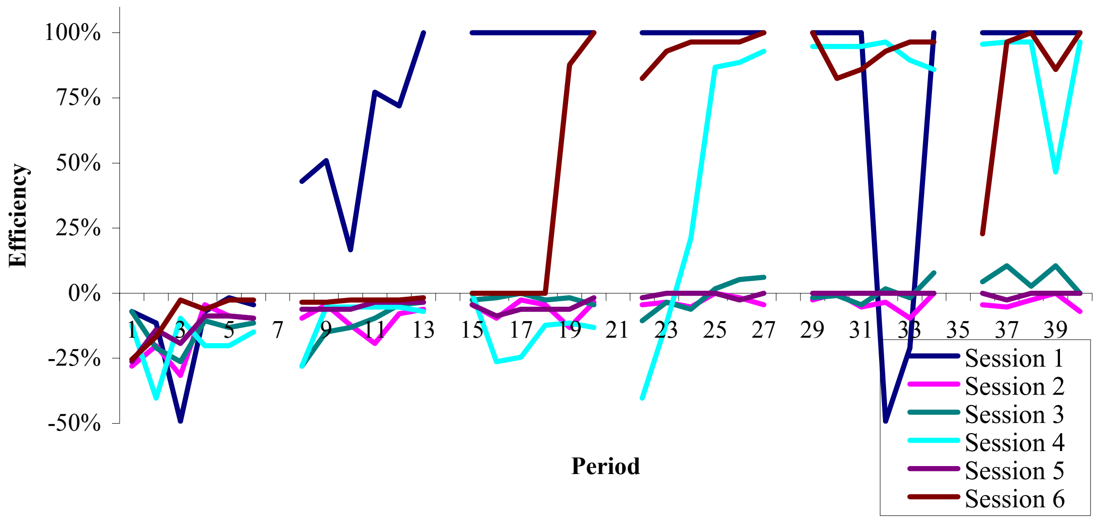
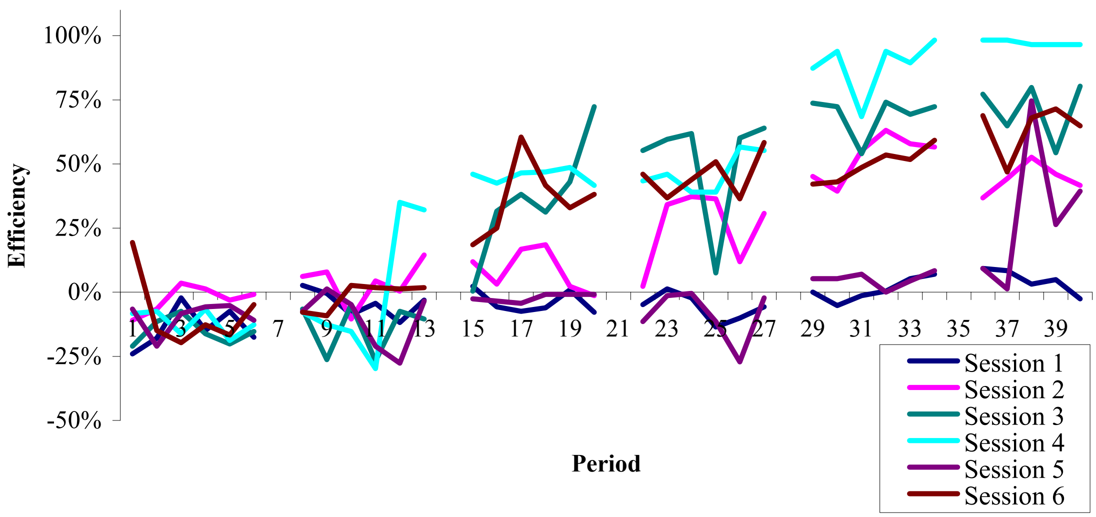


