

# Testing Efficient Risk Sharing with Heterogeneous Risk Preferences\*

Maurizio Mazzocco  
UCLA

Shiv Saini  
University of Wisconsin

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## Abstract

Previous papers have tested efficient risk sharing under the assumption of identical risk preferences. In this paper we show that, if in the data households have heterogeneous risk preferences, the tests proposed in the past reject efficiency even if households share risk efficiently. To address this issue we propose a method that enables one to test efficiency even when households have different preferences for risk. The method is composed of three tests. The first one can be used to determine whether in the data under investigation households have homogeneous risk preferences. The second and third test can be used to evaluate efficient risk sharing when the hypothesis of homogeneous risk preferences is rejected. We use this method to test efficient risk sharing in rural India. Using the first test, we strongly reject the hypothesis of identical risk preferences. We then test efficiency with and without the assumption of preference homogeneity. In the first case we reject efficient risk sharing at the village and caste level. In the second case we still reject efficiency at the village level, but we cannot reject this hypothesis at the caste level. This finding suggests that the relevant risk-sharing unit in rural India is the caste and not the village.

## 1 Introduction

A good understanding of the degree of risk sharing that characterizes households in industrialized and developing countries is important to answer policy questions. To see this consider for instance a village in a developing country. The consumption pattern and hence the welfare of each individual in the village is affected by a variety of idiosyncratic and aggregate shocks. Some examples of such shocks are severe weather conditions, price fluctuations, health problems, unemployment spells, and

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crop diseases. Unless the households in the village can insure themselves against these shocks using the existing institutions, individual consumption will fluctuate in response to them with detrimental effects on individual welfare. The typical risk-sharing institutions available to households in rural villages are gifts and transfers, borrowing from village lenders, saving and storage technologies, and the diversification of crops. In addition to the existing institutions, the local and central government may decide to provide additional insurance for instance by introducing new financial assets, by simplifying the access to the existing financial markets, or by providing crop, health, and unemployment insurance.

A crucial step in evaluating whether there is scope for government reforms is the derivation of tests that enable one to determine whether households are able to share risk efficiently using the existing institutions. In the past three decades several papers have derived such tests and tested efficient risk sharing in industrialized and developing countries. The papers in this literature have one common feature. They assume that households have identical preferences for risk. The first contribution of the present paper is to show that, if in the data the preferences for risk are heterogeneous, the tests used in the past reject efficient risk sharing even if the households under investigation share risk efficiently.

The second contribution of this paper is to provide a method that enables one to test efficient risk sharing even when households have heterogeneous preferences for risk. The method is composed of three tests. The first one is a test of homogeneity in risk preferences. Using this test, the researcher can determine whether the hypothesis of identical risk preferences is rejected for the group of households that is being analyzed. If this hypothesis is not rejected one can use the tests proposed in the past. If this hypothesis is rejected, however, new tests are required that allow for preference heterogeneity. The second and third test derived here enable one to test efficient risk sharing when the null of identical risk preferences is violated.

The method that we propose is based on a new approach. Instead of using the first order conditions to derive the tests, we employ the household expenditure functions which relate household expenditure to aggregate resources. The use of the expenditure functions has four main advantages. First, the heterogeneity in risk preferences can be easily considered in the derivation of the tests. Second, we are able to derive tests that are non-parametric. The only restrictions that the household utility functions must satisfy are monotonicity and concavity. Third, the non-separability between consumption and leisure can be easily incorporated in the tests. Fourth, the method enables one to determine the fraction of households for which homogeneity in risk preferences and efficiency is rejected.

The third contribution of the paper is to show that the two testable implications on which the two efficiency tests are based are the only implications of efficient risk sharing if the following two conditions are satisfied. First, the only assumptions on the household utility functions are monotonicity and concavity. Second, no longitudinal variation in wages is observed or used in the efficiency test. This result implies that any testable implication of efficient risk sharing different from the ones derived here must be the result of additional assumptions on household preferences or longitudinal

variation in wages.

As a final contribution we use the method proposed in this paper to test efficient risk sharing in rural India. We find that it is crucial to allow for heterogeneity in risk preferences to understand the risk-sharing arrangements in Indian villages. Using data from the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) we strongly reject the hypothesis that households have identical preferences for risk. We then test efficient risk sharing at the village and caste level with and without the assumption of homogeneous risk preferences. At the village level, we reject efficiency in both cases. At the caste level, we reject the hypothesis that caste fellows share risk efficiently when we use the standard test. We cannot reject this hypothesis, however, when we use the non-parametric tests that allow for preference heterogeneity.

These findings suggest that the relevant risk-sharing unit in rural India is not the village, as previously suggested, but the caste. This result is consistent with recent evidence reported in Munshi and Rosenzweig (2006), where it is found that to understand migration patterns in rural India one should use the caste as the relevant social unit. In the last part of the paper we provide descriptive evidence on some of the institutions used by caste fellows to share risk. We find that transfers and loans between households that belong to the same caste are important sources of mutual insurance in rural India.

Recently, heterogeneity in risk preferences and its effect on risk sharing have been investigated in other papers. In an interesting project, Schulhofer-Wohl (2006) proposes a test of efficient risk sharing that allows for heterogeneity in risk preferences. His analysis differs from ours in several respects. First, the test is parametric and only valid under a particular specification of the utility function. Second, he does not control directly for heterogeneity in risk preferences. Instead, the heterogeneity is captured by allowing for nuisance parameters in the model. Third, because of this, the author cannot test whether the hypothesis of homogeneity in risk preferences is violated. The main advantage of Schulhofer-Wohl's method relative to ours is that it requires a shorter panel for its implementation. Dubois (2004) also allows for heterogeneity in risk preferences when he analyzes share-cropping agreements. In the paper, households have Constant Relative Risk Aversion (CRRA) utilities and the heterogeneity in risk preferences is introduced by allowing the coefficient of relative risk aversion to depend on observable variables. Mazzocco (2004) investigates the effect of heterogeneity in risk preferences on efficient risk sharing within a group. It is shown that with heterogeneous risk preferences efficient risk sharing can increase group savings even if it reduces the amount of uncertainty faced by the group. Hara, Huang, and Kuzmics (2006) analyze theoretically how the efficient risk sharing rule is affected by the heterogeneity in risk attitude.

The paper proceeds as follows. In section 2 we discuss the literature on efficient risk sharing. In section 3, we present a model of efficient risk sharing. In section 4, we derive the testable implications of homogeneity in risk preferences and efficiency. In section 5, we discuss the semi-parametric esti-

mation of the household expenditure functions required for the implementation of the tests. Section 6 describes the implementation of the tests. In section 7, the data used in the tests are described. Section 8 presents the results of a simulation study. Section 9 reports the results of the tests. Section 10 discusses the results. Section 11 concludes.

## 2 Tests of Efficient Risk Sharing in the Literature

In the past three decades many papers have tested efficient risk sharing using data from developed and developing countries. Some of the papers in this literature are Altug and Miller (1990), Cochrane (1991), Mace (1991), Altonji, Hayashi, and Kotlikoff (1992), Townsend (1994), Attanasio and Davis (1996), Hayashi, Altonji, and Kotlikoff (1996), Ravallion and Chaudhuri (1997), Ogaki and Zhang (2001), and Blundell, Pistaferri, and Preston (2006). The general interpretation of the findings in this literature is that efficient risk sharing is rejected. The large number of papers that develop models of partial insurance are evidence that this is the perception in the economic profession.<sup>1</sup> The papers in the risk sharing literature have one common feature. They assume that the households under investigation have the same risk preferences. In this section we will show that if in the data households have heterogeneous preferences for risk, tests based on the assumption of identical risk preferences reject efficiency even if the households under consideration share risk efficiently.<sup>2</sup>

To describe the effect of the assumption of identical risk preferences on previous tests, we will use the following simple example. Consider an economy composed of two households with heterogeneous preferences for risk which share risk efficiently. It is assumed that their preferences are separable across states of nature, over time, and between consumption and leisure. It is also assumed that they belong to the Harmonic Absolute Risk Aversion (HARA) class, i.e. the marginal utility of consumption can be written in the form  $u_c^i(c) = (a_i + c)^{-\gamma_i}$ . The economy is characterized by both idiosyncratic and aggregate shocks. To simplify the discussion, in this example we will assume that the only possible source of heterogeneity across households is represented by differences between household utility functions.

The intuition behind the efficient allocation of resources in this economy can be provided by dividing risk sharing into two parts. First, if the households share risk efficiently it is optimal for them to pool their resources and hence eliminate the idiosyncratic uncertainty that they face. We will

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<sup>1</sup>See for instance Kocherlakota (1996), Ligon (1998), Attanasio and Rios-Rull (2000), Ligon, Thomas, and Worrall (2002), and Heathcote, Storesletten, and Violante (2007).

<sup>2</sup>The only paper in this literature that partially addresses the issue of heterogeneity in risk preferences is the seminal work by Townsend (1994). Townsend's paper is divided into two parts. In one part, efficiency is tested under the assumption of identical risk preferences. This part is generally cited as evidence against efficient risk sharing. In the second part, the author allows for preference heterogeneity. This part has two limitations. First, it is assumed that preferences belong to the Constant Absolute Risk Aversion (CARA) class, which has been frequently criticized in the literature on decisions under uncertainty because of its inability to describe household behavior. Second, when preference heterogeneity is allowed, the test is performed by estimating the efficiency condition using a small number of observations. As a consequence the estimates on which the tests are based are imprecise and the outcome of the test difficult to interpret.

refer to this component of risk sharing as income pooling. Second, under efficiency the households should insure each other against aggregate shocks by allocating pooled resources according to their individual preferences for risk. This component of risk sharing will be denoted by the term mutual insurance.

Now suppose that the econometrician incorrectly assumes that the two households have identical risk preferences. We will show that in the economy under consideration mutual insurance has a significant effect on household behavior. The econometrician, however, by assuming that preferences are identical is imposing the restriction that there is no mutual insurance in the economy. We will show that this result holds by using two standard sets of conditions: the feasibility conditions and the efficiency conditions. Denote by  $t$  an arbitrary period, by  $\omega_t$  a realization of a state of nature in that period, and by  $h_t = (\omega_1, \dots, \omega_t)$  a history of realizations. Let  $c^i(h_t)$  be consumption of household  $i$ ,  $y^i(h_t)$  be the amount of resources available to household  $i$ , and define  $Y(h_t) = y^1(h_t) + y^2(h_t)$ . The feasibility condition for a history  $h_t$  can then be written as follows:

$$c^1(h_t) + c^2(h_t) = Y(h_t).$$

It indicates that under efficiency only pooled resources can affect household consumption. The efficiency condition for a history  $h_t$  can be written in the following form:

$$\mu_1 u_c^1(c^1(h_t)) = \mu_2 u_c^2(c^2(h_t)),$$

where  $\mu_i$  is the Pareto weight of household  $i$ . The efficiency condition shows that aggregate resources are optimally allocated according to individual preferences and Pareto weights.

Figures 1 and 2 use the feasibility and efficiency conditions to describe the efficient allocation of resources in the economy for different realizations of aggregate income  $Y(h_t)$ . To focus on the effect of preference heterogeneity, we will consider the case of identical Pareto weights. Similar results hold if the Pareto weights are allowed to differ. The dotted curve in figure 1 depicts the weighted marginal utility of the more risk averse household, whereas the solid line describes the weighted marginal utility of the second household. Figure 1 can be used to determine the efficient allocation of resources for each realization of aggregate income using the following two steps. First, for a given  $Y(h_t)$  draw a horizontal line to determine a pair of consumption levels at which the efficiency condition is satisfied. Second, move the horizontal line up or down until the two consumption levels satisfy the feasibility condition. Figure 2 is obtained by repeating this procedure for each realization of aggregate resources. It depicts efficient consumption for the two households as a function of aggregate resources. We will use the term expenditure functions to refer to the functions depicted in Figure 2.

