Extrapolative Expectations and the Equity Premium

James J. Choi
Harvard University

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Abstract

Survey and experimental evidence suggest that most stockholders believe high past market returns predict high future market returns, even though the opposite is true over horizons longer than a few months. I argue that the presence of these extrapolative investors, combined with an elasticity of intertemporal substitution (EIS) greater than unity, can resolve the equity premium and volatility puzzles. Extrapolators overreact to dividend news, generating high return volatility and countercyclical expected returns. Sophisticated investors with rational expectations respond by making their consumption growth more procyclical, justifying the high observed equity premium. Aggregate consumption growth, however, has low covariance with stock returns because it includes the consumption growth of extrapolators, who raise their savings rates in response to positive returns. Extrapolators further increase the equity premium because they do not realize that stocks are a hedge against lower future returns; thus, they have excessive demand for the riskfree asset. A general equilibrium model yields a high countercyclical Sharpe ratio and a low, stable riskfree rate that match U.S. data with a relative risk aversion coefficient of 3 and an EIS of 2 when the number of extrapolators is 75% of the stockholding population—a fraction consistent with the available empirical evidence.

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1 Introduction

There is a growing body of evidence that many investors think high past stock market returns predict high future returns (e.g., De Bondt (1993), Durell (2001)), even though there is no support for this belief in the data (Fama and French (1988b)).[1] This paper argues that the interaction between these extrapolative investors and sophisticated investors with rational expectations can resolve the long-standing equity premium puzzle of Mehra and Prescott (1985) if the elasticity of intertemporal substitution (EIS) is greater than 1.

In the model presented, aggregate consumption has low covariance with market returns and relative risk aversion is low. Nonetheless, the average stock market return in excess of the riskfree rate is high. Furthermore, stock returns are much more volatile than dividend growth (Shiller (1981), LeRoy and Porter (1981)) and do not predict future dividend growth (Cochrane (1992)), which implies that stock returns are mean-reverting (Cochrane (1991), Campbell and Shiller (1988a, 1988b), Fama and French (1988a), Hodrick (1992)). These movements in expected returns are not fully offset by the countercylical movements in the conditional variance of returns, creating countercyclical Sharpe ratios (Harvey (1989), Chou, Engle, and Kane (1992), Whitelaw (1997)). The riskfree rate is low and stable. I find that I can match the moments of the postwar U.S. data with a relative risk aversion coefficient of 3 and an EIS of 2 when the number of extrapolative investors is 75% of the stockholding population—a fraction consistent with the available empirical evidence.

Extrapolators generate an anomalously high equity premium through three channels. First, they make stocks highly risky, even though stocks are claims to a fundamentally low-risk technology. Extrapolators are more willing to buy stocks when there has been recent good news, and they are less willing to buy stocks when there has been recent bad news. In equilibrium, this generates procyclical price-dividend ratios, which lead to countercyclical expected stock returns because dividend growth is independently and identically distributed over time. Sophisticated investors with rational expectations respond by making their consumption more procyclical, for an EIS greater than 1 causes lower expected future returns to increase consumption today. Sophisticates' consumption growth therefore has a high covariance with equity returns, leading sophisticates to demand a high average equity return. This is consistent with Aït-Sahalia, Parker, and Yogo's (2003) finding that the consumption of the rich—who are likely to be disproportionately rational—can better explain the equity premium.

Second, extrapolators obscure the riskiness they create in stocks. Extrapolators' expectations of future returns are moving in the opposite direction of sophisticates' contrarian expectations.

[1] It is true that there is a small amount of postive serial correlation in index returns over a horizon of a few months (Lo and MacKinlay (1988)). However, investor expectation surveys typically ask about longer horizons, where return autocorrelations are negative.

Therefore, when sophisticates are decreasing their savings rate in response to a positive innovation in stock prices, extrapolators are increasing their savings rate because they think investment prospects have improved. Aggregate consumption statistics add together sophisticate and extrapolator consumption, creating a series whose growth has low covariance with stock returns.

Third, extrapolators increase demand for the riskfree asset, driving down the mean riskfree rate. Sophisticates have a higher average demand for the riskfree asset because their consumption growth is made more volatile by extrapolators, increasing their precautionary savings motive. Extrapolators are on average too eager to hold the riskfree asset because they do not realize that stocks are a hedge against deteriorations in the investment opportunity set.[2] The riskfree rate is stable because the procycliality of extrapolator demand for the riskfree asset is offset by the countercyclicality of sophisticate demand. Furthermore, both extrapolator and sophisticate demands are individually stable. Extrapolator demand for the riskfree asset is steady because extrapolators use a long historical data series to estimate future equity returns, so the impact of any given quarter's news on beliefs is small. Since extrapolators are modeled as demanding a constant perceived risk premium, the stability of equity return beliefs translates directly into a stable riskfree asset demand. Sophisticates' riskfree asset demand is stable because their consumption smoothing motive is weak due to their high EIS, and innovations to that motive are negatively correlated with innovations to their precautionary savings motive.

Previous theoretical work has also shown that the presence of irrational noise traders can affect stock prices despite the presence of rational arbitrageurs (De Long et al. (1990), Campbell and Kyle (1993), Shleifer and Vishny (1997), Barberis and Shleifer (2003)). Empirical studies that find important limits to the extent arbitrageurs can move prices towards fundamental values include Lee, Shleifer, and Thaler (1991), Froot and Dabora (1999), Mitchell, Pulvino, and Stafford (2002), Wurgler and Zhuravskaya (2002), and Lamont and Thaler (2003). To date, however, there has been little exploration of the effect noise traders have on the joint behavior of consumption and asset returns.[3]

Section 2 reviews evidence on the prevalence of extrapolative expectations and the EIS. Section 3 presents the model and its baseline parameterization. Section 4 presents the results of the model. Section 5 reports results from alternative parameterizations of the model. Section 6 concludes. An appendix describes the computational procedure used to numerically solve the model.

[2] This does not mean that noise traders are *always* more reluctant than sophisticates to hold stocks. When sentiment is especially high, noise traders are more willing to hold stocks than sophisticates.

[3] An exception is Ingram (1990), who models an economy populated by sophisticates and noise traders who follow rule-of-thumb consumption and portfolio rules. She obtains an equity premium and volatility that are both an order of magnitude below the values observed in the data. De Long et al. (1990) suggest, as I do, that the riskiness of stocks may be both caused and hidden by noise trader consumption. However, they do not formally model this effect.

2 Empirical evidence

There are two key inputs that drive the model's results: the presence of investors who believe high past returns predict high future returns and an elasticity of intertemporal substitution greater than unity. This section presents empirical evidence that supports both these assumptions. I begin by discussing the studies that document widespread extrapolative expectations about stock returns. I then turn to the literature on the EIS. Finally, I argue that a number of empirical findings can be understood as arising from the interaction of extrapolative expectations with an EIS greater than unity.

2.1 Extrapolative expectations

Evidence on the prevalence of extrapolative expectations about stock returns has come from many different sources. De Bondt (1993), Fisher and Statman (2000), Durell (2001), and Vissing-Jørgensen (2003) employ survey evidence to show that individual investors' stock market returns expectations are positively correlated with past returns. Benartzi's (2001) survey shows similar extrapolative beliefs about investors' employer stock in 401(k) accounts. Fisher and Statman (2000), Benartzi (2001), Liang and Weisbenner (2002), Vissing-Jørgensen (2003), and Choi et al. (2004a) find that actual investor portfolio allocations are consistent with their reported beliefs. De Bondt (1993) and Andreassen and Kraus (1990) confirm that in experimental simulations of the stock market, subjects exhibit extrapolative behavior. Field evidence suggests that sophisticated investors are aware of the existence of naive extrapolative beliefs. Khwaja and Mian (2004) find that Pakistani stock brokers manipulate prices in order to exploit positive feedback traders, and Gordon (2000) describes similar behavior in the early 20th century U.S. stock market and the 18th century Dutch market.

These extrapolative expectations are not rational. Durell (2001) shows that the percent of investors who expect the stock market to increase in the next 12 months is a significantly negative predictor of actual 12-month future returns. Furthermore, the typical investor's beliefs systematically deviate from the beliefs of more sophisticated market observers. De Bondt (1991) finds that professional economists are contrarians when forecasting S&P 500 returns, and Clarke and Statman (1998) find the same is true of investment newsletter editors. Shiller (2000) reports that the percent of institutional investors who expect long-horizon market declines is positively correlated with past six-month Dow Jones Industrial Average returns.