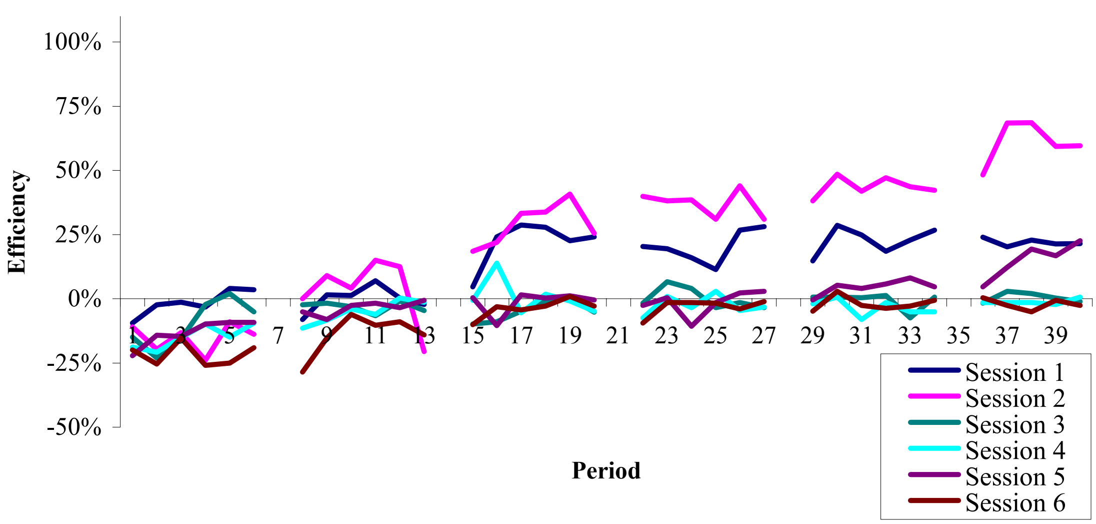
Figure 9a: Efficiency of Competitive Equilibrium Profits Relative to Autarky