The two figures show that if preferences are heterogenous the weighted marginal utilities will generally cross. As a consequence the expenditure functions will also cross. To provide some insight on the meaning of the crossing, consider the realizations of aggregate resources that generate household expenditures that are to the left of the crossing point. In this case the household that is more

risk averse consumes more than half of aggregate resources. The allocations in this region can be interpreted as the outcome of the insurance provided by the less risk averse household against adverse realizations of aggregate resources. Consider now the region to the right of the crossing point. In this case, the household that is less risk averse consumes more than half of aggregate resources as a compensation for the insurance provided against adverse aggregate shocks. To summarize, in this economy the households first eliminate the idiosyncratic uncertainty by pooling their resources and then insure each other against aggregate shocks by allocating the aggregate resources according to their preferences for risk.

We will now describe the efficient allocations of resources under the assumption made by the econometrician that households have identical risk preferences. Figures 3 and 4 depict efficient risk sharing for the general case of different Pareto weights. They show that the weighted marginal utilities and hence the household expenditure functions can never cross. The reason for this result is straightforward. Since it is assumed that the two households have identical risk preferences there is no scope for mutual insurance. As a consequence, the household with higher Pareto weight will always consume more than half of aggregate resources. We can therefore conclude that the assumption of identical preferences for risk is equivalent to the restriction that mutual insurance is an irrelevant component of efficient risk sharing.

Using this result we can determine how the tests developed in the risk sharing literature perform when they are used to test efficiency in an economy characterized by heterogeneous risk preferences. To this end, consider the following generalization of the efficiency test initially proposed by Mace (1991). Under the assumptions made by the econometrician that the two households in the economy have identical HARA preferences and share risk efficiently, it is straightforward to show that the following relationship between household and aggregate consumption must be satisfied:

$$f(c_{t+1}^i) - f(c_t^i) = \frac{1}{2} \sum_{j=1}^2 \left( f(c_{t+1}^j) - f(c_t^j) \right), \quad (1)$$

where  $f(c)$  is a transformation of consumption that varies with the utility function chosen to characterize the household preferences. Specifically,  $f(c) = c$  for CARA preferences,  $f(c) = \log(c)$  for CRRA preferences, and  $f(c) = \log(a + c)$  for the general class of HARA preferences. This generalization of Mace's test is useful because the tests used in the papers cited above are a special case of it. To see this note that under the assumption of CARA preferences, equation (1) establishes that the first difference in household consumption must equal the first difference in aggregate consumption, which is the test used in Mace (1991), Townsend (1994), and Ravallion and Chaudhuri (1997). If preferences are assumed to belong to the CRRA class, according to equation (1) household consumption growth must equal aggregate consumption growth, which is the test used in Cochrane (1991), Mace (1991), and Altonji, Hayashi, and Kotlikoff (1992).<sup>3</sup> Finally if the utility function belongs to

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<sup>3</sup>Altug and Miller (1990), Hayashi, Altonji, and Kotlikoff (1996), and in part of their work Attanasio and Davis (1996)

the general HARA class, equation (1) still equates household consumption growth with aggregate consumption growth, but the growth rate must be computed taking into account the subsistence level  $a$ . This is the test used in Ogaki and Zhang (2001).<sup>4</sup>

Now consider two periods with the following features. In the first period the economy is characterized by an adverse realization of aggregate resources, i.e. aggregate resources are on the left of where the expenditure functions cross. In second period the economy is characterized by a good realization of aggregate resources, i.e. aggregate resources are on the right of the crossing point. Since the two households have different preferences for risk, they will insure each other against aggregate shocks. This implies that between the two periods consumption of the more risk averse household will vary less than aggregate consumption and consumption of the less risk averse household will vary more. Formally, the equality tested in previous papers is replaced by the following inequalities:

$$f(c_{t+1}^1) - f(c_t^1) < \frac{1}{2} \sum_{j=1}^2 \left( f(c_{t+1}^j) - f(c_t^j) \right) < f(c_{t+1}^2) - f(c_t^2).$$

This implies that, if in the data used by previous papers households have heterogeneous risk preferences, efficient risk sharing should have been rejected even if households are sharing risk efficiently.<sup>5</sup>

One additional aspect of the tests used in the past should be discussed before claiming that heterogeneity in risk preferences may explain previous rejections. In the risk sharing literature the efficiency condition (1) is tested by adding a variable that captures idiosyncratic shocks to the equation and by verifying whether the coefficient on this variable is statistically significant. Most of the papers add changes in household income to equation (1) and they find that the coefficient is statistically significant and positive. It is important to understand whether heterogeneity in risk preferences can explain the positive coefficient.

We will show that preference heterogeneity can explain the positive coefficient if less risk averse households have more volatile income processes. For ease of exposition, we will further simplify the economy considered in this section by assuming that the only heterogeneity is represented by the curvature parameter  $\gamma_i$ , i.e.  $u_c^i(c) = (a + c)^{-\gamma_i}$  and the Pareto weights are identical. In this economy the expenditure functions have two main properties. First, they are functions of aggregate resources. Second, the functions differ across households and the variation is captured by the heterogeneity in the curvature parameter  $\gamma_i$ . This implies that the transformation  $f(c)$  of the expenditure functions

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use preferences that are non-separable between consumption and leisure. The intuition provided in this section applies also to those papers. However, a model with nonseparable preferences allows for more general patterns of household consumption. We consider this more general case starting from the next section.

<sup>4</sup>Ogaki and Zhang (2001) apply the test to two different sets of data: the data collected by the International Food Policy Research Institute (IFPRI) for Pakistani households and the ICRISAT data for Indian households. When they use the test initially introduced by Mace (1991) they do not reject efficiency in Pakistani villages, but they reject this hypothesis for Indian villages. These findings are consistent with the discussion of this section. Homogeneity in risk preferences can be a good assumption in some environments, but a bad assumption in others.

<sup>5</sup>The test proposed by Cochrane (1991) is affected by the same problem. The assumption of identical CRRA preferences enables one to include in the constant the terms that capture aggregate quantities,  $(1/\gamma_j) \log(\mu_{t+1}/\mu_t)$  in equation (8) in Cochrane (1991). If preferences are heterogeneous the constant will generally be smaller for more risk averse households and larger for less risk averse households. Therefore the inequality will generally still hold.

can be written in the following form:

$$f(c_t^i) = g(C_t^a, \gamma_i).$$

We can now linearize the transformed expenditure functions by taking a first order Taylor expansion around the average  $\gamma_i$  in the economy. We can then compute the first difference of the linearized functions to obtain the following equation:

$$\Delta f(c_t^i) \simeq \Delta g(C_t^a, \bar{\gamma}) + (\gamma_i - \bar{\gamma}) \left. \frac{\partial}{\partial \gamma_i} \Delta g(C_t^a, \gamma_i) \right|_{\bar{\gamma}}. \quad (2)$$

This equation describes the dynamics of efficient expenditure in an economy with heterogeneity in risk preferences.

As mentioned above, in previous papers the efficiency test has generally been performed by estimating the following equation:

$$\Delta f(c_t^i) - \Delta g(C_t^a, \bar{\gamma}) = \alpha + \xi \Delta y_t^i + \epsilon_t^i, \quad (3)$$

for some function  $g(\cdot)$ , where  $y^i$  is income of household  $i$ . A comparison of equations (2) and (3) clarifies the following two points. First, since in the economy households have heterogeneous risk preferences, there is an omitted variable in equation (3), namely  $(\gamma_i - \bar{\gamma}) \left. \frac{\partial}{\partial \gamma_i} \Delta g(C_t^a, \gamma_i) \right|_{\bar{\gamma}}$ . Second, since the households share risk efficiently, the coefficient  $\xi$  on income should be equal to zero. These two points and standard results on the bias generated by omitted variables imply that the expected value of the least square estimate of the coefficient on income can be written as follows:

$$E(\hat{\xi}) = VAR(\Delta y_t^i)^{-1} Cov\left(\Delta y_t^i, (\gamma_i - \bar{\gamma}) \left. \frac{\partial}{\partial \gamma_i} \Delta g(C_t^a, \gamma_i) \right|_{\bar{\gamma}}\right).$$

Consequently, if the covariance on the right hand side is positive, one should expect  $\hat{\xi}$  to be positive. The sign of the covariance can be determined by observing that mutual insurance implies that  $\Delta f(c_t^i)$  is larger for households with risk aversion below the average. From equation (2) it therefore follows that  $(\gamma_i - \bar{\gamma}) \left. \frac{\partial}{\partial \gamma_i} \Delta g(C_t^a, \gamma_i) \right|_{\bar{\gamma}}$  is larger for households with risk aversion below the average. Now suppose that households that are less risk averse have more volatile income processes. Then it must be that

$$Cov\left(\Delta y_t^i, (\gamma_i - \bar{\gamma}) \left. \frac{\partial}{\partial \gamma_i} \Delta g(C_t^a, \gamma_i) \right|_{\bar{\gamma}}\right) > 0,$$

which implies that the coefficient on individual income will be on average positive. Finally, observe that one should expect households that are less risk averse to choose income processes with higher volatility if the following two conditions are fulfilled: households choose their income process before entering the risk-sharing agreement and the economy is characterized by aggregate shocks.

The discussion in this section indicates that if in the economy under investigation preferences for risk are heterogeneous the tests used in the past reject efficiency even if households share risk

efficiently. It is therefore important to derive a test that enables one to verify whether the null of homogeneity in risk preferences is rejected in the data. If it is not, one can use the standard tests. If it is rejected, a test of efficiency is required that allows for differences in risk preferences. The rest of the paper is devoted to deriving tests that enable one to evaluate the hypotheses of homogeneity in risk preferences and efficiency.

### 3 A Model of Efficient Risk Sharing

We use a standard model to characterize efficient risk sharing. In this section we outline its main features and we derive a new result which is crucial for setting up the homogeneity and efficiency tests. Consider an economy in which households live for  $\tau$  periods. For a given history of realizations  $h_t$ , in period  $t$  household  $i$  is endowed with a wage  $w_t^i(h_t)$  and a total amount of time  $T_t^i(h_t)$  that can be divided between leisure and labor. The aggregate amount of non-labor resources in the economy is denoted by  $Y_t(h_t)$ , where  $Y_t(h_t)$  may include profits and savings. Let  $c_t^i(h_t)$  and  $l_t^i(h_t)$  be, respectively, consumption and leisure of household  $i$  in period  $t$  conditional on the history  $h_t$ . Household preferences are assumed to be separable over time and across states of nature. They are allowed to depend on observable and unobservable heterogeneity, which will be denoted by  $z_t^i(h_t)$  and  $\eta_t^i(h_t)$ . The corresponding utility function  $u^i[c_t^i(h_t), l_t^i(h_t); z_t^i(h_t), \eta_t^i(h_t)]$  is assumed to be strictly increasing, strictly concave, and twice continuously differentiable in consumption and leisure. Households have a common discount factor  $\beta$  and share the same beliefs over histories of realizations, which are denoted by  $P(h_t)$ .

Efficient risk sharing in this economy can be described using a standard Pareto problem. Let  $\mu_i$  be the Pareto weight assigned to household  $i$  with  $\sum_i^n \mu_i = 1$  and suppose for simplicity that  $0 < \mu_i < 1$ . The efficient allocation of resources is then the solution of the following problem:

$$\begin{aligned} & \max_{\{c_t^i(h_t), l_t^i(h_t)\}} \sum_{i=1}^n \mu_i \sum_{t=1}^{\tau} \beta^t \sum_{h_t} P(h_t) u^i[c_t^i(h_t), l_t^i(h_t); z_t^i(h_t), \eta_t^i(h_t)] & (4) \\ \text{s.t. } & \sum_{i=1}^n (c_t^i(h_t) + w_t^i(h_t) l_t^i(h_t)) = Y_t(h_t) + \sum_{i=1}^n w_t^i(h_t) T_t^i(h_t) \quad \text{for each } t, h_t \\ & c_t^i(h_t) > 0, \quad 0 \leq l_t^i(h_t) \leq T_t^i(h_t) \quad \text{for each } t, h_t, \end{aligned}$$

where the right hand side of the resource constraint is full income in the economy.