It is tempting to dismiss studies that find extrapolative expectations because their samples are mostly drawn from a segment of the population that holds only a small fraction of total financial wealth. However, small investors cannot be ignored when relating consumption to asset prices because the consumption distribution in the United States is much flatter than the wealth distribution. For example, the bottom quintile of all households held only 0.8% of the financial wealth in 1998 (Rodriguez et al. (2002)) but had a 9.24% share of nondurable and services consumption in 1997-1998 (Krueger and Perri, 2002). Therefore, unsophisticated stockholders of modest means have significant weight in the consumption data.

Moreover, although it is plausible that wealthy households are more likely than poorer households to hold rational return beliefs, this does not imply that rationality is universal among the rich. Qiu and Welch (2004) find that the correlation between survey measures of wealthy and poor investor expected returns is 97%.[4]

2.2 Elasticity of intertemporal substitution

In an influential paper, Hall (1988) used macroeconomic data and a loglinearized Euler equation to estimate an EIS close to zero. However, subsequent papers have argued that this estimate is biased by the presence of liquidity constraints (Laibson, Repetto, and Tobacman (1998), Carroll (2001)), limited participation in asset markets (Vissing-Jørgensen (2002)), aggregation biases (Attanasio and Weber (1993)), and linearization error induced by time-varying uncertainty (Bansal and Yaron (2004)).

Gourinchas and Parker (2002) address many of these concerns by estimating the EIS on household-level data without linearizing the Euler equation and explicitly modeling labor income uncertainty and liquidity constraints. When they use a robust weighting matrix, their point estimate of the EIS is 2, and this rises to 6.5 when they restrict their consumption measure to nondurables. Attanasio, Banks, and Tanner (2002) use linearized Euler equations but restrict their sample to shareholders and estimate an EIS of 1.54. When Vissing-Jørgensen and Attanasio (2003) use a similar methodology, they find that accounting for taxes yields elasticity estimates between 1.03 and 2.34 with respect to the Treasury bill return. Bansal and Yaron (2004) argue that the negative correlation between price-dividend ratios and future consumption volatility implies that the EIS is greater than unity.

2.3 Interaction of extrapolative expectations with an EIS greater than unity

Several papers have found results consistent with a high EIS and extrapolative expectations that are less prevalent among the rich. Choi et al. (2004b) find that an idiosyncratic capital gain that is orthogonal to true information about future returns causes 401(k) investors to cut consumption growth. Their result can be explained by an EIS greater than unity and Vissing-Jørgensen's (2003) finding that cross-sectional variation in reported past personal portfolio returns strongly predicts beliefs about future market returns. Starr-McCluer (2002) reports that 11.6% of stockholders claim the 1990s bull market caused them to save more, while only 3.4% say that it caused them to save

[4] Many of these wealthy households are likely to invest according to these beliefs. Phoenix Companies (2004) finds that 50% of the millionaires it surveys make their own investment decisions without consulting a professional advisor.

less. In contrast, among households with more than \$250,000 in stock holdings, the results are much more balanced; the bull market caused 12.6% to save more and 12.6% to save less. Aït-Sahalia, Parker, and Yogo (2003) find over a longer time span that the consumption growth of the rich covaries much more with stock market returns than aggregate stockholder consumption growth does, implying that the rich raise their consumption more than the poor when expected returns are low.

3 Model description

Following Lucas (1978) and Mehra and Prescott (1985), I consider an infinite-horizon endowment economy where there is one share of equity that pays the aggregate consumption endowment as a stochastic perishable dividend $\bar{C}_t$ each period. Log dividend growth is independently and normally distributed with mean $\bar{g}$ and variance σ_g^2. There is also a riskfree asset present in infinitesimal positive net supply.[5] The ex-dividend price of the risky asset at time t is P_t, and the riskfree rate is $R_{f,t+1}$. Restricting attention to an endowment economy whose endowment process is identical to the actual consumption process sacrifices no generality in testing an asset-pricing model (Cochrane (2001)).

There are two types of finitely-lived agents: extrapolators, present in measure Q, and sophisticates, present in measure $1 - Q$. Both types have recursive utility of the form derived by Epstein and Zin (1989) and Weil (1989),

$$U_t = \left[(1-\beta) C_t^{(1-\gamma)/\theta} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{1/\theta} \right]^{\theta/(1-\gamma)}, \tag{1}$$

where γ is the coefficient of relative risk aversion, ψ is the EIS, $\theta \equiv (1-\gamma)/(1-1/\psi)$, and $\beta < 1$ is the time discount factor.[6] Investors have no bequest motive.[7]

At the beginning of each period, securities pay a dividend. Securities are then traded and consumption occurs. At the end of the period, a fixed positive fraction $\delta \leq 1 - \beta$ of the population is stochastically chosen to die. New investors are born to replace deceased investors, whose assets are evenly distributed among the newborns.[8] The number of newborns each period is fixed to

[5]The presence of an infinitesimal net supply of the riskfree asset makes more intuitive the use of equation (15) in determining the economy's riskfree rate. In an economy where the net supply of the riskfree asset is exactly zero, equation (15) is the economy's shadow riskfree rate.

[6]When $\gamma = 1/\psi$, Epstein-Zin-Weil utility is equivalent to constant relative risk aversion utility.

[7]Hurd (1989) and Gan et al. (2004) find that nearly all bequests in the U.S. are accidental. In the absence of a bequest motive, there would be a strong desire among investors to annuitize their wealth (Yaari (1965)). Annuitization is exogenously precluded in the model. Empirically, very few people hold private annuities. This outcome could be achieved endogenously by giving individuals private information about their mortality risk, causing adverse selection to dry up the annuity market.

[8]I assume that death occurs suddenly, so that there is no opportunity for an investor to liquidate all of his assets for consumption upon learning that death is imminent. I also assume that a sophisticate's heir is not disproportionately

maintain a constant total population, and the fraction of extrapolators among newborns is Q. Let ω_{t+1} be the total number of risky asset shares held by extrapolators at the end of period t before mortality. After mortality and redistribution, the number of shares extrapolators hold going into period $t+1$ is

$$\tilde{\omega}_{t+1} = (1-\delta)\,\omega_{t+1} + \delta Q. \tag{2}$$

The first term on the right hand side of this equation is the number of shares held by extrapolators who survive into $t+1$. The second term is the total number of shares that are bequested multiplied by the proportion of those shares that go to extrapolators.

A key challenge is choosing the form of the extrapolators' beliefs. The empirical evidence on beliefs cited above suggests that extrapolators react to past returns *per se*. However, modeling returns as the primitive on which extrapolation acts poses two difficulties. First, return-chasers have upward-sloping demand curves for assets, which makes the existence of equilibrium a nontrivial hurdle. Second, the prior period's price must be retained as a state variable in order to calculate the current period's return. This requirement is a serious obstacle because only special cases of heterogeneous investor economies can be solved analytically. Numerical algorithms for computing equilibria require exogenous bounding of the state space before computation. When prior price is a state variable, one needs to specify the range of prior price states in order to compute prices, but one needs to compute prices in order to know the range of prior price states.