Panel (a) *Unknown2*

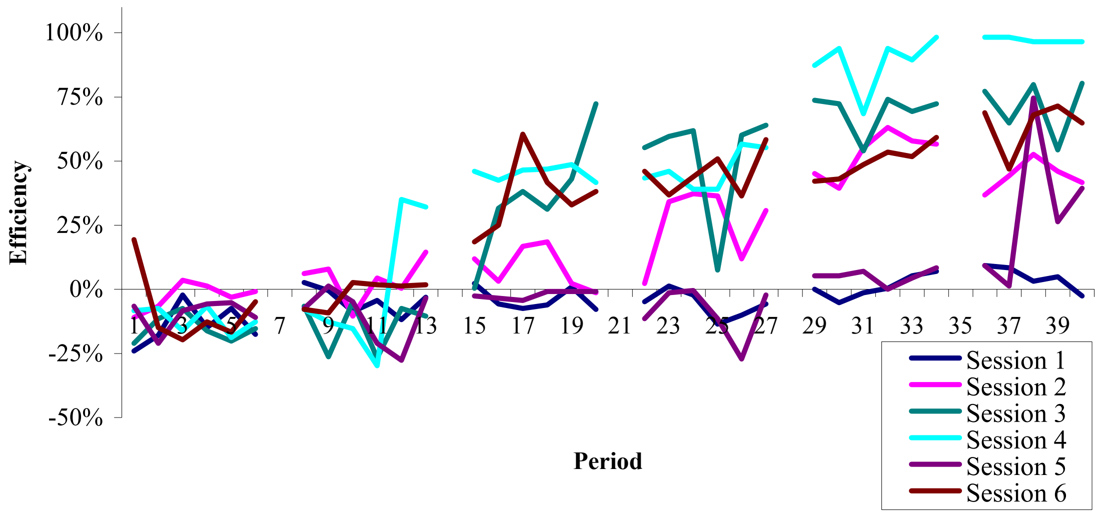


Panel (b) *Unknown4*

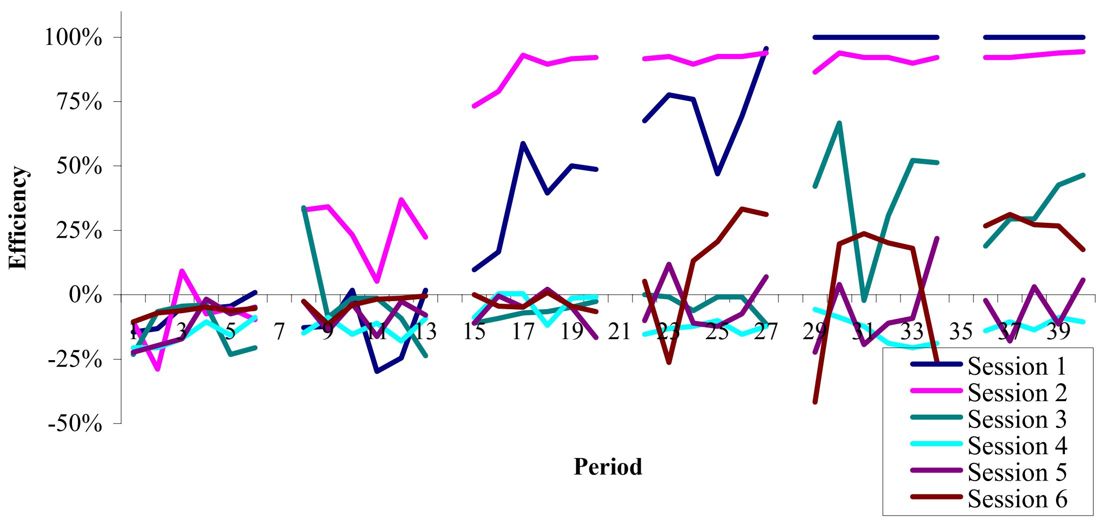


Panel (c) *Unknown8*

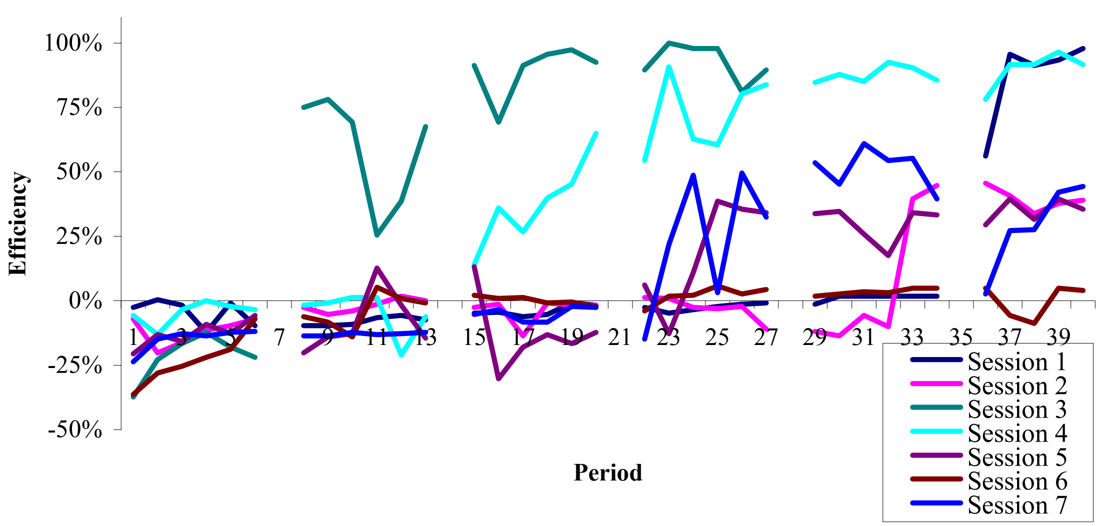
Figure 9b: Efficiency of Competitive Equilibrium Profits Relative to Autarky