The test of homogeneous risk preferences and the two efficiency tests will be derived using the expenditure functions discussed in the previous section and depicted in Figures 2 and 4. With non-separability between consumption and leisure, household expenditure in period  $t$  is equal to  $c_t^i + w_t^i l_t^i$ . One possible approach that can be used to derive the expenditure functions is to solve the Pareto problem (4) with respect to consumption and leisure and then substitute the solution in household expenditure. This approach, however, has one major limitation. The expenditure functions obtained

using this method depend on the wages and heterogeneity variables of each household in the economy, i.e.

$$\rho_t^k = c_t^i + w_t^i l_t^i = \bar{\rho}^k (\bar{Y}_t; w_t^1, \dots, w_t^n, z_t^1, \dots, z_t^n, \eta_t^1, \dots, \eta_t^n) \quad \text{for } k = 1, \dots, n,$$

where  $\bar{Y}$  is full income.<sup>6</sup> A test based on these expenditure functions is therefore not feasible for two reasons. First, in every dataset one only observes a fraction of the households that compose the economy. Some of the variables in the expenditure functions are therefore not observed. Second, even if all the variables were observed it would generally be infeasible to estimate a function that depends on so many variables. It is important to remark that the majority of the tests used in the past are affected by a similar problem. In those tests one has to control for aggregate resources in the economy, but the researcher only observes the resources of the households in the dataset. We solve this problem by using a three-stage formulation of the economy which has the same solution as the standard Pareto program.

We will now describe the three-stage formulation starting from the last stage. Let  $\rho_t^i(h_t)$  be an arbitrary amount of aggregate resources allocated to household  $i$  in period  $t$  conditional on the history  $h_t$ . In the last stage, conditional on  $\rho_t^i(h_t)$  household  $i$  chooses consumption and leisure for period  $t$  and history  $h_t$ , by solving the following individual problem:

$$\begin{aligned} V^i(\rho_t^i(h_t); w_t^i(h_t), z_t^i(h_t), \eta_t^i(h_t)) &= \max_{c_t^i(h_t), l_t^i(h_t)} u^i [c_t^i(h_t), l_t^i(h_t); z_t^i(h_t), \eta_t^i(h_t)] \\ \text{s.t. } c_t^i(h_t) + w_t^i(h_t) l_t^i(h_t) &= \rho_t^i(h_t) \\ c_t^i(h_t) \geq 0, \quad 0 \leq l_t^i(h_t) &\leq T_t^i(h_t). \end{aligned}$$

Consider now the second stage. Let  $\rho_t^{i,j}(h_t)$  denote an arbitrary amount of aggregate resources allocated to the pair composed of households  $i$  and  $j$ . In the intermediate stage, conditional on  $\rho_t^{i,j}(h_t)$  the pair chooses the optimal amount of resources to allocate to households  $i$  and  $j$  in period  $t$  and history  $h_t$  by solving the following problem:<sup>7</sup>

$$\begin{aligned} V^{i,j}(\rho_t^{i,j}; w_t^i, w_t^j, z_t^i, z_t^j, \eta_t^i, \eta_t^j) &= \max_{\rho_t^i, \rho_t^j} \mu_i V^i(\rho_t^i; w_t^i, z_t^i, \eta_t^i) + \mu_j V^j(\rho_t^j; w_t^j, z_t^j, \eta_t^j) \\ \text{s.t. } \rho_t^i + \rho_t^j &= \rho_t^{i,j}. \end{aligned}$$

It is worth discussing two features of the second-stage problem. First, since  $\rho_t^k = c_t^k + w_t^k l_t^k$ , its solution provides the expenditure functions on which the tests will be based. Second, the expenditure functions obtained using this stage are only function of wages and heterogeneity of households  $i$  and  $j$ , i.e.

$$\rho_t^k = \rho_t^k(\rho_t^{i,j}, w_t^i, w_t^j; z_t^i, z_t^j, \eta_t^i, \eta_t^j) \quad \text{for } k = i, j.$$

<sup>6</sup>Note that the assumption that preferences are separable over time and across states enables us to write the expenditure functions only as a function of variables in period  $t$  and history  $h_t$ .

<sup>7</sup>Unless required for expositional clarity, the dependence on  $h_t$  will be suppressed in the rest of the paper.

Since in many datasets one observes wages and heterogeneity variables for all pairs of households in the sample, tests based on expenditure functions are feasible as long as one uses the expenditure functions obtained in the intermediate stage.

In the first stage full income is optimally allocated to each pair of households. Conditional on the amount of resources available in the economy in period  $t$  and history  $h_t$ , each pair receives an allocation that is the solution of the following problem:<sup>8</sup>

$$V \left( Y_t + \sum_{i=1}^n w_t^i T_t^i; w_t, z_t, \eta_t \right) = \max_{\{\rho_t^{2i-1, 2i}\}} \sum_{i=1}^{n/2} V^{2i-1, 2i} \left( \rho_t^{2i-1, 2i}; w_t^{2i-1}, w_t^{2i}, z_t^{2i-1}, z_t^{2i}, \eta_t^{2i-1}, \eta_t^{2i} \right)$$

$$s.t. \quad \sum_{i=1}^{n/2} \rho_t^{2i-1, 2i} = Y_t + \sum_{i=1}^n w_t^i T_t^i,$$

where  $w_t$ ,  $z_t$ , and  $\eta_t$  are the vectors of wages and heterogeneity variables.

Under the standard assumptions that preferences are separable over time and across states of nature, it follows that the solution of the three-stage problem is equivalent to the original Pareto problem, i.e.

$$\sum_{t=1}^{\tau} \beta^t \sum_{h_t} P(h_t) V \left( Y_t(h_t) + \sum_{i=1}^n w_t^i(h_t) T_t^i(h_t); w_t(h_t), z_t(h_t), \eta_t(h_t) \right) =$$

$$\max_{\{c_t^i(h_t), l_t^i(h_t)\}} \sum_{i=1}^n \mu_i \sum_{t=1}^{\tau} \beta^t \sum_{h_t} P(h_t) u^i [c_t^i(h_t), l_t^i(h_t); z_t^i(h_t), \eta_t^i(h_t)]$$

$$s.t. \quad \sum_{i=1}^n (c_t^i(h_t) + w_t^i(h_t) l_t^i(h_t)) = Y_t(h_t) + \sum_{i=1}^n w_t^i(h_t) T_t^i(h_t) \quad \text{for each } t, h_t$$

$$c_t^i(h_t) \geq 0, \quad 0 \leq l_t^i(h_t) \leq T_t^i(h_t) \quad \text{for each } t, h_t.$$

The intuition for this result is straightforward. Under the assumptions of this paper, the conditions of the second welfare theorem are fulfilled. As a result the solution of the Pareto problem can be decentralized using transfers.

## 4 Testable Implications

In this section we derive the testable implications on which the test of homogeneity in risk preferences and the efficiency tests are based. They are all derived using the expenditure functions obtained from the intermediate stage of the efficiency problem described in the previous section. The discussion will be divided into three parts. In the first part we will consider an economy where households share risk efficiently and derive a restriction that will enable us to test whether preferences are homogeneous

<sup>8</sup>The Pareto problem can be decomposed in three stages by pairing households in different ways. Here we consider one possible set of pairs under the assumption that there is an even number of households in the economy. If  $n$  is odd, three households will have to be arranged in one group.

across households. In the second part we will recover a necessary condition of efficient risk sharing that can be used to test efficiency even if preferences are heterogeneous. In the last part, we will show that the necessary condition is also sufficient if no intertemporal variation in real wages is observed or used.

We will first derive a testable implication for homogeneity in risk preferences. The implication is based on the following idea. Consider households  $i$  and  $j$  and suppose that they share risk efficiently. The discussion in section 2 implies that if their expenditure functions cross, mutual insurance must be a significant component of efficient risk sharing. If mutual insurance is an important feature of risk sharing, the two households must have heterogeneous preferences. One can therefore conclude that under efficiency if the expenditure functions cross the hypothesis of homogeneous risk preferences can be rejected. This idea is formalized in the following proposition.

**Proposition 1** *Suppose that household  $i$  and  $j$  share risk efficiently. If there exist two realizations of aggregate resources  $\rho^{i,j}$  and  $\bar{\rho}^{i,j}$  such that*

$$\rho^i(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j) > \rho^j(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$$

and

$$\rho^i(\bar{\rho}^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j) < \rho^j(\bar{\rho}^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j),$$

household  $i$  and household  $j$  cannot have identical preferences.

**Proof.** In the appendix. ■

Three remarks are in order. First, Proposition 1 can easily be used to set up a test of preference homogeneity. For each pair of households in the data one can recover their expenditure functions and verify whether they cross or not. Second, the preference homogeneity test is valid only under the maintained assumption that households share risk efficiently. The test is therefore useful only if the final goal is to test efficiency. Third, if the test does not reject homogeneity in risk preferences one can test for efficient risk sharing using the standard approach. However, if the hypothesis is rejected new tests of efficiency are required that allow for heterogeneity in risk preferences.

We will now derive a necessary condition of efficient risk sharing that is satisfied even if households have different risk preferences. Consider households  $i$  and  $j$  and observe that under efficiency the following two restrictions must be fulfilled. First, after controlling for differences across households in real wages, observable and unobservable heterogeneity, only aggregate resources should affect the expenditure of households  $i$  and  $j$ . Hence, conditional on  $w^i, w^j, z^i, z^j, \eta^i$ , and  $\eta^j$ , for each realization of  $\rho^{i,j}$  only one value should be observed for  $\rho^i(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  and  $\rho^j(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$ . Second, an increase in aggregate resources should increase expenditure of both households. If one of these restrictions is not satisfied, household behavior is not only affected by changes in aggregate

resources as predicted by efficient risk sharing, but also by idiosyncratic shocks. These two restrictions imply that a necessary condition for efficient risk sharing is that  $\rho^i(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  and  $\rho^j(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  must be strictly increasing functions of aggregate resources. The following proposition formalizes this result.

**Proposition 2** *Suppose that the utility functions of households  $i$  and  $j$  are strictly increasing and concave. Then, if households  $i$  and  $j$  share risk efficiently,  $\rho^i(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  and  $\rho^j(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  are strictly increasing functions of aggregate resources.*

**Proof.** In the appendix. ■

The result presented in Proposition 2 contains two testable implications. First, after controlling for aggregate resources, wages, observable and unobservable heterogeneity the expenditure functions should not depend on other variables that capture idiosyncratic shocks. This testable implication is a generalization of the standard test of efficiency to an environment with heterogeneous risk preferences. The second testable implication can be described as follows. After controlling for wages, observable and unobservable heterogeneity, the expenditure functions should be increasing in aggregated resources. To the best of our knowledge this is the first paper to propose this restriction as a test of efficiency. It has the advantage relative to the first implication that there is no need to use variables related to idiosyncratic shocks, the choice of which is often arbitrary.

The condition described in Proposition 2 is also sufficient for efficient risk sharing if no intertemporal variation in real wages is observed or used to test efficiency. There are two situations in which this assumption on wages is satisfied. First, only cross-sectional variation in wages is observed or used by the researcher. Second, household preferences are assumed to be separable between consumption and leisure, which implies that the variation in wages is not exploited in the efficiency tests. These two situations are the ones considered by the risk sharing literature. Mace (1991), Townsend (1994), Altonji, Hayashi, and Kotlikoff (1992), Ravallion and Chaudhuri (1997), and Ogaki and Zhang (2001) assume separability between consumption and leisure. Cochrane (1991) uses a cross-section of households and hence ignores any longitudinal variation in real wages. Altug and Miller (1990) and Hayashi, Altonji, and Kotlikoff (1996) exploit longitudinal consumption variation after controlling for longitudinal variation in leisure. This is equivalent to using longitudinal variation in consumption after having removed the portion that is explained by longitudinal variation in real wages.

The next proposition formally describes the sufficient condition for efficiency. It shows that if  $\rho^i(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  and  $\rho^j(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  are strictly increasing functions of  $\rho^{i,j}$ , then it is always possible to find increasing and concave utility functions and Pareto weights such that efficiency is satisfied.

**Proposition 3** *Suppose that preferences are separable between consumption and leisure or  $w^i$  and  $w^j$  do not vary. Then, if  $\rho^i(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  and  $\rho^j(\rho^{i,j}, w^i, w^j; z^i, z^j, \eta^i, \eta^j)$  are strictly in-*

creasing functions of aggregate resources, there exist utility functions that are strictly increasing and concave such that the two households share risk efficiently.

**Proof.** In the appendix. ■

Proposition 3 implies that if only variation in expenditure is used, the only testable implications of efficient risk sharing are the two implications described in Proposition 2. Any other restriction is the outcome of the particular functional form selected for household preferences.<sup>9</sup>

## 5 Estimation of the Expenditure Functions

The testable implications derived in the previous section can be used to set up tests of homogeneity in risk preferences and efficiency only if an estimate of the household expenditure functions is available. In this section we will discuss the method used in this paper in the estimation of these functions.