To avoid these problems, I model extrapolators as projecting past consumption dividend growth into the future. I will show in Section 4.2 that in equilibrium, the resulting stock return beliefs mimic those of an investor who directly extrapolates from past returns. At time t, extrapolators estimate the expected growth rate of the consumption dividend going forward by calculating a geometrically weighted average of present and past gross dividend growth realizations, $\{G_t, G_{t-1}, G_{t-2}, ...\}$. Using the $\hat{E}_t$ operator to denote extrapolator expectations at time t,

$$\hat{E}_t G_{t+j} = (1-\phi)\sum_{\tau=0}^{\infty} \phi^{\tau} G_{t-\tau}, 0 < \phi < 1,\ j = 1, ..., \infty. \tag{3}$$

This inference procedure is rational if the expected dividend growth rate is a random walk. (See Barsky and De Long (1993))

Given $\hat{E}_t G_{t+j}$, extrapolators estimate the expected gross return on equities, $\hat{E}_t R_{t+j}$, by observing the price-dividend ratio and applying the Gordon (1962) pricing model, a procedure that assumes a constant expected return over all horizons going forward. Many market observers, including Fama and French (2002), use this approach to estimate the expected equity return. I am able to define $\hat{G}_t \equiv \hat{E}_t G_{t+j}$ and $\hat{R}_t \equiv \hat{E}_t R_{t+j}$ for all $j = 1, ..., \infty$ because these expectations do not

likely to be sophisticated him or herself.

vary with j. The resulting expression for extrapolators' return expectation is

$$\hat{R}_t = \hat{G}_t \left(1 + \frac{\bar{C}_t}{P_t}\right). \tag{4}$$

In order for equilibrium to exist in infinite-horizon economies, agents' budget sets must be constrained to prevent Ponzi schemes. A convenient way to do this when markets are incomplete is to impose a debt constraint.[9] I require that at every point in time and in all states of the world, each investor's wealth must be positive. This is a natural assumption, since in this economy, there is no future non-financial income against which to borrow in order to purchase current consumption.

Both sophisticates and extrapolators know that the distribution of log consumption growth has full support on $\mathbb{R}$, so they are unwilling to borrow or leverage their portfolios for fear of ending up with negative wealth and infinite marginal utility in the next period. In other words, investors are unable to issue riskfree debt. I make this assumption for technical expedience, as allowing trade in two assets generates serious computational difficulties that I discuss in Section 5. However, in practice, the quantity of riskless securities outstanding is small: in the second quarter of 2004, U.S. households and nonprofit organizations held less than 2.8% of their net worth (including human capital) in riskfree assets.[10]

Because there is only an infinitesimal net supply of the riskfree asset which nobody sells short, there is no asset allocation decision to be made by any of the investors; the only decision made is how much to save each period. Extrapolators solve a simplified problem in order to translate expected equity returns into a consumption-wealth ratio. First, they transform $\hat{R}_t$ into an expected log stock return by assuming that log returns are normally distributed with variance $\hat{\sigma}_s^2$. The resulting expression is $\hat{E}_t \log R_t = \log \hat{R}_t - \hat{\sigma}_s^2/2 \equiv \hat{r}_t - \hat{\sigma}_s^2/2$. They then solve for their desired consumption growth assuming that future stock returns are independently and identically distributed.[11] Adopting the convention for the rest of the paper that variables in lowercase are the logs

[9] A conventional transversality condition cannot be used because market incompleteness implies that even when all investors' expectations are rational, there are multiple possible valuations of a given future income stream in equilibrium. See, for example, Magill and Quinzii (1994).

[10] I use Flow of Funds statistics (Board of Governors of the Federal Reserve System (2004)) to calculate this figure. I consider bank deposits, open market paper, Treasury securities, and agency- and GSE-backed securities to be riskfree assets. Note that some of these securities are long-lived securities that are subject to inflation and interest rate risk. I estimate the market value of human capital by considering employee compensation as a dividend on human capital that has an expected annual growth rate of 3% and a standard deviation of 10% (Viceira (2001)). If the required return on human capital is constant, then a discount rate of 6.49% on wages makes the Sharpe ratios of human capital and equities equal.

[11] Limited cognitive ability might lead extrapolators to adopt this assumption, which considerably simplifies the consumption problem. A consumption rule that takes into account revisions to future returns expectations would require modeling noise traders' beliefs about the correlation between innovations to price-dividend ratios and innovations to dividends. For the sake of parsimony, I do not pursue this course.

of the corresponding variables in uppercase,

$$\hat{E}_t \Delta \hat{c}_{t+j} = \psi \left(\log \beta + \hat{r}_t - \hat{\sigma}_s^2/2 \right), \; j = 1, ..., \infty. \tag{5}$$

This solution also obtains if extrapolators choose consumption growth assuming future log returns will equal the expected log return with certainty. Much of the popular financial advice literature adopts this approach: authors use the average historical stock return to calculate the expected return—a procedure that is necessary only if stock returns are stochastic—but then drop the assumption of return uncertainty in order to calculate recommended savings rates (e.g. Chatzky (2001), Garner et al. (2000), Loeb (2001)).

Now armed with a time series of expected stock returns and desired expected consumption growth, extrapolators choose their log consumption-wealth ratio using the Campbell (1993) Taylor approximation to the intertemporal budget constraint around the log consumption-wealth ratio $1 - \beta$, which is the optimal solution when $\psi = 1$:

$$\hat{c}_t - \hat{w}_t = \hat{E}_t \sum_{j=1}^{\infty} \beta^j \left(r_{t+j} - \Delta \hat{c}_{t+j} \right) + \frac{\beta k}{1-\beta} \tag{6}$$

$$= \frac{\beta \left[(1-\psi) \left(\hat{r}_t - \hat{\sigma}_s^2/2 \right) - \psi \log \beta + k \right]}{1-\beta}, \tag{7}$$

where $k \equiv \log(\beta) + (1-\beta) \log(1-\beta)/\beta$.

The laws of motion governing extrapolator assets (before mortality) and beliefs are

$$\omega_{t+1} = \left(\frac{\bar{C}_t}{P_t} + 1 \right) \left(1 - \frac{\hat{C}_t}{\hat{W}_t} \right) \tilde{\omega}_t \tag{8}$$

$$\hat{G}_{t+1} = \phi \hat{G}_t + (1-\phi) G_{t+1} \tag{9}$$

Equation (8) expresses the number of equity shares held by extrapolators after consumption as being equal to the extrapolators' wealth at the beginning of the period times the savings rate divided by the price of an equity share. Equation (9) is the recursive form of equation (3).

The risky asset is priced so that sophisticates' Euler equation is exactly satisfied when they clear the consumption market. The market-clearing condition is $C_t^* = \bar{C}_t - \hat{C}_t$. Rewriting extrapolator consumption as being equal to extrapolator wealth times the extrapolator consumption-wealth ratio and dividing by the consumption dividend, the expression for aggregate sophisticate consumption growth is

$$\frac{C_{t+1}^*}{C_t^*} = \left(\frac{1 - \left[\left(1 + P_{t+1}/\bar{C}_{t+1} \right) \tilde{\omega}_{t+1} \right] \left(\hat{C}_{t+1}/\hat{W}_{t+1} \right)}{1 - \left[\left(1 + P_t/\bar{C}_t \right) \tilde{\omega}_t \right] \left(\hat{C}_t/\hat{W}_t \right)} \right) \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right). \tag{10}$$

We cannot use aggregate sophisticate consumption growth C^*_{t+1}/C^*_t to price assets, since that incorporates the consumption of sophisticates who have not yet been born when assets are trading in period t. However, the consumption growth of sophisticates who survive into the next period is part of a valid stochastic discount factor. Using $\tilde{X}_{t+1}$ to denote the t to $t+1$ consumption growth of sophisticates who are alive in both periods t and $t+1$, the equilibrium first-order condition is

$$E_t\left[\beta^\theta \tilde{X}_{t+1}^{-\frac{\theta}{\psi}}\left(\frac{P_{t+1}/\bar{C}_{t+1}+1}{P_t/\bar{C}_t}\right)^\theta G_{t+1}^\theta\right]=1. \tag{11}$$

The homotheticity of the Epstein-Zin-Weil utility function implies that surviving sophisticates' share of aggregate sophisticate consumption each period equals their share of aggregate sophisticate wealth. Survivors hold $(1-\delta)(1-\omega_{t+1})$ equity shares coming into period $t+1$, whereas on aggregate sophisticates hold $1-\tilde{\omega}_{t+1}$ shares coming into period $t+1$. Coming into period t, sophisticates who survive into period $t+1$ hold $(1-\delta)(1-\tilde{\omega}_t)$ shares, whereas aggregate sophisticate shares coming into period t is $1-\tilde{\omega}_t$. Therefore, we can derive the expression

$$\tilde{X}_{t+1}=\left(\frac{1-\omega_{t+1}}{1-\tilde{\omega}_{t+1}}\right)\frac{C^*_{t+1}}{C^*_t}. \tag{12}$$