Panel (a) ***Unknown4***



Panel (b) ***Recall4***



Panel (c) ***Known4***

Figure 10: Subjects Who Trade after Ability to Exchange is Discovered

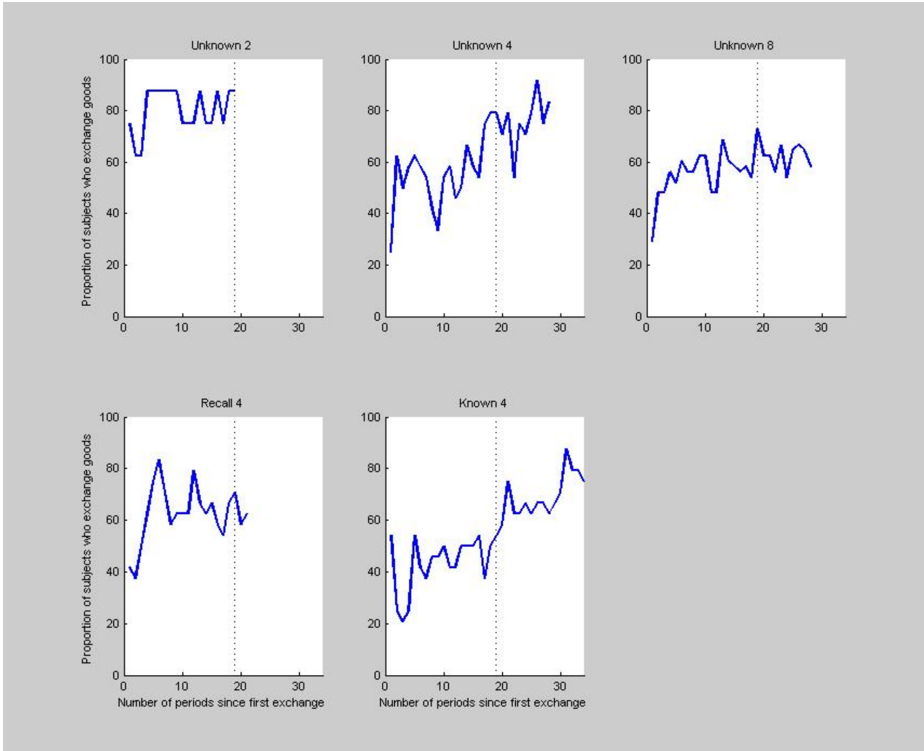


Figure 11: Ex-post Efficiency and Waste

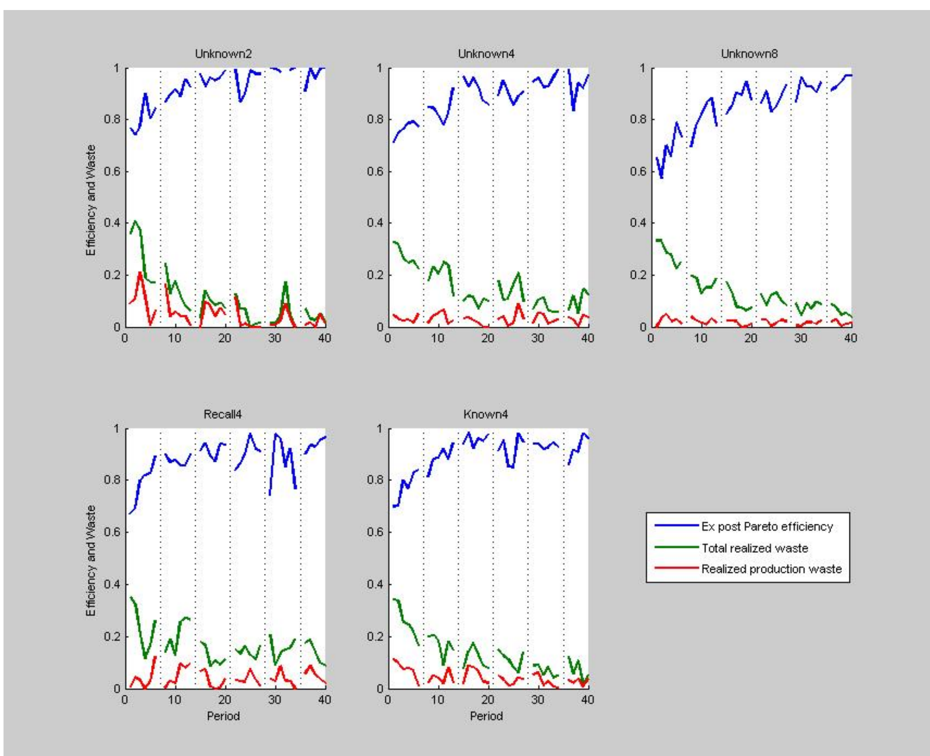
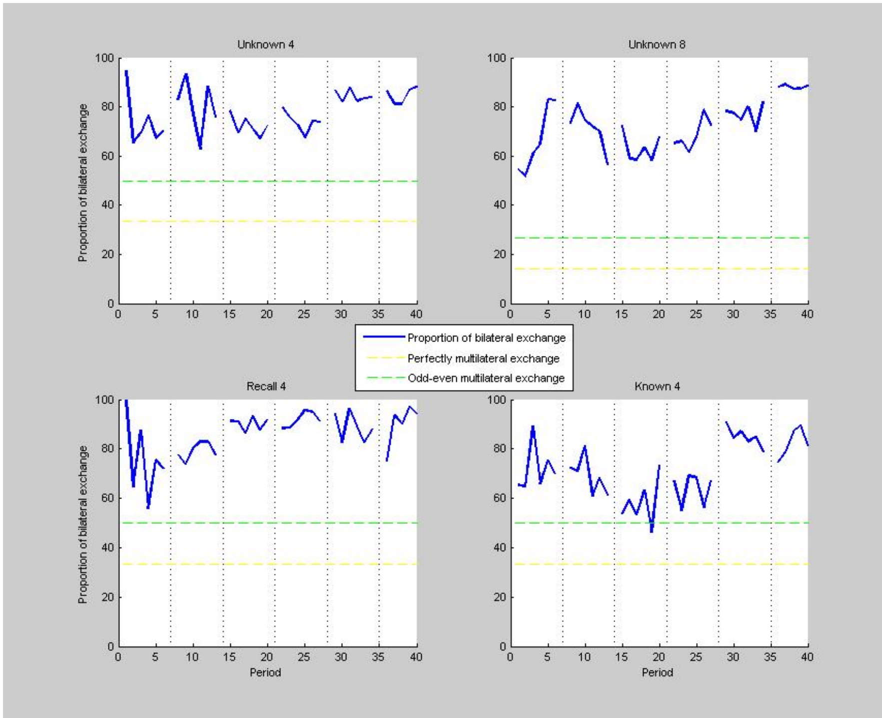