The estimation approach must allow for heterogeneity in risk preferences. In an environment with different preferences, there are two features of the expenditure functions that must be considered. First, households with heterogeneous preferences have expenditure functions with different functional forms. To address this issue we will estimate the expenditure functions separately for each household. Second, if risk preferences are heterogeneous there is no close form solution for the expenditure functions except for the frequently criticized case of CARA utilities. For this reason we will employ a semi-parametric estimator.

Without data limitations, the expenditure functions can be estimated non-parametrically separately for each household without making additional assumptions. The dataset employed in this paper has, however, some limitations that will be discussed in a later section. To overcome them, we will impose four restrictions on the way observable and unobservable heterogeneity and measurement errors enter the expenditure functions. First, we will assume that the observable and unobservable heterogeneity enter the expenditure functions only as a linear combination of the difference between  $z^i$  and  $z^j$ , and  $\eta^i$  and  $\eta^j$ , i.e. only  $d^{i,j} = \theta_{i,j}(z^i - z^j) + \eta^i - \eta^j$  affects  $\rho^k$ . This assumption simplifies significantly the estimation since all the variation in heterogeneity is captured by the single index  $d^{i,j}$ . Second, it is assumed that the unobservable component of heterogeneity does not change over time. Under this assumption, the effect of  $\eta^i - \eta^j$  can be captured by adding a constant for the pair of households  $i$  and  $j$  to the vector of observables  $z$ . The constant captures unobservable preference shocks that are very persistent.<sup>10</sup> As a third assumption, we will impose the restriction that the coefficients on the observable heterogeneity variables are common across households to increase the

<sup>9</sup>Hara (2006) derives a similar result under the assumption that preferences are separable between consumption and leisure. Our result is more general in the following sense. We show that without separability between consumption and leisure the condition that expenditure is an increasing function of aggregate resources is sufficient for efficient risk sharing only if there is no intertemporal variation in wages.

<sup>10</sup>The estimator can be modified to accommodate a term  $\eta^i - \eta^j$  that varies over time. In this case, one can follow Blundell and Powell (2001) and consider the expectation of the expenditure functions taken over  $\eta^i - \eta^j$ .

precision of our estimates. Finally, we will allow for measurement errors  $m$  in individual expenditure, which are assumed to be additive and independent of  $w^i$ ,  $w^j$ , and  $d^{i,j}$ . Under these assumptions, the expenditure function of household  $k$  can be written as follows:<sup>11</sup>

$$\rho^k = g^k(\rho^{i,j}, w^i, w^j, d^{i,j}) + m^k. \quad (5)$$

Three issues are worth a discussion. First, there is a large class of utilities that generates the expenditure function described in (5). For instance, all the utility functions that can be written in the following form:

$$u^i(c^i, l^i, z^i, \eta^i) = v(c^i, l^i) \exp(\theta z^i + \eta^i)$$

produce expenditure functions consistent with (5). Second, in the model  $\rho^{i,j}$  is equal to the sum of individual expenditures. In the estimation,  $\rho^{i,j}$  will be computed using this relationship. As a consequence, if the data on individual expenditure are affected by measurement errors, the data on aggregate resources will also be affected by the same problem. The method used in the estimation must therefore allow for measurement errors in  $\rho^{i,j}$ . Third, the assumptions on observable and unobservable heterogeneity are needed only if one is interested in the economic meaning of  $z$  and  $\eta$  and in whether an economic model can generate the particular functional form imposed on the expenditure functions. If one is not interested in these issues, observable and unobservable heterogeneity can be introduced by simply adding a polynomial in  $z$  and  $\eta$  to  $g^k(\rho^{i,j}, w_t^i, w_t^j)$ .

Under the assumptions listed above, the expenditure functions can be estimated using the combination of two estimators available in the non-parametric literature. The first one is the estimator developed by Newey et al. (1999). For a given single index  $d^{i,j}$  it enables us to estimate non-parametrically the expenditure functions controlling for the endogeneity of  $\rho^{i,j}$ . The second one is the estimator proposed by Ichimura (1993), which will be used to estimate the parameters of the single index  $d^{i,j}$ .

We will briefly describe how the two estimators can be employed to recover the expenditure functions. Suppose first that the parameters defining the heterogeneity term  $d^{i,j}$  are known. Observe that the expenditure functions cannot be estimated using standard non-parametric methods because  $E[m^k | \rho^{i,j}] \neq 0$ . To address this issue, let  $q$  be a set of instruments in the sense that the following conditions are satisfied:

$$\rho^{i,j} = h(q^{i,j}) + u^{i,j}, \quad E[u^{i,j} | q^{i,j}] = 0, \quad \text{and} \quad E[m^k | u^{i,j}, q^{i,j}] = E[m^k | u^{i,j}].$$

Then, we have that

$$\begin{aligned} E[\rho^k | \rho^{i,j}, w^i, w^j, d^{i,j}, q^{i,j}] &= g^k(\rho^{i,j}, w^i, w^j, d^{i,j}) + E[m^k | \rho^{i,j}, w^i, w^j, d^{i,j}, q^{i,j}] \\ &= g^k(\rho^{i,j}, w^i, w^j, d^{i,j}) + E[m^k | u^{i,j}, q^{i,j}] = g^k(\rho^{i,j}, w^i, w^j, d^{i,j}) + \lambda(u^{i,j}), \end{aligned}$$

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<sup>11</sup>The time subscript will be suppressed here and in the remaining sections to simplify the notation.

where  $\lambda(u) = E[m^k|u]$ . Newey et al. (1999) propose to estimate the function  $g^k$  in two steps. In the first step the error term  $u$  is estimated non-parametrically as  $\hat{u}^{i,j} = \rho^{i,j} - \hat{h}(q^{i,j})$ . In the second step, the function  $g^k(\rho^{i,j}, w^i, w^j, d^{i,j}) + \lambda(u^{i,j})$  is estimated using the estimated residuals in place of the true ones. An estimator of the function  $g^k$  can then be recovered by isolating the components that do not depend on the residuals  $u$ . The estimation will be performed using the series estimator proposed by Newey et al. (1999) with polynomials.

The parameters of the heterogeneity term  $d^{i,j}$  are not known, but they can be estimated using one of the semi-parametric methods developed for the estimation of single-index models. We use the semi-parametric least square approach proposed by Ichimura (1993).

## 6 The Tests

In this section we discuss how the tests of preference homogeneity and efficiency can be implemented using the testable implications derived in section 4. The discussion will be divided into two parts. In the next three subsections, we will describe how one can derive test statistics for each pair of households in the data. These test statistics can be used to evaluate preference heterogeneity and efficiency separately for each observed pair of households. The final goal, however, is to set up tests that can reject the null hypotheses for the entire group of households. In the last subsection we discuss how the test statistics computed for each pair can be combined to construct these tests.

### 6.1 Test of Homogeneous Risk Preferences For a Pair of Households

The test of homogeneity in risk preferences for a pair of households is based on Proposition 1, which states that under efficiency if two households have identical risk preferences their expenditure functions should not cross. To simplify the notation we will suppress the dependence of the expenditure functions on wages, observable and unobservable heterogeneity.

The test can be constructed using the following idea. Consider the pair composed of households  $i$  and  $j$ , suppose that they share risk efficiently, and denote by  $g^{i,j}$  the difference in their expenditure functions. Under the null of identical preferences,  $g^{i,j}$  as a function of aggregate resources should always be either positive or negative since there cannot be a crossing. As a consequence, the area below the positive part of  $g^{i,j}$  multiplied by the area below the negative part of  $g^{i,j}$  is equal to zero under the null, but it is positive under the alternative. Formally,

$$\xi_1^{i,j} = - \left( \int_{\{u: g^{i,j}(u) \geq 0\}} g^{i,j}(u) du \right) \left( \int_{\{u: g^{i,j}(u) < 0\}} g^{i,j}(u) du \right) \begin{cases} = 0 & \text{under } H_0 \\ > 0 & \text{under } H_A \end{cases} .$$

Thus, if  $\xi_1^{i,j}$  is positive the null hypothesis of homogeneous risk preferences can be rejected.<sup>12</sup>

<sup>12</sup>We have also experimented with the simpler test statistic  $\bar{\xi}^{i,j} = \max\{g^{i,j}\} \min\{g^{i,j}\}$ . The simulation study that will be discussed later in the paper suggests, however, that a test based on  $\bar{\xi}^{i,j}$  has less power and control than a test based on  $\xi^{i,j}$ . The difference in power and control is especially large in the case of measurement errors with high variance.

Using this idea the test can be implemented in three steps. In the first step, the difference between expenditure functions is estimated using the method discussed in the previous section. Let  $\hat{g}^{i,j}$  be the estimated difference. In the second step, the test statistic  $\hat{\xi}_1^{i,j}$  is computed as follows:

$$\hat{\xi}_1^{i,j} = - \left( \sum_{l=1}^n \mathbf{1}_{\{\hat{g}^{i,j}(\rho_l^{i,j}) \geq 0\}} \hat{g}^{i,j}(\rho_l^{i,j}) \right) \left( \sum_{l=1}^n \mathbf{1}_{\{\hat{g}^{i,j}(\rho_l^{i,j}) < 0\}} \hat{g}^{i,j}(\rho_l^{i,j}) \right),$$

where  $\mathbf{1}_{\{\cdot\}}$  is an indicator function. In the final step, the distribution of  $\hat{\xi}_1^{i,j}$  is recovered by bootstrap. To increase the power of the test, we follow Hall and Wilson (1991) and compute the bootstrap distribution by resampling  $\hat{\xi}_1^{i,j*} - \hat{\xi}_1^{i,j}$  instead of  $\hat{\xi}_1^{i,j*}$ , where  $\hat{\xi}_1^{i,j*}$  is the estimated test statistic obtained using a bootstrap sample. The null is then rejected for the pair composed of households  $i$  and  $j$  if  $\hat{\xi}_1^{i,j}$  is too large, i.e. if

$$\hat{\xi}_1^{i,j} > q^{i,j*}(0.95),$$

where  $q^{i,j*}(0.95)$  is the 95-th percentile of  $\hat{\xi}_1^{i,j*} - \hat{\xi}_1^{i,j}$ .

## 6.2 Test of Efficiency with Omitted Variables for a Pair of Households

The first test of efficiency is based on the first testable implication of Proposition 2 according to which any variable that captures idiosyncratic shocks should not enter the expenditure functions. This implication is tested by employing the semi-parametric approach proposed by Fan and Li (1996). We will briefly describe the method.

Consider the pair composed of households  $i$  and  $j$  and suppose that the expenditure function of household  $i$  depends on an omitted variable  $y^i$ . Then under the assumptions of sections 5 the function can be written in the following form:

$$\rho^i = g^i(\rho^{i,j}, w^i, w^j, d^{i,j}, y^i) + \lambda(u^{i,j}) + \epsilon^i = f^i(X^{i,j}, y^i) + \epsilon^i,$$

where  $E[\epsilon^i | X^{i,j}, y^i] = 0$ . Under the null hypothesis of efficiency,  $y^i$  should not enter the expenditure function. As a consequence

$$f^i(X^{i,j}, y^i) = E[\rho^i | X^{i,j}, y^i] = E[\rho^i | X^{i,j}] = r^i(X^{i,j}).$$

But under the alternative hypothesis we have that

$$f^i(X^{i,j}, y^i) \neq r^i(X^{i,j}).$$

Let  $\nu^i = \rho^i - r^i(X^{i,j})$ . Then,  $E[\nu^i | X^{i,j}, y^i] = f^i(X^{i,j}, y^i) - r^i(X^{i,j}) = 0$  under the null and  $E[\nu^i | X^{i,j}, y^i] \neq 0$  under the alternative. Now observe that

$$E[\nu^i E[\nu^i | X^{i,j}, y^i]] = E[\{E[\nu^i | X^{i,j}, y^i]\}^2] \geq 0,$$

where the inequality follows from the previous discussion and the inequality is replaced by an equality if and only if the null hypothesis is correct. Fan and Li propose to test for the omitted variable  $y^i$  using this inequality and the following idea. If  $\nu^i$  and  $E[\nu^i|X^{i,j}, y^i]$  were known, one could estimate  $E[\nu^i E[\nu^i|X^{i,j}, y^i]]$  using its sample analog  $n^{-1} \sum_i \nu^i E[\nu^i|X^{i,j}, y^i]$ . The authors suggest to replace the residuals  $\nu^i$  with estimated ones and  $E[\nu^i|X^{i,j}, y^i]$  with its kernel estimator. Finally, to overcome the random denominator problem in the kernel estimation, they propose to replace the sample analog with its density weighted version:

$$\frac{1}{n} \sum_i [\nu^i f(X^{i,j})] E[\nu^i f(X^{i,j}) | X^{i,j}, y^i] f(X^{i,j}, y^i),$$

where  $f(\cdot)$  denotes the probability density function. The test statistic for each household can then be determined by dividing the estimated sample analog by its estimated standard deviation and by multiplying it by  $nh^{d/2}$ , where  $n$  is the number of observations and  $h$  is a smoothing parameter in the kernel estimator. In the present paper we estimate the residuals using the series estimator described in section 5 and the densities using a standard gaussian kernel estimator.