The sophisticates' shadow price for the riskfree asset is given by the sophisticates' expected stochastic discount factor. Inverting the relation to yield a shadow riskfree rate gives

$$R^*_{f,t+1}=1\Bigg/E_t\left[\beta^\theta \tilde{X}_{t+1}^{-\frac{\theta}{\psi}}\left(\frac{P_{t+1}/\bar{C}_{t+1}+1}{P_t/\bar{C}_t}\right)^{\theta-1} G_{t+1}^{\theta-1}\right]. \tag{13}$$

Since investors are constrained to hold 100% of their portfolio in the risky asset, the extrapolators' shadow riskfree rate is the rate that makes them happy to hold such an allocation. Consistent with their approximating assumption that future returns are independently and identically distributed, extrapolators use a mean-variance portfolio rule,[12] so their shadow riskfree rate is

$$\hat{R}_{f,t+1}=\exp\left[\hat{r}_t-\gamma\hat{\sigma}_s^2\right]. \tag{14}$$

The price of the infinitesimal supply of the riskfree asset is determined by the investor who is willing to pay the most for it:

$$R_{f,t+1}=\min\{R^*_{f,t+1},\hat{R}_{f,t+1}\}. \tag{15}$$

I solve for a recursive equilibrium defined over the state space $S\equiv(\tilde{\omega},\hat{G})$. Let $Y(S_t)$ be the price-dividend ratio function, and $\tilde{X}(S_t,S_{t+1})$ be the function giving the consumption growth of

[12] Notice that if noise traders were to incorporate intertemporal hedging into their portfolio rule, the riskfree rate would fall further, since noise traders believe that low stock returns predict low future returns and hence would have an even higher demand for the safe asset.

surviving sophisticates between states S_t and S_{t+1}, defined by equations (10) and (12). S_t and S_{t+1} are linked by equations (2), (8), and (9). The pricing problem involves solving for $Y(S)$ which satisfies

$$E_t\left[\beta^{\theta}\tilde{X}\left(S_t,S_{t+1}\right)^{-\frac{\theta}{\psi}}\left(\frac{Y\left(S_{t+1}\right)+1}{Y\left(S_t\right)}\right)^{\theta}G_{t+1}^{\theta}\right]=1 \tag{16}$$

for all points in S.

There are nine parameters that must be chosen for the model, which is simulated at a quarterly frequency. These parameters are summarized in Table 1, where I have annualized numbers when appropriate. I choose the mean of log consumption dividend growth, $\bar{g}$, to match the U.S. postwar aggregate per capita mean for nondurables and services. The annual standard deviation of aggregate postwar per capita consumption growth is 1.07%, but numerous studies have found that stockholder consumption is significantly more volatile than nonstockholder consumption. This paper is concerned with stockholder behavior, since one does not expect the Euler equation for equities to hold for agents whose portfolios are at a corner solution. Therefore, I wish to choose consumption dividend volatility to match stockholder consumption volatility.[13] Mankiw and Zeldes (1991) estimate an annual standard deviation of 3.1%, but they measure only food consumption, which is likely to be less volatile than total consumption. Vissing-Jørgensen (2002) estimates a standard deviation of 5.80%, but some portion of that is due to measurement error. I choose the midpoint of these two figures for $2\sigma_g$.

The parameter ϕ, which is the autocorrelation of $\hat{G}$, is chosen to match the first-order autocorrelation of the postwar price-dividend ratio because I find that the simulated price-dividend ratio autocorrelation is usually very close to ϕ. I set the quarterly death rate δ so that each agent has an expected investing lifetime of $1/4\delta = 50$ years, roughly corresponding to the number of years a U.S. resident lives after reaching age 21. There is little guidance from data on the exact fraction of extrapolators in the stockholding population. However, many different studies have consistently found that stock returns expectations are extrapolative on average when subjects are not selected based upon their financial sophistication. If only a small fraction of the population were extrapolators, then one would expect at least a few studies to find acyclical or contrarian expectations. Furthermore, two of the surveys cited in Section 2.1 that find extrapolative expectations on average draw nationally representative samples. One therefore suspects that extrapolators outnumber contrarians, leading me to set $Q = 0.75$.

Finally, I choose the preference parameters and the extrapolators' belief about stock return volatility. I fix the EIS ψ at 2, which is the point estimate of Gourinchas and Parker (2002). I then vary risk aversion γ as a free parameter until the model's results match the data. β and $\hat{\sigma}_s$ are chosen for a given value of γ to match the mean postwar riskfree rate and the model's realized

[13] Mimicking the more volatile stockholder consumption series makes the equity premium puzzle easier to resolve because the puzzle is fundamentally about the smoothness of consumption growth.

standard deviation of stock returns.

I solve the model numerically using the time-iteration algorithm described in Judd, Kubler, and Schmedders (2000). The computational algorithm is described in greater detail in the Appendix. Once solutions have been obtained, I draw a 207-quarter period of the economy (corresponding to the length of the post-war period reported in Campbell (2003)) by starting the system at $\omega_t = Q$ and $\hat{G}_t = \exp\left(\bar{g} + \sigma_g^2/2\right)$, simulating a 1,207-quarter sample, and then discarding the first 1,000 quarters of data. I repeat this procedure 1,000 times in order to obtain a distribution of return paths. On a 2.8 gigahertz Pentium 4 computer running Matlab, it takes two to four hours to solve for the price-dividend ratio function and a couple of minutes to run the simulation.

4 Results

4.1 Unconditional moments

Table 2 shows the model's simulated moments for the baseline parameterization, where agents' coefficient of relative risk aversion is 3 and the annualized time discount factor is 0.953. With these reasonable preference parameters, the model can match the data's mean riskfree rate in its median simulation run. In addition, the riskfree rate is very stable. Campbell (2003) observes that the true riskfree rate must be more stable than the real riskfree rate calculated from the data, since the latter includes *ex post* inflation shocks. 90% of the simulation runs yield riskfree rate standard deviations between 0.33% and 0.87%, a range considerably below the 1.75% standard deviation observed in the postwar period.

Table 3 gives the log riskfree rate at selected points in the state space. Numbers in bold indicate that the marginal investor in the riskfree asset at that point is a sophisticate. It is apparent that on average, extrapolators have a higher demand for the riskfree asset than sophisticates. This is because unlike sophisticates, extrapolators do not realize that equity returns are mean-reverting, so they have no positive hedging demand for stocks. The stability of the riskfree rate comes the fact that extrapolator demand for the riskfree asset is countercyclical, whereas sophisticate demand is procyclical. This reflects the cyclicality of each investor type's optimism and prevents the interest rate from rising too high at the extremes of the state space. In addition, each type's demand is relatively stable. Extrapolators' expectation of stock returns has low volatility, due to the positive correlation between dividend growth shocks and price-dividend ratios, coupled with the fact that they use a long historical series to form their estimate of dividend growth. This translates directly into low volatility of the riskfree rate at points where the marginal investor is a extrapolator, since extrapolators demand a constant perceived risk premium. Sophisticate riskfree asset demand is relatively stable because their high EIS leads to a weak consumption smoothing motive and, as described below, innovations to their consumption smoothing motive are negatively correlated with innovations to their precautionary savings motive.

The third and fourth rows of Table 2 give the equity return premium and volatility. Both numbers are high but a little below the moments of the postwar data. This is to be expected, since equity in the model is a claim to the entire economy's consumption, whereas in reality, equity is a leveraged claim to consumption. Masulis (1988) reports that corporate debt as a fraction of total U.S. corporate market value fluctuated between 13% and 44% over a sixty year period. Rather than explicitly modeling equity as a leveraged claim, which would increase both the mean and variance of its returns, I concentrate on matching the Sharpe ratio in the data. The fifth row of Table 2 shows that the Sharpe ratio in the median simulation run of the model is very close to its empirical value.