Figure 12: Evolution of Bilateral Exchange



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Appendix I

Definitions of Pareto optimality

Definition: *Ex post Pareto optimality.* Let $W(\bar{t})=[w_1,w_2,\dots w_N]$ be the output matrix which corresponds to production determined by $\bar{t}=\bar{t}_1,\bar{t}_2,\dots,\bar{t}_N$. An allocation X such that $\sum_{i=1}^{N} x_i=\sum_{i=1}^{N} w_i$ is ex post Pareto optimal if and only if there does not exist an allocation Y such that (1) $\sum_{i=1}^{N} y_i=\sum_{i=1}^{N} w_i$; (2) $u(y_i)\geq u(x_i)$ for each $i = 1,2,\dots,N$; (3) $u(y_i)>u(x_i)$ for some i.

Definition: *Ex ante Pareto optimality.* Let $W(\bar{t})=[w_1,w_2,\dots w_N]$ be the output matrix which corresponds to production determined by $\bar{t}=t_1,t_2,\dots,t_N$. An allocation X such that $\sum_{i=1}^{N} x_i=\sum_{i=1}^{N} w_i$ is ex ante Pareto optimal if and only if there does not exist a feasible allocation Y such that $u(y_i)\geq u(x_i)$ for each $i = 1,2,\dots,N$ and $u(y_i)>u(x_i)$ for some i. Here feasibility implies $\sum_{i=1}^{N} y_i=\sum_{i=1}^{N} \omega_i$ for any $W(t)=[\omega_1,\omega_2,\dots \omega_N]$ such that $t_i\in[0,10]$, $i = 1,2,\dots, N$.