At this point we have one test statistic for each household in the pair. To compute the test statistic for the pair observe that efficiency is rejected if  $y^i$  affects the expenditure function of at least one household. We can therefore compute the test statistic for the pair  $\hat{\xi}_2^{i,j}$  by taking the maximum of the individual test statistics. Similarly to the homogeneity test, the distribution of the test statistic for the pair is obtained using bootstrap by resampling  $\hat{\xi}_2^{i,j*} - \hat{\xi}_2^{i,j}$ . The null hypothesis is then rejected for the pair composed of households  $i$  and  $j$  if  $\hat{\xi}_2^{i,j}$  is too large, i.e. if

$$\hat{\xi}_2^{i,j} > q^{i,j*}(0.95).$$

### 6.3 Test of Efficiency with Increasing Expenditure Functions For a Pair

The second test of efficient risk sharing is based on the implication that under efficiency the expenditure functions should be increasing in total resources. To implement the test, we employ a generalization of the monotonicity test introduced by Hall and Heckman (2000) to a model with multiple regressors and endogeneity.

We will provide the intuition underlying the test using a simpler version of the economy considered in this paper. Suppose that preferences are separable between consumption and leisure, there is no observable and unobservable heterogeneity, and no measurement errors. In this case, household  $i$ 's expenditure is only a function of  $\rho^{i,j}$  and there is no endogeneity issue, i.e.

$$\rho_i = g(\rho^{i,j}) + \epsilon. \tag{6}$$

Let  $\left\{ \left( \rho_t^i, \rho_t^{i,j} \right), 1 \leq t \leq T \right\}$  be data generated by equations (6) and denote by  $\left\{ \left( \bar{\rho}_t^i, \bar{\rho}_t^{i,j} \right), 1 \leq t \leq T \right\}$  the same data sorted in increasing order of aggregate resources  $\rho^{i,j}$ . Consider a subset of the sorted

data  $\left\{ \left( \bar{\rho}_t^i, \bar{\rho}_t^{i,j} \right), r \leq t \leq s \right\}$  and estimate the slope of the linear regression of  $\rho^i$  on  $\rho^{i,j}$ . Repeat the last step for any subset of the sorted data that contains enough information to estimate the slope. Hall and Heckman's idea is that under the hypothesis that the function  $g(\rho^{i,j})$  is increasing, the minimum over all the estimated slopes should not be negative.

Formally the test is implemented as follows. For a given integer  $m$  that will be defined later, let  $r$  and  $s$  be integers that satisfy  $0 \leq r \leq s - m \leq T - m$  and let  $a$  and  $b$  be scalars. Denote by  $h(w^i, w^j, d^{i,j})$  a polynomial in the wages and heterogeneity term, and by  $\delta(u^i)$  a polynomial in the first stage residuals in the estimator proposed by Newey et al. (1999). Define

$$S(a, b, h, \delta | r, s) = \sum_{i=r+1}^s \left\{ \rho^i - [a + b\rho^{i,j} + h(w^i, w^j, d^{i,j}) + \delta(u^i)] \right\}^2.$$

For each choice of  $r$  and  $s$ , let  $\hat{a}(r, s)$ ,  $\hat{b}(r, s)$ ,  $\hat{h}(r, s)$ , and  $\hat{\delta}(r, s)$  be the solution of the following least square problem:

$$\left( \hat{a}, \hat{b}, \hat{h}, \hat{\delta} \right) = \operatorname{argmin} S(a, b, h, \delta | r, s).$$

The variance matrix of the estimated coefficients is equal to  $\sigma^2 (X'X)^{-1}$ , where  $\sigma^2$  is the variance of the residuals in the expenditure function and  $X$  is the matrix of regressors. This implies that the variance of  $\frac{\hat{b}}{\sqrt{(X'X)_{b,b}^{-1}}}$  is equal to  $\sigma^2$ , where  $(X'X)_{b,b}^{-1}$  is the diagonal element of the inverse matrix that corresponds to  $\hat{b}$ . The test statistic for each household in the pair can then be defined as

$$\hat{\xi}_3^i = \max \left\{ -\frac{\hat{b}(r, s)}{\sqrt{(X'X)_{b,b}^{-1}}} : 0 \leq r \leq s - m \leq T - m \right\}.$$

Note that the integer  $m$  plays the role of a smoothing parameter in the sense that larger values of  $m$  reduce the effect of outliers. Similarly to the first efficiency test, the test statistic for the pair  $\hat{\xi}_3^{i,j}$  can be computed by taking the maximum of the two individual test statistics. The test rejects the null if  $\hat{\xi}_3^{i,j}$  is too large.

The distribution of the test statistic is derived using the bootstrap method suggested by Hall and Heckman (2000). According to this method, the bootstrap distribution should be derived under the hypothesis that the function under investigation is constant in  $\rho^{i,j}$  because it is the most difficult nondecreasing function for which to test. As in the previous tests, the bootstrap distribution is obtained by resampling  $\hat{\xi}_2^{i,j*} - \hat{\xi}_2^{i,j}$ . We can then reject the null for the pair composed by households  $i$  and  $j$  if

$$\hat{\xi}_3^{i,j} > q^{i,j*}(0.95).$$

## 6.4 Tests at the Economy Level

In this subsection we will explain how the test statistics derived in the previous three subsections can be combined to obtain one test of preference homogeneity and two tests of efficiency with the

following two features. First, each null hypothesis is tested simultaneously for all households in the economy. Second, if the null is rejected, the tests enable us to determine the fraction of households for which the hypothesis is rejected.

The tests at the economy level are based on the multiple testing procedure developed in Romano and Wolf (2005) and Romano, Shaikh, and Wolf (2006).<sup>13</sup> Consider  $n$  hypotheses  $H_1, \dots, H_n$  and let  $T_1, \dots, T_n$  be the associated test statistics. Suppose that one is interested in a null hypothesis  $H_0$  which is equal to the intersection of  $H_1, \dots, H_n$ , in the sense that  $H_0$  is not rejected only if each individual hypothesis  $H_k$  is not rejected. Romano and Wolf (2005) and Romano, Shaikh, and Wolf (2006) propose two methods for testing  $H_0$ . The first method controls the familywise error rate (FWE), i.e. the probability of rejecting at least one of the true hypotheses. Romano, Shaikh, and Wolf (2006) provide evidence that if the number of individual hypotheses is large, the method that controls for the FWE rate is too conservative. They propose an alternative method called  $k$ -stepM method which controls the  $k$ -FWE rate, i.e. the probability of rejecting at least  $k$  true hypotheses. Since in our paper the number of individual hypotheses is between 396 and 428, we adopt the  $k$ -stepM method.

We will now describe the multiple testing method. Let  $T_{r_1} \geq T_{r_2} \geq \dots \geq T_{r_n}$  be the test statistics ordered from the highest to the lowest. For any subset of individual hypotheses  $D$ , denote by  $k\text{-max}\{T_i\}$  the  $k$ -th largest test statistic and by  $c_D(1 - \alpha, k)$  the  $1 - \alpha$  percentile of its sampling distribution. In the first step, all the individual hypotheses are considered, i.e.  $D = D_1 = \{H_{r_1}, \dots, H_{r_n}\}$ , and the following generalized confidence region is constructed:

$$[T_{r_1} - c_{D_1}, \infty) \times \dots \times [T_{r_n} - c_{D_1}, \infty).$$

The individual hypothesis  $T_{r_i}$  is then rejected if  $0 \notin [T_{r_i} - c_{D_1}, \infty)$ , or equivalently  $T_{r_i} > c_{D_1}$ . If in the first step no individual hypothesis is rejected, the null  $H_0$  is also not rejected and the procedure stops. If at least one hypothesis is rejected  $H_0$  is also rejected. If the number of rejections is smaller than  $k$ , the procedure stops. Otherwise, Romano, Shaikh, and Wolf (2006) show that the power is increased by proceeding to the second step. Let  $R_1$  be the number of individual hypotheses rejected in the first step. In the second step, one considers the individual hypotheses not yet rejected, i.e.  $D = D_2 = \{H_{r_{R_1+1}}, \dots, H_{r_{R_n}}\}$ , and construct the corresponding generalized confidence region:

$$\left[ T_{r_{R_1+1}} - c_{D_2}, \infty \right) \times \dots \times \left[ T_{r_{R_n}} - c_{D_2}, \infty \right),$$

where the threshold  $c_{D_2}$  is constructed using the following method. Construct all the possible subsets that contain the  $n - R_1$  hypotheses that were not rejected plus  $k - 1$  of the rejected hypotheses. Denote by  $D_{2,i}$  the  $i$ -th subset. For each subset compute the  $1 - \alpha$  percentile of the sampling distribution of the  $k$ -th largest test statistic  $c_{D_{2,i}}(1 - \alpha, k)$ . The threshold  $c_{D_2}$  is the maximum of all  $c_{D_{2,i}}$ . The

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<sup>13</sup>There are other procedures that enable one to test multiple hypotheses. See for instance Holm (1979), Hochberg and Tamhane (1987), and Hommel (1988). The advantage of the methods proposed by Romano and Wolf (2005) and Romano, Shaikh, and Wolf (2006) is that they allow for dependence between the individual hypotheses.

hypothesis  $T_{r_{R_i}}$  is then rejected if  $T_{r_{R_i}} > c_{D_1}$ . If the number of rejected hypotheses is smaller than  $k$  one should stop. Otherwise, one continues in this stepwise fashion until less than  $k$  hypotheses are rejected. The  $1 - \alpha$  percentile of the sampling distribution of the  $k$ -th largest test statistic is computed using the bootstrap method illustrated in Romano, Shaikh, and Wolf (2006).

## 7 The ICRISAT Dataset

The three tests developed in this paper are used to understand risk sharing in rural India using the Village Level Studies (VLS) started by the ICRISAT. This dataset has been chosen for two reasons. First, a good understanding of the effect of idiosyncratic and aggregate shocks on household welfare is particularly important in developing countries, where shocks may have devastating effects on household resources because of the small number of formal markets. Second, several papers in the past have tested efficient risk sharing using this dataset. The results obtained here can therefore be compared with the findings of previous papers.

The ICRISAT started the VLS at six locations in rural India on July 1975. The study added four villages in 1981. In each village 40 households were selected to represent families in four land holding classes: 10 from landless laborers; 10 from small farmers; 10 from medium farmers; 10 from large farmers. The sample used in the estimation is composed of households from only 3 of the 10 villages: Aurepalle, Shirapur, and Kanzara. We focus our analysis on these three villages for two reasons. First, data for these villages are available for the entire sample period. Second, previous studies have only considered these villages. The VLS records data on production, labor supply, assets, price of goods, rainfall, monetary and non-monetary transaction, household size, age, education, and three different caste rankings from 1975 to 1985. Townsend (1994) gives a detailed description of the data. We will therefore discuss only the issues that are specific to our paper.

In the estimation we need data on consumption, labor supply, wages, demographic variables, and non-labor income. The ICRISAT collects information on these variables approximately every month. The expenditure functions can therefore be estimated using monthly data. We will now discuss how these variables are constructed.