Table 4 gives the conditional moments of surviving sophisticate consumption growth. The volatility of sophisticate consumption growth increases dramatically with extrapolator wealth. This rise in volatility occurs because extrapolator consumption growth is less volatile than aggregate consumption growth. Therefore, in order to clear the consumption market, sophisticate consumption growth must be more volatile than aggregate consumption growth. When extrapolators with stable consumption constitute a larger share of aggregate consumption, the volatility of the residual (sophisticate consumption) must increase. Average extrapolator wealth share was 68.4% in the median simulation run, 60.8% in the 5th percentile simulation run, and 73.5% in the 95th percentile simulation run. The wealth distribution is not highly volatile; the difference between the minimum and maximum extrapolator wealth share was 5.5 percentage points in the median simulation run, 2.8 percentage points in the 5th percentile simulation run, and 10.3 percentage points in the 95th percentile simulation run. These wealth distributions produce an unconditional surviving sophisticate consumption growth volatility of 21.1% a year in the median simulation run. This is not far above Aït-Sahalia, Parker, and Yogo's (2003) average estimated consumption growth volatility among the wealthy (a group that includes some extrapolators) of 15.8%. On the other hand, the median unconditional standard deviation for extrapolator consumption growth was only 3.1%. Regressing the change in surviving extrapolator consumption on a constant and the contemporaneous change in surviving extrapolator wealth, I find the annualized marginal propensity to consume out of capital gains was -0.78% in the median simulation run. This accords well with the empirical work of Choi et al. (2004), whose estimates of the marginal propensity to consume out of capital gains for 401(k) investors range from -0.37% to -1.82%.

We also see that sophisticate consumption volatility is countercyclical. This is because when extrapolators are pessimistic, their low savings rate leads to a low marginal utility of current consumption. Therefore, a given change in their belief about the value of investing changes their current consumption level a great deal. On the other hand, when extrapolators are optimistic, their savings rate is high, so they are more reluctant to cut consumption further in response to improvements in expected returns.

The second column of Table 2 shows return moments for an equivalently calibrated economy where there are only sophisticates. Because the stochastic process governing such an economy is independently and identically distributed, there is no variation in the price-dividend ratio or the riskfree rate, which means that equity return volatility is equal to consumption growth volatility. The low volatility of sophisticate consumption growth leads to a low risk premium, a low Sharpe ratio, and a high riskfree rate.

4.2 Conditional moments

Extrapolators have been modeled as extrapolating dividend growth, not returns. However, in equilibrium, their return beliefs mimic those of investors who extrapolate based upon past returns. Table 5 shows the coefficients from a regression of current stock return beliefs on the most recent eight quarters of realized returns. The effect of a given quarter's return is strongest at the end of the quarter and decays slowly as it recedes into the past.

Unconditionally, extrapolators' expectations of stock returns are close to correct. In the median simulation run, the geometric mean quarterly return on equities is 1.52%, and the average extrapolator forecast for returns across all simulation runs is 1.55%. However, extrapolators are poor conditional forecasters. Table 6 regresses the current extrapolator expectation for quarterly returns on the geometric mean quarterly return over the next 1, 2, and 4 years. Consistent with the findings of Durell (2001), extrapolator optimism is a significant negative predictor of future returns, and this predictive power increases with horizon.

Table 7 presents price-dividend ratios at selected points in the state space. Price-dividend ratios are positively correlated with extrapolator optimism. This procyclicality explains the poor conditional performance of extrapolator forecasts: because there are no bubbles in this economy and dividend growth is independently and identically distributed, higher price-dividend ratios imply lower expected returns. Countercyclical expected returns are necessary to clear the consumption market; extrapolators want to save more during booms, so in order to induce sophisticates to save less, expected returns must fall.

Table 8 presents regressions that predict excess log returns over the next 1, 2, and 4 years using the log price-dividend ratio. The R^2 in the median simulation run increases with horizon and almost exactly matches the empirical value. The coefficients on the log price-dividend ratio in the median simulation runs are somewhat larger in magnitude than the values observed in the postwar period, but the empirical values fall comfortably within the middle 90 percentile range of simulation outcomes.

Table 9 shows that equity return volatility is countercyclical and increasing in extrapolator wealth. This is because sophisticate consumption growth volatility is countercyclical and increasing in extrapolator wealth. In order for sophisticates to be happy to accept increasing consumption volatility, the predictability of stock returns must increase, implying increased stock return volatility. Table 10 shows that the countercyclicality of volatility is not exactly offset by changes in expected excess return, so the Sharpe ratio is countercyclical. Sharpe ratios also increase as extrapolator wealth decreases, but this is an artifact of the homoskedasticity assumption I have imposed on the extrapolators; when extrapolators hold only 10% of the wealth, the standard deviation of stock returns is in the neighborhood of 6%, but extrapolators continue to believe that the standard deviation is 12.5% and demand a correspondingly high premium. A more realistic (but complicated) specification of extrapolator beliefs would allow their volatility expectations to adjust according to recently realized volatility. This strategy would mitigate, if not eliminate, the large increase in Sharpe ratios as sophisticate wealth increases. In practice, simulated economies spend very little time in states where extrapolators have low wealth because of the continuous flow of bequests.

5 Sensitivity analysis

In this section, I explore the sensitivity of the results to different parameterizations of the model. The results of this investigation are summarized in Table 11.

The first perturbation I study is an increase in the average investing lifetime to 80 years rather than 50 years. Increasing expected lifetime slows the transfer of wealth from sophisticates to extrapolators, yielding lower average extrapolator wealth. All the desirable stylized facts generated by the model come from the nonfundamental risk that extrapolators bring to the economy. Therefore, if average extrapolator wealth is lowered enough by the slowing of bequests, their economic influence will be diminished, and the model may not be able to match the empirical data.

The first row of Table 11 shows that these fears are unfounded; raising the expected lifetime by 30 years has a minimal effect upon the results. The reason is that when sophisticate consumption volatility falls due to lower extrapolator wealth, the average riskfree rate rises due to the reduction in the precautionary savings motive. In order to keep the average riskfree rate equal to its empirical value, agents must be made more patient. However, when extrapolators are more patient, their consumption becomes more responsive to changes in their return beliefs because future payoffs are more heavily weighted. Therefore, the drop in sophisticate consumption volatility induced by lower extrapolator wealth is offset by the accompanying rise in β.

I next examine the effect of lowering the EIS, which causes extrapolators' consumption to become less sensitive to their return beliefs, thus lowering sophisticate consumption volatility as well. The second row of Table 11 reveals that dropping the EIS to 1.5 reduces the equity premium and volatility, but the change is not dramatic. Again, the higher β necessary to match the mean riskfree rate mostly offsets the effect of reducing the EIS.

Third, I try lowering the proportion of extrapolators in the population to 25%. I find that in order to match the empirical Sharpe ratio, relative risk aversion must rise to 4. Because of extrapolators' smaller economic influence, the equity premium and volatility are considerably smaller than those obtained in the baseline parameterization. In order to match the empirical equity premium, the average return must almost double. While the amount of leverage required to achieve this may be implausible, the fact that such a dramatic drop in the number of extrapolators still generates a fairly high equity premium and volatility reassures us that the results are not sensitive to smaller changes in extrapolator frequency.

Finally, I force the marginal investor in the riskfree asset to always be a sophisticate, so that both equities and the riskfree asset are always priced by the same stochastic discount factor. That is, the riskfree rate is always determined by equation (13). Price-dividend ratios are unaffected by this choice. Such a setup is able to match the data's moments with a relative risk aversion of 9. Remarkably, the riskfree rate remains more stable than the *ex post* realizations in the postwar data. This is because even though expected sophisticate consumption growth is countercyclical, reflecting the countercyclical sophisticate savings rate, sophisticate consumption growth volatility is also countercyclical, so that innovations in the consumption smoothing and precautionary savings motives work against each other. Furthermore, the consumption smoothing motive is not strong because the EIS is high.

In an economy where the support of log dividend growth is finite, short sales of the riskfree asset would endogenously arise, and the region of the state space where sophisticates' stochastic discount factor endogenously prices both assets would correspondingly increase. Unfortunately, computing equilibria when there are two traded assets is a formidable task. In order to keep the state space compact in an infinite-horizon economy with aggregate consumption growth, one must normalize next period's payout of one riskfree share by a variable that grows with the economy; otherwise, the number of riskfree shares demanded on average will approach infinity as the economy grows without bound. A natural candidate for this normalizing variable is the current period's consumption dividend. This means that in order to compute the distribution of wealth in period t, one must know not only the number of riskfree shares held by each type of investor coming into t, but also the dividend growth between $t-1$ and t. Therefore, adding a tradeable riskfree asset requires the addition of two state variables to the two already in place, raising the computational burden exponentially.