Characterizing the ex ante Pareto set

Fact 1: *No more than one subject of each type may choose to produce both goods.* Consider two odd subjects with interior production times $t_1\leq t_2$. If these subjects would instead spend $t_1-\varepsilon$ and $t_2+\varepsilon$ in production (where $\varepsilon\leq min\{t_1,10-t_2\}$), the total amount of blue produced would remain the same, as blue is a linear function of time for odd subjects. However, by convexity of red production, the net addition of reds produced by subject 2 is strictly greater than the net loss of reds produced by subject 1, so more reds are produced overall. By continuity of production, there exists $\delta>0$ such that subject 2 could spend $t_2+\varepsilon-\delta$ time producing reds, and the total output of each good would be greater than at t_1 and t_2. But then there trivially exists a Pareto-improving allocation. The argument is similar for even subjects. ■

Fact 2: *It cannot be the case that both an odd subject specializes in blue goods and an even subject specializes in red goods.* If two subjects specialize opposite their comparative advantage, their total pair wise output is roughly 22.5 reds and 25.3 blues. The pair wise competitive output is 130 reds and 110 blues, which can Pareto-dominate any allocation resulting from specialization opposite of comparative advantage. ■

Fact 3: *If R and B are the total amounts of red and blue goods produced, then 0.5B≤R≤3B.* If $R<0.5B$, then if even subjects consume all output, some blue units will be wasted. If odd subjects (who consume blues in lesser proportion to reds) consume some of the output, even more blue units will be wasted, so this total output cannot support an ex ante Pareto optimal allocation. Similarly, if $R>3B$, then some red units must be wasted. ■

Fact 4: *An odd and even subject may not each produce both red and blue goods, unless both almost completely specialize in producing the same good.* Suppose $t_{odd}, t_{even} \in (0,10)$, and let ε be the quantity of blue goods that the odd subject would forego in moving from t_{odd} to 10. The gain in the quantity of red goods is:

$$\Delta R_{odd} = 130 - \frac{13}{10\sqrt{10}} \cdot \frac{100 - \varepsilon\left[10 - \left(\frac{300\sqrt{10}}{13}\right)^{2/5}\right]^{5/2}}{10}$$

For the even subject to accommodate the loss of blue units, he must decrease his time producing red. Since his production of blue is convex, the most time it could take to create ε new units of blue is for $t_{even} = 10$, and since his red production technology is linear, at this time is reflected the largest cost in reds of adding ε new units of blue. The maximum loss of red units is:

$$\Delta R_{even} = -\frac{13\sqrt{\frac{10\varepsilon}{11}}}{10 - \left(1 - \sqrt{\frac{13}{55}}\right)}$$

It can be shown numerically that $\Delta R_{odd} + \Delta R_{even} > 0$ for ε greater than (approximately) 0.0284, which corresponds to $t_{odd} < 9.9876$. By continuity of production, this number can be decreased by an arbitrarily small amount so that the total amount of blue and red produced by the two subjects can be increased relative to t_{odd} and t_{even}. Thus, the odd subject must produce at least 129.25 units of red and no more than 0.03 units of blue in order to potentially support an ex ante Pareto optimum in which an even subject produces both goods, and this only when the even subject is producing a negligible amount of blue. Note that in the experiment a subject can only choose times in increments of 0.1, so a supportable pair of non-specialists requires that the odd subject spend less time in producing blues than the grid in the experiment allows.

To complete the proof, we consider a quantity δ of red goods that the even subject would forego in moving from t_{even} to 0. The gain in blue goods is:

$$\Delta B_{even} = 110 - \frac{11}{10}\left[10 - \frac{10 - \sqrt{\frac{260}{11}}}{13}\delta\right]^2$$

For the odd subject to accommodate the loss of red units, the most time it could possibly take to create δ new units of red is if $t_{odd} = 0$. The ensuing loss of blue is:

$$\Delta B_{odd} = -\frac{10\left(\frac{10\sqrt{10}}{13}\delta\right)^{2/5}}{10 - \left(\frac{300\sqrt{10}}{13}\right)^{2/5}}$$

It can be shown numerically that $\Delta B_{even} + \Delta B_{odd} > 0$ for δ greater than (approximately) 0.1918, which corresponds to $t_{even} > 0.0758$. Analogous to the previous case, the even subject must therefore spend less time producing red than the grid in the experiment allows. ■

It is thus with little loss of generality to state that $N - 1$ subjects must be fully specialized to support an ex ante Pareto optimal allocation. It is possible for $N - 2$ subjects to be fully specialized when 2 subjects of opposite type are almost completely specialized in the same good, but this circumstance requires a production grid finer than the one adopted in the experiment.