Monthly household consumption is calculated using the transaction data from the ICRISAT Household Transaction Schedule. The consumption variable is the sum of consumption on grain, consumption on other food items, namely oil, animal products, fruits and vegetables, and consumption on other non-durable goods. There are two main problems with using the transaction files: the frequency of the interviews varies; the dates of the interviews differ across households. For example, a household in Aurepalle was interviewed on January 11, February 10, and March 21 in 1980, whereas a different household in the same village was interviewed on January 17, February 13, and March 25 in the same year. It is therefore difficult to compare expenditures across households and over time. To overcome this problem we assume that each interview is characterized by a constant rate of

consumption. Under this assumption we can compute monthly consumption using the consumption data from consecutive interviews. The measure of consumption that we obtain using this method corresponds to non-durable consumption for the entire household. Since different households have different size and gender-age structure, we divide household consumption by an age-gender weight which is constructed following Townsend (1994).<sup>14</sup>

The construction of the wage and labor supply variables requires a separate discussion. Three different types of employment and wages are recorded by the ICRISAT. The Labor, Draft Animal, and Machinery Utilization Schedule contains information on hours, days of employment, and wages of individuals entering daily employment outside their own farm. In the Household Transaction Schedule, labor income of individuals with regular jobs outside their own farm is recorded, but there is no information on the days and hours of employment. We assume that the data on regular labor income refer to the period covered by the interview and that the individual with the regular job works 8 hours a day for 5 days a week. In the Plot Cultivation Schedule, the ICRISAT records data on the number of hours supplied by men, women, and children to their own farm and the value of their labor. The value of own labor is imputed by the ICRISAT on the basis of the village-specific market wages. The data in these three schedules are collected every interview. We employ this information to construct the daily wages and labor supply that correspond to each month using the method described for consumption. Daily labor supply is therefore the average number of hours of employment on daily jobs, regular jobs, and jobs on own farm supplied by adult members. Daily wages are the average of total labor income earned on any job by adult members divided by the total number of hours. In the construction of leisure we follow Rosenzweig (1988) and Townsend (1994) and compute the time endowment  $T$  by assuming that each individual has 26 days per month and 14 hours per day that can be divided between labor and leisure. The remaining days and hours account for sleep, sickness, and holidays. The first efficiency test is implemented using non-labor income as the omitted variable. It is constructed as the sum of income from gifts, dowries, pension, theft, and profits.

The Households Member Schedule records data on demographic variables. We use this information to construct the vector of observable heterogeneity variables, which is composed of the mean age of adult household members, the number of infants, the age-gender weight, and the caste ranking considered in Behrman (1988). Consumption and wages are deflated using the consumer price index published by the Labour Bureau of India. The set of instruments is constructed using information on lagged rainfall, lagged total expenditure, and lagged savings.

The sample period is July 1975 to July 1985 for Aurepalle and July 1975 to July 1984 for Shirapur and Kanzara. We drop the households that leave the sample before 1985 for Aurepalle and 1984 for Shirapur and Kanzara. This implies that we have up to 126 observations for each household in

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<sup>14</sup>The age-gender weight is computed by adding the following numbers: for adult males, 1.0; for adult females, 0.9; for males aged 13-18, 0.94; for females aged 13-18, 0.83; for children aged 7-12, 0.67; for children aged 4-6, 0.52; for Toddlers 1-3, 0.32; and for infants 0.05.

Aurepalle and up to 114 observations for each household in Shirapur and Kanzara. In all tests, we drop a household if it has fewer than 80 data points. Table 1 reports the summary statistics of the main variables.

## 8 Simulation Study

In this section we will study the performance of the three tests proposed in this paper using simulations. The results will enable us to improve the power and control of the tests when they are implemented using the ICRISAT data. Following Romano, Shaikh, and Wolf (2006) we will focus on three measures of test performance: the average number of false hypotheses rejected; the average number of true hypotheses rejected; the empirical FWE rate.

All the simulations share the following features. It is assumed that the group under investigation is composed of ten households. All households have a utility function which is nonseparable between consumption and leisure and has the following form:

$$u^i(c, l; z, \eta) = \frac{(c^{\sigma_i} l^{1-\sigma_i} + a_i)^{1-\gamma_i}}{1-\gamma_i} \exp\{\theta z + \eta\}.$$

This utility generates expenditure functions that satisfy the restrictions discussed in section 5. The parameters  $\sigma_i$  and  $a_i$  are assumed to be identical across households with  $\sigma_i = 0.5$  and  $a_i = 1$ . The risk aversion parameter  $\gamma_i$  is allowed to vary across households. Five households are assumed to have  $\gamma_i = 1.2$ , whereas the corresponding parameter for the other five households is set to 2.5. Households can save using a risk-free asset with no constraint on their borrowing ability. The interest rate is fixed at 0.05 and the discount factor is set equal to 0.95. Each household can draw a high or low daily wage with equal probability. The high and low wages are set equal to 3 and 5 rupees, respectively. We allow for unobservable heterogeneity in the form of a pair fixed effect and for observable heterogeneity using the following four variables: mean adult age, caste ranking, age-gender weight, and number of infants. Household decisions are simulated for 160 periods. To approximate the length of the panel in the ICRISAT, the test is then performed using the 120 periods that are between  $t = 21$  and  $t = 140$ . The simulation is repeated 500 times. The distribution of the test statistics is determined using 500 bootstraps.

To evaluate the effect of measurement errors on the outcome of the tests, we add measurement errors to household expenditure  $\rho^i$ . We consider two types of measurement errors. In the first case, they are drawn from a normal distribution with mean zero and a negligible standard deviation ( $\sigma_m = 0.1$ ). In the second case they are drawn from a normal distribution with the same mean but a standard deviation that is equal to half the standard deviation of the simulated household expenditure. We expect the two standard deviations used in the simulation to be a lower and upper bound for the standard deviation of the measurement errors in the data.<sup>15</sup> In the homogeneity in risk

<sup>15</sup>Ravallion and Chaudhuri (1997) convincingly argue that there are measurement errors in the ICRISAT. We could

preferences test, the effect of measurement errors depends also on the quality of the instruments. In the simulation of that test we consider two sets of instruments. The first set contains a larger number of instruments and generates an average  $R^2$  for total expenditure of about 0.97, where the average is computed across pairs. The second set contains fewer instruments and produces an average  $R^2$  of about 0.5.

The implementation of tests requires the researcher to choose  $k$  in the  $k$ -FWE rate and the order of the polynomial in  $\rho^{ij}$ . We experimented with several values for  $k$ . We report the results for  $k$  equal to 1, 3, and 5, since generally for  $k$  greater than 5 the loss in control more than dominates the gain in power. We also experimented with polynomials of order 1, 2, 3, and 5. For each test, we will report only the results that are useful to understand the relationship between the order of the polynomial and the test performance.

To implement the test, one must control for the variation in wages, and in observable and unobservable heterogeneity. We control for this variation in two steps. We first estimate semi-parametrically the expenditure functions and their differences. We then fix wages and the heterogeneity term at the household mean and perform the tests.

## 8.1 Simulations for the Test of Homogeneity in Risk Preferences

In this subsection we evaluate the performance of the test of homogeneity in risk preferences. To that end, we simulate the decisions of the ten households under the maintained assumption of efficient risk sharing. The Pareto weights are chosen so that the expenditure functions of households with heterogeneous risk preferences cross. The goal of the simulation study is therefore to evaluate whether different specifications of the test can detect these crossings. The actual data may correspond to Pareto weights for which the household expenditure functions do not cross even if preferences are heterogeneous. The results should therefore be interpreted as an upper bound for the power of the test.

The results indicate that the performance of the test depends on four features of the simulated data: the severity of the measurement error problem; the quality of the instruments used to address it; the order of the polynomial in total expenditure; the choice of  $k$  in the  $k$ -FWE rate. The results are reported in table 2. To understand them it is important to remember that the risk aversion parameter is equal to 1.2 for five households and to 2.5 for the remaining five. In the simulation there are therefore 25 pairs with heterogeneous preferences and expenditure functions that cross, and 20 pairs with homogeneous risk preferences.

We start the description of the results by discussing the effect of the measurement errors on the outcome of the test. If the standard deviation of the measurement errors is negligible, the test can easily detect the existence of heterogeneity in risk preferences. The average rejection of false

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not find a paper, however, that estimates their standard deviation.

hypotheses is 25 out of 25. If the standard deviation is large the average number of false hypotheses rejected by the test drops significantly for some specifications of the test. The lowest average is obtained when we employ the small set of instruments, a polynomial of order 2, and  $k = 1$ . For this specification the average number of correct rejections is just 0.4 out of 25. The number increases significantly if we use the large set of instruments, a polynomial of order 1, and we set  $k$  equal to 5. In this case the average number of correct rejections is 24.3 out of 25. To understand why measurement errors have a substantial effect on the power of the test, observe that the test statistic is constructed using the difference in expenditure functions. In the simulation the difference is generally much smaller than the actual expenditures. When the measurement errors are drawn from a distribution with a standard deviation that equals half the standard deviation of household expenditure, the difference in measurement errors dominates the difference in true expenditures in many instances. It is therefore difficult for the test to detect a crossing unless a strong set of instruments is used and  $k$  is increased. The measurement errors have very little effect on the number of true hypotheses rejected by the test. The average number of false rejections is always very small with the highest number being 1 for the large set of instruments, high  $\sigma_m$ ,  $k = 5$ , and a polynomial of order 2.

The order of the polynomial in  $\rho^{ij}$  also has a significant effect on the outcome of the test. The best performance is obtained when the order is set equal to 1. This result can be explained by noting that for HARA preferences the efficient expenditure functions are approximately linear in aggregate expenditure. The difference in expenditure functions is therefore also approximately linear. As a consequence the standard errors of the estimated difference increase with the order of the polynomial, which explains the results.

The simulation study also indicates that the choice of  $k$  has an important effect on the power of the test. When the standard deviation of the measurement errors is large the test is too conservative if  $k$  is set equal to 1. When we increase  $k$  to 3 or 5 we observe a substantial gain in power with little loss in control. For instance when we use the large set of instruments and a polynomial of order 1, an increase in  $k$  from 1 to 3 raises the average number of correct rejections from 15.2 to 23.5. At the same time the average number of false rejections increases only slightly from 0 to 0.4. An additional increase in  $k$  to 5 has only a small effect on the outcome of the test. When we use a polynomial of order 2 and the small set of instruments  $k = 5$  appears to be the optimal choice.

The test of homogeneity in risk preferences in the ICRISAT will be set up taking into account the results of the simulation study. In the ICRISAT we expect HARA preferences to be only an approximation of household preferences. We also expect the standard deviation of the measurement errors to be between the ones considered in the simulation study. Moreover, the set of instruments that will be used produces an average  $R^2$  of about 0.71. Because of all this, we will implement the test using two specifications. In the first one we set  $k$  equal to  $\frac{3}{45}\%$  of the total number of individual hypotheses and the order of the polynomial to 1. In the second specification we change the order of

the polynomial to 2.

## 8.2 Simulations for the Efficiency Test with Non-labor Income

In this subsection we will discuss the performance of the efficiency test with non-labor income. Its evaluation requires the simulation of an economy in which a first group of households share risk efficiently, whereas efficiency is violated for a second group. We will assume that five households behave efficiently and that the remaining five are in autarky and can insure themselves against income shocks only by using the risk-free asset. We therefore have 45 individual hypotheses, 35 of which are false. It is important to point out that autarky with a risk-free asset is only one possible alternative to efficiency. Other alternatives are autarky without savings, cooperation without commitment, and cooperation with asymmetric information. We have chosen autarky with a risk-free asset because it has been shown in the finance literature that in this environment households can achieve a degree of insurance similar to the degree that can be achieved in an economy with efficient risk sharing.

The ability of households in autarky to insure themselves against income shocks using the risk-free asset depends on the properties of the non-labor income process. In the simulation we consider a process that attempts to replicate non-labor income in the ICRISAT data. It is assumed that the process is distributed according to a normal distribution with a mean that depends on lagged non-labor income, mean adult age, caste, number of infants, and age-gender weight. This specification enables us to capture the fact that non-labor income is highly persistent in the data: everything else equal, an increase in lagged non-labor income by 100% increases current non-labor income by about 50%. Using this specification and the ICRISAT data we can estimate the mean and variance of the process. We can then compute the probability of drawing different realizations for non-labor income.