Computational limitations require, then, that the range of possible riskfree share holdings be stringently limited in order to keep the state space manageable. I have tried to solve, using the time-iteration algorithm described in the Appendix, for a recursive equilibrium when sophisticates are allowed to sell short up to $\bar{C}_t$ of the riskfree asset at time t. I find that the algorithm does not converge, but ends up cycling between two candidate solutions. Recursive equilibria do not always exist in economies with heterogeneous agents and incomplete markets (Kubler and Schmedders (2002)). It is not known whether the time-iteration algorithm fails because of the non-existence of recursive equilibria in this case, or if a more advanced technique would be able to identify such an equilibrium. Expanding the state space in order to search for non-recursive equilibria is computationally infeasible because of the curse of dimensionality.

6 Conclusion

This paper has argued that the equity premium and volatility puzzles arise from the presence of extrapolative investors who believe that high past stock market returns predict high future returns. When the elasticity of intertemporal substitution is greater than unity, this belief causes extrapolator savings rates to vary procyclically, pushing up price-dividend ratios during booms and pushing down price-dividend ratios during recessions. The resulting predictability of stock returns causes sophisticated investors with rational expectations to make their savings rates countercyclical, increasing the covariance between their consumption growth and stock returns. This high covariance justifies the high average return on equities. However, aggregate consumption statistics include both extrapolator and sophisticate consumption, hiding from the econometrician the stock market's consumption risk to sophisticates. The equity premium is further increased by extrapolators' ignorance of mean reversion in stock returns, which creates excessive demand for the riskfree asset.

I numerically solve a general equilibrium model of an endowment economy that is populated by sophisticated investors and extrapolators. I find that I can generate a low, stable riskfree rate and a high, countercyclical Sharpe ratio consistent with the postwar U.S. data with a relative risk aversion coefficient of 3 and an elasticity of intertemporal substitution (EIS) of 2 when extrapolators constitute 75% of the population.

Numerous empirical studies have documented that extrapolative beliefs about stock returns are commonly held in the population, even among the rich. However, it seems *a priori* plausible that the rich are more likely to hold rational beliefs than the poor. Consistent with this notion, other empirical studies have shown that the covariance of consumption growth with stock returns is higher for the rich. The model in this paper predicts that identifying the subset of investors who understand that stock returns are mean-reverting and measuring their consumption growth will yield an even higher covariance. Furthermore, the unconditional average portfolio share in cash and Treasury bills will be increasing with the strength of a household's extrapolative tendencies, since knowledge of mean reversion in stock returns generates a positive hedging demand for equities.

7 Appendix: Computational algorithm

I solve the model numerically using the time-iteration algorithm described in Judd, Kubler, and Schmedders (2000). Consumption dividend shocks are discretized using a 9-point Gauss-Hermite quadrature, which Tauchen and Hussey (1991) show can be interpreted as a Markov approximation to the underlying continuous process. I approximate the function Y by using a cubic spline with knots at the grid points $\{\tilde{\omega}_k\} \times \{\hat{G}_k\}$ and the "not-a-knot" end condition. (See de Boor (2001) for an introduction to the theory of splines.) There are 29 knots in $\{\tilde{\omega}_k\}$ between 0 and 1 inclusive: five knots evenly distributed between 0 and 0.04 inclusive, 19 knots evenly distributed between 0.05 and 0.95 inclusive, and five knots evenly distributed between 0.96 and 1 inclusive. I use 21 knots in $\{\hat{G}_k\}$, where there is one knot at $\exp\left(\bar{g} + \sigma_g^2/2\right)$ and 10 knots spaced evenly in each direction around the center knot, such that a six-standard-deviation range of $\hat{G}$ is covered. I have also used 7 knots in $\{\hat{G}_k\}$ covering a four-standard-deviation range and found the resulting solutions to be similar.

Let Y_i be the solution function that was obtained in the ith iteration. Then at iteration $i+1$, I use a trust-region dogleg algorithm to solve for Y_{i+1} such that for every point S_t in the state space grid, the following relationship is satisfied:

$$E_t\left[\beta^\theta \tilde{X}\left(S_t, S_{t+1}\right)^{-\frac{\theta}{\psi}} \left(\frac{Y_i\left(S_{t+1}\right)+1}{Y_{i+1}\left(S_t\right)}\right)^\theta G_{t+1}^\theta\right] - 1 = 0. \tag{17}$$

This approach transforms the problem from finding a continuous function that satisfies the Euler equation at an infinite number of points to solving a system of $29 \times 21 = 609$ nonlinear equations with 609 unknowns. When the system is being solved at a boundary of $\hat{G}_k$, $\hat{G}_{t+1}$ is constrained to remain at the boundary if a dividend realization would take it off the grid according to equation (9). The trust-region dogleg stops when the left hand side of equation (17) is less than 10^{-6}, which indicates that the welfare loss to the sophisticate as the result of optimization error is less than a millionth of the marginal value of consumption in period t.

The algorithm continues to iterate until the maximum absolute difference between Y_{i+1} and Y_i evaluated at any grid point is less than 10^{-5}. When $\tilde{X}$ is evaluated at points where extrapolators hold almost all the wealth in the economy, small approximation errors in the interpolation of Y_i can cause extrapolators to consume more than the endowment in period $t+1$, implying negative sophisticate consumption. In order to avoid this situation, I restrict $t+1$ sophisticate consumption to be bounded below at 0.01% of the $t+1$ dividend. This constraint is sometimes required for the algorithm to converge to a solution. However, in all the results presented in this paper, the constraint is never active at any of the grid points when the Euler equation is evaluated at the final solution.

The time-iteration algorithm is not globally convergent; like most numerical algorithms, it requires a starting point that is not too far from the final solution. My initial guess for Y is the solution when there are only extrapolators in the economy. In that case, market clearing requires that

$$\frac{1}{P_t/\bar{C}_t + 1} = \exp\left[\frac{\beta\left[(1-\psi)\left(\hat{r}_t - \hat{\sigma}_s^2/2\right) - \psi \log \beta + k\right]}{1-\beta}\right]. \tag{18}$$

This initial guess guarantees that extrapolator consumption does not exceed the endowment at time $t+1$ in the first iteration. I also fix the value of Y to equal this initial guess in all subsequent iterations when $\omega_t = 1$.

Riskfree rates for the simulations are determined by calculating the riskfree rate at each grid point and using cubic spline interpolation to approximate the riskfree rate for points not on the grid.[14]

8 References

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[14] The sophisticates' shadow riskfree rate cannot be calculated when noise traders own 100% of the wealth. When $\omega_t = 1$, I assume that $R_{f,t+1} = \hat{R}_{f,t+1}$.

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Table 1: Baseline Parameter Choices

This table gives the baseline parameter values used for the model. Where appropriate, numbers have been annualized, and any calculations used to annualize the numbers are reflected in the second column. For example, in order to annualize mean log consumption dividend growth, the variable $\overline{g}$ has been multiplied by 4, so the corresponding entry in the second column is $4\overline{g}$.

Parameter	Variable	Value
Mean log consumption dividend growth	$4\,\overline{g}$	1.964%
Standard deviation of log consumption dividend growth	$2\sigma_g$	4.449%
Extrapolator belief persistence coefficient	ϕ^4	0.784
Expected investing lifespan	$1/4\delta$	50 years
Proportion of extrapolators in population	Q	0.75
Elasticity of intertemporal substitution	ψ	2
Coefficient of relative risk aversion	γ	3
Time discount factor	β^4	0.953
Extrapolators' perceived log stock return standard deviation	$2\hat{\sigma}_s$	12.5%

Table 2: Simulated Annualized Moments

The second column gives annualized moments from simulating the model for 1,207 quarters and discarding the first 1,000 quarters. The top number in each cell is the median moment from 1,000 simulation runs. The numbers in brackets below are the 5th and 95th percentile moments from the simulation runs. The third column gives the moments from an economy populated only by sophisticated investors. The fourth column gives actual moments from postwar U.S. data, taken from the data in Campbell (2003). To obtain the log equity premium, I calculate the mean of the equity return divided by the riskfree rate, and then I take the log of that mean. The log equity Sharpe ratio is the log equity premium divided by the log equity return standard deviation.