We further characterize the ex ante contract set, which is the subset of the ex ante Pareto set that yields at least autarky utility to each subject. In two-subject economies, it is now trivial to show that at least one subject must specialize in his comparative advantage. We have two cases: (1a) The odd subject specializes in red and the even subject spends less than $t \approx 2.2092$ time producing red; (1b) The even subject specializes in blue and the odd subject spends at least $t \approx 8.3126$ time producing red (CE is a special case of (1a) and (1b)). The allocation implied by (1a) and (1b) is uniquely determined by the allocation of time by the non-specialist, with the specialist earning greater than competitive utility.

In 4-subject economies, it is easy to show that at least 3 subjects must fully specialize in their comparative advantage to support an allocation in the ex ante contract set. We have two cases for the fourth subject: (2a) If the subject is even, he must spend less than $t \approx 5.2848$ time producing red; (2b) If the subject is odd, he must spend at least $t \approx 3.8470$ time producing red (CE is a special case of (2a) and (2b)). When output satisfies (2a) or (2b), there is a unique allocation of red and blue goods to odd and even types, although the distribution within-type must only satisfy the no waste condition and provide each subject with at least autarky utility.

When N = 8, at least two subjects of each type must specialize in their comparative advantage, and all subjects of one type must specialize in their comparative advantage. There are four cases: (3a) 4 odd specialists in red, 3 even specialists in blue, and 1 even specialist unrestricted; (3b) 4 even specialists in blue, 3 odd specialists in red, and 1 odd subject unrestricted; (3c) 4 odd and 1 even subject specializing in red, 2 even subjects specializing in blue, and 1 even subject spending less than $t \approx 3.1894$ time in red; and (3d) 4 even and 1 odd subject specializing in blue, 2 odd subjects specializing in red, and 1 odd subject spending at least $t \approx 7.9521$ time in red. Competitive equilibrium is a special case of (3a) and (3b).

Appendix II

Solving for *Waste* and *Realized Production Waste*

Definition of Waste (W): We have the following four cases:

- **Case 1:** $r_w\, b_w = 0$, or $r_w = 1$, $b_w = 1,2$: $W = 0$ (no inefficient waste)
- **Case 2:** $r_w \geq 3b_w$: $W = 3b_w$
- **Case 3:** $b_w \geq 2r_w$: $W = 2\, r_w$
- **Case 4:** $3b_w > r_w > 0.5b_w$: W solved algorithmically (all waste is inefficient)

For simplicity, we have assumed that wasted goods are not matched to goods that were only partially matched during a subject's consumption (e.g., the half unit of the red good in the example above). To solve for waste in Case 4, we begin by assigning the entire amounts r_w and b_w to an odd subject. Note that this allocation would give the subject an additional r_w cents, since $3b_w > r_w$. Now, consider giving an even agent 2 units of blue and 1 unit of red from this reallocated waste. If $r_w \geq 3b_w - 3$, we do not give these proposed units to the even subject, because the marginal social cost is greater than the marginal social benefit (the cost to the odd subject would be between 3 and 5 cents, and the benefit to the even subject would be 2 cents). If $r_w \geq 3b_w - 5$, we do reallocate the goods to the even subject, because the benefit to the even subject is 2 cents, and the cost to the odd subject is only 1 cent. If $r_w = 3b_w - 4$, the net social benefit of the proposed exchange is zero. Without the exchange, the odd subject wastes 4/3 units of blue, and with the exchange he will waste 1 unit of red, so we assume the exchange is made, thus minimizing the number of wasted goods. This process is iterated until the number of wasted reds held by the odd agent is greater than or equal to three times the number of wasted blues minus 4. W is then the total social benefit of the final allocation of wasted goods. At most, 1 unit of red or 1 unit of blue will be wasted under this final allocation. We would obtain roughly the same allocation of the wasted goods by assigning all waste to the even subject initially.