The simulation results for the first efficiency test differ from the results obtained from the homogeneity test in two respects. First, in the efficiency test the measurement errors have a smaller effect on the outcome of the test. To provide the intuition behind this result note that the present test is based on the estimated expenditure functions and not on their differences. Consequently, measurement errors drawn from the same income process have a smaller effect on the test. For this reason we report the outcome of the simulation study for only one set of instruments which has an average  $R^2$  slightly lower than 0.9. The second difference is that the best performance is obtained with a polynomial in  $\rho^{i,j}$  of order 3. To understand why this test requires a polynomial of higher order, observe that households that share risk efficiently have expenditure functions that are approximately linear in total expenditure. A polynomial of order 1 is therefore the optimal choice for this group. The group of inefficient households, however, have expenditure functions that are non-linear in total expenditure. As a consequence they require a polynomial of higher order. A polynomial of order 3 enables us to approximate in the best possible way the expenditure functions of both groups of households. We only report the results for this specification.

Table 3 describes the outcome of the simulation study. If the standard deviation of the measurement errors is negligible, the test is able to reject almost all false hypotheses. When  $k = 1$  we reject on average 34.5 false hypotheses out of 35. When  $k$  is set equal to 3 or 5 we reject all false hypotheses. When we increase the standard deviation, the average number of correct rejections decreases but only slightly with 28.2 average rejections for  $k = 1$ , 34.9 rejections for  $k = 3$ , and 35 rejections for  $k = 5$ . The average number of true hypotheses rejected is small for both specifications of the measurement errors and it is between 0 and 0.56.

The results of the simulation study are used to set up the test that will be used to evaluate efficiency in Indian villages. The test will be implemented using a polynomial of order 3 in  $\rho^{ij}$  and a  $k$  that corresponds to  $\frac{3}{45}$ % of total hypotheses.

### 8.3 Simulations for the Efficiency Test with Increasing Functions

In this section we will describe the performance of the efficiency test with increasing functions. Similarly to the non-labor income test, we simulate an economy in which five households share risk efficiently and five households are in autarky.

The computation of the test statistics for the present test requires a choice for the smoothing parameter  $m$ . We have experimented with different values for  $m$ . When the measurement errors have a negligible standard deviation the power of the test is maximized without sacrificing control when  $m$  is set as low as possible. When the measurement errors have a large standard deviation, however, we reject too many true hypotheses if  $m$  is set too low because the test cannot reduce the impact of outlying data values. With high standard deviation we obtain the best balance between power and control when we set  $m = 15$ . We report the results for this value of the smoothing parameter.

After having set  $m = 15$ , the performance of the test depends on two features of the simulated data: the choice of  $k$  for the  $k$ -FWE rate; the order of the polynomial in  $\rho^{i,j}$ . We report our findings in table 4 for the two specifications of the measurement errors, for a polynomial in  $\rho^{ij}$  of order 3, and a polynomial of order 5. The results indicate that the efficiency test with increasing functions has less power than the test with non-labor income. They also indicate that in this test it is crucial to increase  $k$  to 5 and the order of the polynomial to 5. For this specification, we have the largest number of correct rejections.

We will now provide the intuition for the lower power of this test. If efficiency is violated the expenditure functions decrease with aggregate expenditure only if aggregate expenditure is the sum of a high level of expenditure for one household and a low level of expenditure for the other household. This event occurs if one household is characterized by a high level of non-labor income, wages, and savings, whereas non-labor income, wages, and savings are low for the second household. Not all households experience this type of event in a given simulation. As a consequence the number of false hypotheses rejected by the test is lower than 35 in some simulations.

It is also important to understand why a higher order polynomial increases the power of this efficiency test. The explanation is related to the bootstrap method used to recover the distribution of the test statistics. The bootstrap distribution is computed using the approach proposed by Hall and Heckman (2000). According to it, the bootstrap should be implemented under the assumption that the function under investigation is constant in  $\rho^{ij}$ . The constant function must be computed using a non-parametric estimator of the household expenditure functions. Households for which efficiency is violated have non-linear expenditure functions. If the non-linearities are not properly captured by the non-parametric estimator, the constant function used in the bootstrap will contain variation in  $\rho^{ij}$  that will increase the variance of the bootstrap distribution. As a result the number of false hypotheses rejected drops.

Given the results obtained with the simulations, we will test efficiency with increasing functions in the ICRISAT by setting the smoothing parameter to 15, the order of the polynomial to 3 and 5, and  $k$  equal to  $\frac{5}{45}\%$  of total individual hypotheses.

## 9 Results

In this section, we report the outcome of the tests for the three Indian villages considered in this paper. We will first describe the outcome of the test of homogeneity in risk preferences. The estimation of the household expenditure functions indicates that every pair of households can be assigned to one of three different categories: (i) pairs whose expenditure functions do not cross; (ii) pairs with expenditure functions that cross once; (iii) pairs whose expenditure functions cross twice. Figure 5 depicts one pair of households for each category. This finding represents a first and informal indication that heterogeneity in risk preferences is a significant feature of Indian villages. The outcome of the formal test, which is reported in Table 5, supports this evidence. At the village level, homogeneity in risk preferences is strongly rejected in all three villages. When we use a polynomial of order 1 in  $\rho^{i,j}$  we reject the null in about 25% of possible cases in Aurepalle, in about 16% of possible cases in Shirapur, and in about 35% of possible cases in Kanzara. When we employ a polynomial of order 2, we obtain a similar number of rejections in Aurepalle and Shirapur. In Kanzara the number of rejections drops to about 25% of possible cases. When we perform the test at the caste level, we reject the hypothesis of identical risk preferences in eight out of thirteen observed castes. These results imply that previous tests should have rejected efficient risk sharing in Indian villages as long as aggregate shocks have a significant impact on household behavior and the variable used to capture idiosyncratic shocks is correlated with risk preferences.

We will now discuss the outcome of the efficiency tests. Before considering the tests proposed in this paper, we describe the results obtained using the standard test employed by the risk-sharing literature. The results, which are reported in table 6, are obtained using the specification employed

by Townsend (1994). It corresponds to the following efficiency condition:

$$\Delta\rho_t^i = \alpha_0 + \alpha_1\Delta X_t + \alpha_2\Delta\rho_t^a + \alpha_3\Delta y_t^i,$$

where  $\Delta\rho_t^i$ ,  $\Delta\rho_t^a$ , and  $\Delta y_t^i$  are the first differences in household expenditure, village expenditure, and household income, and  $X_t$  is a vector of control variables that includes wages, mean household age, caste, number of infants, and age-gender weight. To be able to compare the results of the standard test with the results of the tests developed in the present paper, expenditure is defined as expenditure on non-durable consumption plus expenditure on leisure. We reject efficiency in all three villages, which is consistent with the panel estimates reported in Townsend (1994). The test is also performed at the caste level for castes that have at least two households in the sample. In this case  $\Delta\rho_t^a$  measures the first difference in caste expenditure. In Aurepalle we reject the null in two out of six castes, in Shirapur in two out of three castes, and in Kanzara in two out of four observed castes.

We will now describe the outcome of the two efficiency tests derived in this paper. The results are reported in Table 7. At the village level, both tests reject efficient risk sharing in all three villages. Using the non-labor income test, efficiency is rejected for 12 pairs in Aurepalle, 18 pairs in Shirapur, and 10 pairs in Kanzara. These numbers correspond to about 3% of possible cases in Aurepalle, 5% of possible cases in Shirapur, and 2% of possible cases in Kanzara.<sup>16</sup> The results obtained using the efficiency test with increasing functions are consistent with the ones obtained using non-labor income. The only difference worth noting is that with the second test we observe a larger number of rejections in Kanzara, where the null is violated in about 6% of the cases. The rise in number of rejections is explained by the increase in  $k$  in the k-FWE rate from 3 to 5. When we use  $k$  equal to 3, the number of rejections is always lower for the test with increasing functions. To summarize, at the village level we observe a small number of rejections, which implies that there is a significant degree of mutual insurance in Indian villages. But the test can detect a substantial number of rejections even if one allows for heterogeneity in risk preferences and a general class of utility functions.

Allowing for heterogeneity in risk preferences and a general class of utility functions makes a significant difference when we test risk-sharing at the caste level. Contrary to the outcome of the standard test, we find that castes are able to perfectly insure their members. In Aurepalle we cannot reject efficiency in any of the six observed castes. The same result holds for Shirapur. Kanzara is the only village in which we observe one caste for which efficiency is rejected. For this caste, the null is rejected for 1 pair out of 78 possible cases using the non-labor income test and for 5 pairs using the increasing function test.

One additional point should be discussed. For the caste with ranking 86.25 in Aurepalle and for the caste with ranking 76.25 in Kanzara we cannot reject the hypothesis of homogeneity in risk

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<sup>16</sup>Observe that our tests control the k-FWE rate. This implies that in our tests the probability of rejecting at least  $k$  true hypotheses is controlled to be smaller than or equal to 0.05. The statement that the tests should reject at least 5% of total hypotheses is therefore incorrect in this context.

preferences, we reject efficiency using the standard test, but we cannot reject efficient risk sharing using the semi-parametric tests that allow for heterogeneity in risk preferences. In these two cases the different outcome obtained using our efficiency tests should be attributed to the general class of utility functions allowed by our tests and not to the presence of heterogeneous risk preferences.

The results presented in this section indicate that it is crucial to allow for heterogeneity in risk preferences and a general class of preferences if the goal is to understand the risk-sharing institutions in rural India. The next section attempts to interpret these results using data on transfers and loans.

## 10 Discussion of Results

Two of the findings reported in the previous section are worth a discussion. First, households that belong to the same caste are able to perfectly insure each other against risk. Second, efficient risk sharing is rejected at the village level. The number of rejections is small, which suggests that the degree of insurance is high. But there is a non-negligible number of cases in which the data indicate that households that belong to different castes are affected by caste-specific shocks. These two findings suggest that the relevant social unit in rural India is not the village, as indicated in previous papers, but the caste. These results are consistent with recent evidence reported in Munshi and Rosenzweig (2006), where it is shown that to understand mobility in India one should use the caste as the social unit.<sup>17</sup>

We will now provide some descriptive evidence about the type or risk-sharing institutions that castes use to insure their members against different types of shocks. A formal analysis would require a precise description of each institution, which is beyond the scope of this paper. Using the ICRISAT data, however, we can give some insight on two institutions that appear to be important as risk-sharing devices at the caste level: transfers and loans. We use two types of information available in the ICRISAT. First, the ICRISAT collects data on the amount received by or given to a household as a transfer or loan. In the ICRISAT questionnaire a transfer is defined as a transaction in which resources or money change ownership without compensation. A loan is a similar transaction with a compensation. Second, in the ICRISAT we observe the partner in the transaction, where the list of partners include caste fellows. We can therefore determine whether a loan or a transfer took place between two households that belong to the same caste.

Table 8 reports average real per-capita transfers and loans given by and received from households in the three villages under investigation. There are four patterns that are worth a discussion. First, transfers are a substantial fraction of non-durable expenditure. In Aurepalle, real per-capita transfers given and received are on average 14.5% and 13% of non-durable expenditure. In Shirapur, the transfers given and received are also a high percentage of expenditure at 9.5% and 15%. The village

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<sup>17</sup>According to their paper, the correct social unit should be the sub-caste or *jati*. Our tests of efficiency, however, cannot reject the hypothesis that the caste is the correct risk-sharing unit.

with the lower amount of transfers is Kanzara where transfers given and received are 4.8% and 11.6%. A second pattern to consider is that the average value of loans received is a large fraction of household expenditure, whereas the same number for loans given is negligible.

A third feature of the data is that most of the transfers are given to and received from households that belong to the same caste. In Aurepalle, the transfers given to caste fellows are 83% of total transfers and the transfers received from caste fellows are 70%. Shirapur and Kanzara are characterized by similar percentages. These findings are different from the results reported by Munshi and Rosenzweig (2006), where it is suggested that transfers between caste members are negligible. Our results are more in line with Townsend's view that transfers are an important source of insurance. The data on loans given display a similar pattern. In all cases in which we observe some resources being lent, the loan is given to a caste fellow. When we condition on giving a loan, it is also evident that the amount of resources transferred to the caste fellow is a large fraction of expenditure. The data on loans received display a different pattern. Most of the loans are received from households that do not belong to the same caste.