	Model simulation results	Sophisticate-only economy	U.S. data, 1947.2-1998.4
Mean log riskfree rate	0.90%	5.39%	0.90%
	[0.39, 1.20]		
Log riskfree rate	0.41%	0%	1.75%
standard deviation	[0.30, 0.71]		
Log equity premium	5.92%	0.60%	7.19%
	[5.01, 7.21]		
Log equity return	12.53%	4.45%	15.74%
standard deviation	[11.43, 13.84]		
Log equity Sharpe ratio	0.477	0.135	0.457
	[0.391, 0.581]		
Log price-dividend ratio	0.730	1	0.784
autocorrelation	[0.557, 0.846]		

Table 3: Annualized Log Riskfree Rate

This table presents the annualized log riskfree rate at selected points in the state space. As one moves down the rows, the proportion of wealth held by extrapolators, ω, changes. As one moves across the columns, extrapolators' expectation of the future dividend growth rate changes. $\mu_{\hat{G}}$ is the unconditional mean of $\hat{G}$, extrapolators' belief about the future dividend growth rate. $\sigma_{\hat{G}}$ is the standard deviation of $\hat{G}$. A number in bold indicates that at that point in the state space, a sophisticate is the marginal investor in the riskfree asset.

	Extrapolators' expected future dividend growth rate				
	$\mu_{\hat{G}} - 2\sigma_{\hat{G}}$	$\mu_{\hat{G}} - \sigma_{\hat{G}}$	$\mu_{\hat{G}}$	$\mu_{\hat{G}} + \sigma_{\hat{G}}$	$\mu_{\hat{G}} + 2\sigma_{\hat{G}}$
$\omega = 0.0$	-1.83%	-0.27%	1.29%	2.84%	4.38%
$\omega = 0.1$	-1.61%	-0.16%	1.31%	2.80%	4.29%
$\omega = 0.2$	-1.39%	-0.05%	1.33%	2.76%	**4.07%**
$\omega = 0.3$	-1.18%	0.06%	1.36%	2.72%	**3.11%**
$\omega = 0.4$	-0.98%	0.16%	1.39%	**2.67%**	**2.15%**
$\omega = 0.5$	-0.79%	0.27%	1.42%	**2.30%**	**1.16%**
$\omega = 0.6$	-0.61%	0.38%	1.45%	**1.51%**	**-0.07%**
$\omega = 0.7$	-0.43%	0.48%	1.48%	**0.73%**	**-1.51%**
$\omega = 0.8$	-0.27%	0.59%	1.52%	**0.06%**	**-3.10%**
$\omega = 0.9$	-0.10%	0.69%	1.56%	**-0.77%**	**-5.06%**

Table 4: Surviving Sophisticates' Consumption Growth

Annualized Mean and Standard Deviation

This table presents, at selected points in the state space, the annualized conditional mean and conditional standard deviation of consumption growth for sophisticates who will survive into the next period. The top number in each cell is the mean consumption growth, and the number in parentheses is the standard deviation. As one moves down the rows, the proportion of wealth held by extrapolators, ω, changes. As one moves across the columns, extrapolators' expectation of the future dividend growth rate changes. $\mu_{\hat{G}}$ is the unconditional mean of $\hat{G}$, extrapolators' belief about the future dividend growth rate. $\sigma_{\hat{G}}$ is the standard deviation of $\hat{G}$.

	Extrapolators' expected future dividend growth rate				
	$\mu_{\hat{G}} - 2\sigma_{\hat{G}}$	$\mu_{\hat{G}} - \sigma_{\hat{G}}$	$\mu_{\hat{G}}$	$\mu_{\hat{G}} + \sigma_{\hat{G}}$	$\mu_{\hat{G}} + 2\sigma_{\hat{G}}$
$\omega = 0.0$	0.31%	1.14%	1.75%	2.19%	2.52%
	(4.62%)	(4.58%)	(4.54%)	(4.52%)	(4.50%)
$\omega = 0.1$	3.78%	2.31%	1.56%	1.29%	1.29%
	(8.80%)	(7.81%)	(7.02%)	(6.39%)	(5.91%)
$\omega = 0.2$	6.78%	3.48%	1.44%	0.37%	-0.04%
	(12.28%)	(10.79%)	(9.46%)	(8.35%)	(7.45%)
$\omega = 0.3$	9.36%	4.64%	1.39%	-0.56%	-1.49%
	(15.16%)	(13.50%)	(11.89%)	(10.41%)	(9.14%)
$\omega = 0.4$	11.57%	5.81%	1.40%	-1.51%	-3.09%
	(17.53%)	(15.99%)	(14.29%)	(12.59%)	(11.01%)
$\omega = 0.5$	13.53%	6.96%	1.50%	-2.47%	-4.89%
	(19.47%)	(18.21%)	(16.65%)	(14.89%)	(13.12%)
$\omega = 0.6$	15.30%	8.11%	1.69%	-3.43%	-6.91%
	(21.04%)	(20.20%)	(18.94%)	(17.33%)	(15.51%)
$\omega = 0.7$	17.01%	9.32%	1.96%	-4.40%	-9.24%
	(22.29%)	(21.94%)	(21.10%)	(19.84%)	(18.19%)
$\omega = 0.8$	18.94%	10.63%	2.30%	-5.50%	-12.08%
	(23.34%)	(23.49%)	(23.22%)	(22.48%)	(21.23%)
$\omega = 0.9$	22.00%	12.33%	2.53%	-7.11%	-16.17%
	(24.10%)	(24.67%)	(25.05%)	(25.16%)	(24.83%)

Table 5: Extrapolator Return Expectations and Past Returns

This table presents the results of a regression of extrapolators' expectations at time t of future stock returns on realized returns in the most recent eight quarters. The regression was run on each of 1,000 simulation runs of the model. For each simulation run, 1,207 quarters were simulated and the first 1,000 quarters were discarded. The top number in each cell of the second column is the median coefficient from the 1,000 regressions. The numbers in brackets below are the 5th and 95th percentile coefficients.

Explanatory variable	Coefficient
R_t	0.0099
	[0.0076, 0.0125]
R_{t-1}	0.0094
	[0.0073, 0.0119]
R_{t-2}	0.0089
	[0.0070, 0.0115]
R_{t-3}	0.0085
	[0.0069, 0.0108]
R_{t-4}	0.0081
	[0.0066, 0.0103]
R_{t-5}	0.0076
	[0.0060, 0.0101]
R_{t-6}	0.0072
	[0.0054, 0.0099]
R_{t-7}	0.0068
	[0.0049, 0.0095]
Intercept	0.9483
	[0.9305, 0.9577]
R^2	33.9%
	[23.1, 48.0]

Table 6: Extrapolator Return Expectations and Future Equity Returns

This table presents the results of regressions that predict future equity returns using extrapolators' expectation at time t of future equity returns. The dependent variables are the geometric mean of realized quarterly returns over the next 1, 2, and 4 years. The explanatory variables are extrapolators' return expectation and a constant. The regression was run on each of 1,000 simulation runs of the model. For each simulation run, 1,207 quarters were simulated and the first 1,000 quarters were discarded. In the last three columns, the top number in each cell is the median coefficient from the 1,000 regressions. The numbers in brackets below are the 5th and 95th percentile coefficients.

Return horizon (years)	$\hat{R}_t$	Intercept	R^2
1	-3.8808	0.0769	9.3%
	[-7.4623, -1.5576]	[0.0407, 0.1320]	[2.8, 19.6]
2	-3.4304	0.0692	16.4%
	[-6.2668, -1.3841]	[0.0372, 0.1153]	[4.6, 33.0]
4	-2.8149	0.0580	26.7%
	[-4.5984, -1.0677]	[0.0319, 0.0867]	[6.7, 49.1]

Table 7: Price-Dividend Ratios

This table presents, at selected points in the state space, the price-dividend ratio of equities. Dividends have been annualized. As one moves down the rows, the proportion of wealth held by extrapolators, ω, changes. As one moves across the columns, extrapolators' expectation of the future dividend growth rate changes. $\mu_{\hat{G}}$ is the unconditional mean of $\hat{G}$, extrapolators' belief about the future dividend growth rate. $\sigma_{\hat{G}}$ is the standard deviation of $\hat{G}$.