Definition of *Realized Production Waste*: In Case 1 as defined above, the economy is exchange-efficient, so *realized production waste* = *realized waste*. In Case 4, all waste is exchange-inefficient, so *realized production waste* = 0. In Cases 2 and 3, we reassign the inefficient waste algorithmically, giving priority to odd subjects in Case 2 and evens in Case 3. We sketch the algorithm for Case 2; Case 3 is solved analogously. First note that $r_w \geq 3b_w$. Therefore, we must rid the individual waste accounts of all blue units and three times that many red units (i.e., after the reallocation, $b^{a}{}_{w_i} = 0$ for all i and $\sum_{\{i\}} r^{a}{}_{w_i} = r_w - 3b_w$). There are 5 potential steps, but the algorithm halts once all wasted blues have been reallocated. In each step, we cycle sequentially through subjects of the appropriate type (e.g., for odd subjects, beginning with 1, 3, 5, 7) for practical reasons. (1) Let each odd subject eliminate waste he could have cleared himself, in clusters of three reds for one blue. For example, if an odd subject wasted 15 units of red and 3 units of blue, delete 3 blues and 9 reds from his waste account, leaving him with 6 wasted units of red. (2) Let each odd subject eliminate the remaining wasted red units with the wasted blue units of others, in a 3:1 ratio. Now all (but perhaps 1 or 2) wasted red units for odd subjects have been eliminated. The algorithm will typically halt by this stage, since $r_w \geq 3b_w$ and wasted red units are generally held in largest quantities by odd subjects. (3) Let each odd subject eliminate his remaining wasted blue units with the remaining wasted red units of others, in a 1:3 ratio. Now all wasted blue accounts for odd subjects have been wiped clean. (4) Let each even subject eliminate remaining waste he could have cleared himself, in batches of two blues and one red. (5) Let each even subject eliminate remaining wasted red units with the remaining wasted blue units of other even subjects, in a 1:2 ratio. If this stage is completed, now evens can only hold wasted blues, and odds can hold at most 2 wasted reds each. (6) Finally, let each even subject eliminate his remaining wasted blue units with the remaining wasted red units of the odd subjects, combining red units of odd subjects, if necessary.

Instructions for "Exchange and Specialization as a Discovery Process"

Ex Ante Unknown Treatment with 4 Agents

<page 1>

This is an experiment in the economics of decision making. The instructions are simple, and if you follow them carefully and make good decisions you may earn a considerable amount of money which will be paid to you in CASH at the end of the experiment.

In this experiment you are **Person 1**. You and the other **3** people in this experiment each have the ability to produce two fictitious items: red and blue. For the first **10** seconds of each period, you will produce items in the upper left portion of your screen. Using the scroll bar, you can change the proportion of each second allocated to producing red and blue. Each person's production is displayed in the domino-shapes at the bottom of your screen.

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After the production phase ends, the period continues for another **90** seconds. When the clock expires, you earn cash based upon the number of red and blue items that have been moved to your house. To select items to be moved, *left* click on an item or click on the red or blue buttons at the top of the screen. The yellow highlighted items can be moved by dragging with the *right* mouse button. (You cannot move items until the experiment has started.)

The specific information on how the red and blue items in your house generate earnings is given in the upper right corner of your screen. You personally earn (in cents) the minimum of the following two numbers:

number of red items,
3 times the number of blue items.

Or, think of it this way. You earn by consuming what's in your house in the proportion of 3 red to 1 blue items. For every 1 unit of red you need 3 units of blue to earn 3 cents. Your potential profit updates as items, unit by unit, are moved into your house.

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Everyone in this experiment can send text messages. Everyone can read all posted messages. In the center of the screen, you can type a message in the line in either of two chatrooms and click on the Send button.

You are free to discuss all aspects of the experiment, with the following exceptions: you may not reveal your name, discuss side payments, make threats, or engage in inappropriate language (including such shorthand as 'WTF'). If you do, you will be excused and you will forfeit your earnings.

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During the experiment, every 7 periods will be a "break period" in which nobody produces anything but that the chatrooms are still open.

This is the end of the instructions. If you wish to explore how you produce red and blue items, click the Practice button. You may change the proportion of time allocated to producing red and blue items using the scroll bar, and you may Practice as many times as you wish. (You will not be able to move items until the experiment has begun.)

If you wish to review the instructions, you may go back at this time. If you feel you are prepared to proceed with the actual experiment, click on the Start button. The experiment will begin once everyone has clicked on the Start button. If you have a question that you feel was not adequately answered by the instructions, please raise your hand and ask the monitor before proceeding.