	Extrapolators' expected future dividend growth rate				
	$\mu_{\hat{G}} - 2\sigma_{\hat{G}}$	$\mu_{\hat{G}} - \sigma_{\hat{G}}$	$\mu_{\hat{G}}$	$\mu_{\hat{G}} + \sigma_{\hat{G}}$	$\mu_{\hat{G}} + 2\sigma_{\hat{G}}$
$\omega = 0.0$	25.44	25.44	25.44	25.44	25.44
$\omega = 0.1$	24.09	24.78	25.31	25.71	26.01
$\omega = 0.2$	22.87	24.11	25.14	25.96	26.61
$\omega = 0.3$	21.81	23.49	24.96	26.22	27.24
$\omega = 0.4$	20.89	22.90	24.79	26.48	27.93
$\omega = 0.5$	20.08	22.35	24.62	26.76	28.70
$\omega = 0.6$	19.36	21.83	24.44	27.06	29.55
$\omega = 0.7$	18.72	21.34	24.24	27.35	30.50
$\omega = 0.8$	18.14	20.86	24.02	27.60	31.51
$\omega = 0.9$	17.62	20.41	23.79	27.85	32.61

Table 8: Predicting Excess Log Returns with the Log Price-Dividend Ratio

This table presents the results of regressions that predict future excess equity returns using the log price-dividend ratio. The dependent variables are the log of the mean of equity returns divided by the riskfree rate over the next 1, 2, and 4 years, scaled to give an average quarterly excess return. The explanatory variables are the log price-dividend ratio and a constant. The first column gives the results from running the regression on each of 1,000 simulation runs of the model. For each simulation run, 1,207 quarters were simulated and the first 1,000 quarters were discarded. The top number in each cell of the second column is the median coefficient from the 1,000 regressions. The numbers in brackets below are the 5th and 95th percentile coefficients. The third column gives the results from running the regression on postwar U.S. data taken from Campbell (2003).

	Model simulation	U.S. data, 1947.2-1998.4
	Panel A: 1-year future returns	
Slope	-0.3504	-0.1709
	[-0.6666, -0.1320]	
R^2	9.6%	9.6%
	[2.2, 21.0]	
	Panel B: 2-year future returns	
Slope	-0.6159	-0.3459
	[-1.1249, -0.2165]	
R^2	17.1%	20.2%
	[3.3, 35.0]	
	Panel C: 4-year future returns	
Slope	-1.0111	-0.5526
	[-1.6412, -0.3061]	
R^2	27.8%	29.9%
	[4.1, 51.8]	

Table 9: Annualized Standard Deviations of Equity Returns

This table presents, at selected points in the state space, the annualized conditional standard deviation of equity returns. As one moves down the rows, the proportion of wealth held by extrapolators, ω, changes. As one moves across the columns, extrapolators' expectation of the future dividend growth rate changes. $\mu_{\hat{G}}$ is the unconditional mean of $\hat{G}$, extrapolators' belief about the future dividend growth rate. $\sigma_{\hat{G}}$ is the standard deviation of $\hat{G}$.

	Extrapolators' expected future dividend growth rate				
	$\mu_{\hat{G}} - 2\sigma_{\hat{G}}$	$\mu_{\hat{G}} - \sigma_{\hat{G}}$	$\mu_{\hat{G}}$	$\mu_{\hat{G}} + \sigma_{\hat{G}}$	$\mu_{\hat{G}} + 2\sigma_{\hat{G}}$
$\omega = 0.0$	4.53%	4.51%	4.49%	4.48%	4.48%
$\omega = 0.1$	6.58%	6.09%	5.71%	5.40%	5.17%
$\omega = 0.2$	8.31%	7.58%	6.94%	6.40%	5.96%
$\omega = 0.3$	9.72%	8.93%	8.15%	7.43%	6.80%
$\omega = 0.4$	10.87%	10.14%	9.33%	8.51%	7.75%
$\omega = 0.5$	11.80%	11.24%	10.50%	9.66%	8.81%
$\omega = 0.6$	12.57%	12.23%	11.66%	10.90%	10.01%
$\omega = 0.7$	13.18%	13.10%	12.79%	12.23%	11.44%
$\omega = 0.8$	13.67%	13.86%	13.85%	13.60%	13.08%
$\omega = 0.9$	14.07%	14.52%	14.87%	15.05%	14.97%

Table 10: Conditional Annualized Sharpe Ratios

This table presents, at selected points in the state space, the annualized conditional log Sharpe ratio of equities. The numerator of the log Sharpe ratio is the log of the (true) expected stock return divided by the riskfree rate. The denominator is the standard deviation of the log stock return. As one moves down the rows, the proportion of wealth held by extrapolators, ω, changes. As one moves across the columns, extrapolators' expectation of the future dividend growth rate changes. $\mu_{\hat{G}}$ is the unconditional mean of $\hat{G}$, extrapolators' belief about the future dividend growth rate. $\sigma_{\hat{G}}$ is the standard deviation of $\hat{G}$.

	Extrapolators' expected future dividend growth rate				
	$\mu_{\hat{G}}-2\sigma_{\hat{G}}$	$\mu_{\hat{G}}-\sigma_{\hat{G}}$	$\mu_{\hat{G}}$	$\mu_{\hat{G}}+\sigma_{\hat{G}}$	$\mu_{\hat{G}}+2\sigma_{\hat{G}}$
$\omega = 0.0$	1.546	1.299	1.022	0.729	0.423
$\omega = 0.1$	1.332	1.061	0.800	0.540	0.272
$\omega = 0.2$	1.239	0.934	0.655	0.392	0.137
$\omega = 0.3$	1.193	0.864	0.564	0.289	0.030
$\omega = 0.4$	1.166	0.821	0.502	0.210	-0.058
$\omega = 0.5$	1.149	0.793	0.459	0.151	-0.131
$\omega = 0.6$	1.137	0.776	0.430	0.105	-0.195
$\omega = 0.7$	1.128	0.765	0.410	0.071	-0.247
$\omega = 0.8$	1.121	0.757	0.398	0.047	-0.289
$\omega = 0.9$	1.115	0.752	0.389	0.029	-0.324

Table 11: Alternative Parameterizations

This table gives simulated moments from alternative parameterizations of the model. The first column describes the parameter that was altered from the baseline case. The second, third, and fourth columns give the coefficient of relative risk aversion, annualized time discount factor, and extrapolators' perception of the annualized log equity return standard deviation. These parameters are adjusted so that the mean riskfree rate and Sharpe ratio match the postwar U.S. data and extrapolators' beliefs about equity volatility match the observed unconditional equity volatility in the median simulation run. The last five columns give the annualized log riskfree rate mean and standard deviation, log equity premium, log equity return standard deviation, and log Sharpe ratio. The top number in each cell is the median moment from 1,000 simulation runs of the model. The numbers below in brackets give the 5th and 95th percentile simulation moments. For each simulation run, 1,207 quarters were drawn and the first 1,000 quarters were discarded. The log Sharpe ratio is the log equity premium divided by the log equity return standard deviation.

Altered parameter	RRA	β^4	$2\hat{\sigma}_s$	$4E(r_f)$	$2\sigma(r_f)$	$4\log E(R/R_f)$	$2\sigma(r)$	Log Sharpe ratio
Investing lifetime is 80 years	3	0.9542	12.35%	0.91%	0.40%	5.74%	12.32%	0.471
				[0.35, 1.24]	[0.30, 0.66]	[4.80, 7.23]	[0.109, 0.138]	[0.381, 0.590]
EIS = 1.5	3	0.9622	11.40%	0.91%	0.44%	5.03%	11.35%	0.444
				[0.42, 1.23]	[0.33, 0.66]	[4.17, 6.37]	[10.42, 12.36]	[0.363, 0.565]
Extrapolators are 25% of	4	0.9700	8.90%	0.90%	0.46%	3.78%	8.82%	0.424
population				[0.24, 1.31]	[0.26, 0.65]	[3.16, 4.47]	[7.73, 10.00]	[0.336, 0.541]
Sophisticate is always	9	0.9542	13.00%	0.87%	1.52%	6.10%	13.10%	0.465
marginal riskfree investor				[-1.28, 3.28]	[1.13, 2.11]	[2.87, 8.97]	[11.88, 14.29]	[0.224, 0.679]