A last point that should be discussed is that a large fraction of the transfers between caste fellows is given to and received from households living outside the village boundaries. This pattern is consistent with previous findings, see for instance Rosenzweig and Stark (1989), which indicate that the relationship between caste members extends beyond the village.

## 11 Conclusions

In the paper we show that, if in the data households have heterogeneous risk preferences, the standard test of efficient risk sharing rejects efficiency even if households share risk perfectly. To address this problem in the paper we propose a method that enables one to test for efficiency even if risk preferences are heterogeneous. We apply this method to Indian villages. We find strong evidence against the common assumption of identical risk preferences. We also find that efficient risk sharing is rejected in Indian villages, but it is not rejected for the castes that compose those villages. This finding implies that the only policies that can improve the welfare of individuals living in Indian villages are policies that insure households against aggregate shocks at the caste, village, or state level.

The method proposed in this paper can be used in other environments provided that a long panel is available. For instance, the method can be used to test efficient risk sharing across countries using the long panels that exist at the country level. The method can also be used to test efficient risk sharing in the US at the country, state, extended-family, or household level using the CEX and synthetic cohorts.

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## A Proofs

In all the proofs we will suppress the dependence on observable and unobservable heterogeneity to simplify the notation.

### A.1 Proof of Proposition 1

**Proof.** This proposition will be proved by contradiction. Suppose household  $i$  and  $j$  share risk efficiently and that there exist two income realizations  $\rho^{i,j}$  and  $\bar{\rho}^{i,j}$  such that

$$\rho^i(\rho^{i,j}; w^i, w^j) > \rho^j(\rho^{i,j}; w^i, w^j)$$

and

$$\rho^i(\bar{\rho}^{i,j}; w^i, w^j) < \rho^j(\bar{\rho}^{i,j}; w^i, w^j).$$

Assume that household  $i$  and  $j$  have identical risk preferences. This implies that  $V^i = V^j = V$ , where  $V^k$  is the value function of household  $k$  introduced in the three-stage formulation of section 3. We will show that under these conditions efficiency is violated.

Without loss of generality assume that  $\rho^{i,j} < \bar{\rho}^{i,j}$ . Strict concavity of the household utility functions implies that the corresponding value functions are strictly concave in income.<sup>18</sup> Consequently,

$$V_\rho(\rho^j(\rho^{i,j}; w^i, w^j)) > V_\rho(\rho^i(\rho^{i,j}; w^i, w^j)).$$

This implies,

$$\mu_i V_\rho(\rho^j(\rho^{i,j}; w^i, w^j)) > \mu_i V_\rho(\rho^i(\rho^{i,j}; w^i, w^j)) = \mu_j V_\rho(\rho^j(\rho^{i,j}; w^i, w^j)),$$

where the equality follows from efficient risk sharing. Hence,  $\mu_i > \mu_j$ .

Now consider the realization  $\bar{\rho}^{i,j}$ . Strict concavity of  $V$  implies that

$$V_\rho(\rho^i(\bar{\rho}^{i,j}; w^i, w^j)) > V_\rho(\rho^j(\bar{\rho}^{i,j}; w^i, w^j)). \quad (7)$$

Consequently,

$$\mu_i V_\rho(\rho^i(\bar{\rho}^{i,j}; w^i, w^j)) > \mu_i V_\rho(\rho^j(\bar{\rho}^{i,j}; w^i, w^j)) > \mu_j V_\rho(\rho^j(\bar{\rho}^{i,j}; w^i, w^j)),$$

where the first inequality follows from (7) and the second one from  $\mu_i > \mu_j$ . This implies that efficiency is not satisfied and hence that the household utility functions cannot be identical. ■

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<sup>18</sup>For a proof see for instance Proposition 3.6 in Kreps (1990).

## A.2 Proof of Proposition 2

**Proof.** The proposition will be proved by contradiction. Suppose that households  $i$  and  $j$  share risk efficiently and assume that the expenditure function of household  $i$  is decreasing in some interval  $I = [\rho_l, \rho_u]$ . We will show that efficiency cannot be satisfied.

Consider two realizations of aggregate income  $\rho^{i,j} \in I$  and  $\bar{\rho}^{i,j} \in I$  with  $\rho^{i,j} > \bar{\rho}^{i,j}$ . Since the expenditure function of household  $i$  is decreasing in  $I$  we have,

$$\rho^i(\rho^{i,j}; w^i, w^j) \leq \rho^i(\bar{\rho}^{i,j}; w^i, w^j).$$

As a consequence,

$$\mu^j V_\rho^j(\rho^j(\rho^{i,j}; w^i, w^j)) = \mu^i V_\rho^i(\rho^i(\rho^{i,j}; w^i, w^j)) \geq \mu^i V_\rho^i(\rho^i(\bar{\rho}^{i,j}; w^i, w^j)) = \mu^j V_\rho^j(\rho^j(\bar{\rho}^{i,j}; w^i, w^j)),$$

where the two equalities follow from efficiency and the inequality follows from the strict concavity of  $V^k(\rho^k)$ . Hence, by strict concavity of  $V^k(\rho^k)$ , we have

$$\rho^j(\rho^{i,j}; w^i, w^j) \leq \rho^j(\bar{\rho}^{i,j}; w^i, w^j).$$

This implies that

$$\rho^i(\rho^{i,j}; w^i, w^j) + \rho^j(\rho^{i,j}; w^i, w^j) \leq \rho^i(\bar{\rho}^{i,j}; w^i, w^j) + \rho^j(\bar{\rho}^{i,j}; w^i, w^j) \leq \bar{\rho}^{i,j} < \rho^{i,j},$$

which cannot be the efficient allocation of resources since preferences are strictly increasing. ■

## A.3 Proof of Proposition 3

**Proof.** Suppose that preferences are separable between consumption and leisure or that  $w^i$  and  $w^j$  are constant. Then,

$$\rho^i = \rho^i(\rho^{i,j}; w^i, w^j) = \hat{\rho}^i(\rho^{i,j}), \quad (8)$$

$$\rho^j = \rho^j(\rho^{i,j}; w^j, w^i) = \hat{\rho}^j(\rho^{i,j}), \quad (9)$$

for some functions  $\hat{\rho}^i$  and  $\hat{\rho}^j$ .

Under the assumption that  $\rho^i$  and  $\rho^j$  are strictly increasing functions of  $\rho^{i,j}$ , equations (8) and (9) can be solved for  $\rho^{i,j}$  and equated to obtain,

$$(\hat{\rho}^i)^{-1}(\rho^i) = (\hat{\rho}^j)^{-1}(\rho^j), \quad (10)$$

where  $(\hat{\rho}^k)^{-1}$  is the inverse function of  $\hat{\rho}^k$ . Let  $g: \mathbb{R}^+ \rightarrow \mathbb{R}^+$  be a strictly decreasing function and  $\mu_k$  a scalar satisfying the following conditions:  $0 < \mu_k < 1$ , for  $k = 1, \dots, n$ , where  $n$  is the number of households in the economy;  $\sum_{i=1}^n \mu_k = 1$ . Then, equation (10) implies that

$$\mu_i \frac{g\left((\hat{\rho}^i)^{-1}(\rho^i)\right)}{\mu_i} = \mu_j \frac{g\left((\hat{\rho}^j)^{-1}(\rho^j)\right)}{\mu_j}. \quad (11)$$

Let the function  $V^k(\rho^k)$  be defined as follows:

$$V^k(\rho^k) = \frac{1}{\mu_k} \int_0^{\rho^k} g\left((\hat{\rho}^i)^{-1}(t)\right) dt.$$

The function  $V^k$  satisfies the following two properties:  $V_\rho^k = g\left((\hat{\rho}^k)^{-1}(\rho^k)\right)/\mu_k > 0$  and  $V_{\rho\rho}^k = g_\rho < 0$ . The first inequality follows from the second fundamental theorem of calculus and  $g > 0$ . The second inequality follows from  $g$  being decreasing and the expenditure functions being increasing, which implies that  $(\hat{\rho}^k)^{-1}$  is also increasing. Consequently,  $V^k(\rho^k)$  is a well-defined strictly increasing and concave utility function over  $\rho^k$ .

Under separable preferences  $\rho^k$  is equal to consumption of household  $k$ . Under the assumption that real wages are constant, the composite commodity theorem shows that consumption and leisure can be treated as a single good and that only the utility function over the composite good  $\rho^k = c^k + w^k l^k$  is relevant to describe household behavior.<sup>19</sup> Hence, by (11) there exist strictly increasing and concave utility functions  $V^i(\rho)$  and  $V^j(\rho)$  such that the efficiency condition is satisfied, i.e.:

$$\mu_i V_\rho^i(\rho^i) = \mu_j V_\rho^j(\rho^j).$$

■

## B Tables

Table 1: Summary Statistics.

Variable	Aurepalle		Shirapur		Kanzara	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Grain	18.4	16.1	21.1	16.4	20.7	25.5
Food (minus Grain)	7.7	7.1	16.0	17.5	17.2	14.2
Non-Durable (minus Grain, Food)	6.0	27.8	10.1	15.0	12.1	18.2
Household size	7.7	3.0	7.3	3.1	7.4	3.5
Number of Infants	0.13	0.35	0.07	0.28	0.14	0.36
Mean Adult age	38.4	6.6	39.4	6.9	37.0	5.5
Age-Gender Weight	6.4	2.7	6.0	2.5	5.9	2.8
Daily Wage	2.5	3.0	3.9	4.2	4.0	3.5
Daily Labor Supply	5.0	3.6	6.0	3.3	5.4	2.7
Non-labor Income	16.8	110.0	50.0	309.1	28.8	154.8
N. of Households	30		29		30	

Note: The sample period corresponds to 1975-1985 for Aurepalle and to 1975-1984 for Shirapur and Kanzara.

<sup>19</sup>See Hicks (1990) for a proof of the composite commodity theorem.

Table 2: Simulation Results for the Test of Homogeneity in Risk Preferences

Null Hypothesis	Homogeneity in Risk Preferences		
	Average N. of False Hypotheses Rejected	Average N. of True Hypotheses Rejected	Empirical Familywise Error Rate
Order of Polynomial in $\rho^{ij} = 1$			
Large Instrument Set			
$\sigma_m = 0.01, k=1$	25.0/25.0	0.0/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	15.2/25.0	0.0/20.0	0.0
$\sigma_m = 0.01, k=3$	25.0/25.0	0.1/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	23.5/25.0	0.4/20.0	0.04
$\sigma_m = 0.01, k=5$	25.0/25.0	0.5/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	24.3/25.0	0.8/20.0	0.0
Small Instrument Set			
$\sigma_m = 0.01, k=1$	25.0/25.0	0.0/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	3.6/25.0	0.0/20.0	0.0
$\sigma_m = 0.01, k=3$	25.0/25.0	0.1/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	15.9/25.0	0.2/20.0	0.01
$\sigma_m = 0.01, k=5$	25.0/25.0	0.5/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	19.2/25.0	0.5/20.0	0.0
Order of Polynomial in $\rho^{ij} = 2$			
Large Instrument Set			
$\sigma_m = 0.01, k=1$	25.0/25.0	0.03/20.0	0.03
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	3.6/25.0	0.0/20.0	0.0
$\sigma_m = 0.01, k=3$	25.0/25.0	0.2/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	15.5/25.0	0.3/20.0	0.02
$\sigma_m = 0.01, k=5$	25.0/25.0	0.6/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	19.0/25.0	1.0/20.0	0.04
Small Instrument Set			
$\sigma_m = 0.01, k=1$	25.0/25.0	0.01/20.0	0.01
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	0.4/25.0	0.0/20.0	0.0
$\sigma_m = 0.01, k=3$	25.0/25.0	0.2/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	5.6/25.0	0.2/20.0	0.01
$\sigma_m = 0.01, k=5$	25.0/25.0	0.7/20.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	9.7/25.0	0.8/20.0	0.02

Note: The results are obtained by simulating an economy with ten households that share risk efficiently. Five households have a coefficient of risk aversion equal to 1.2 and five households have a coefficient of risk aversion equal to 2.5. There are therefore 20 pairs for which the null of identical risk preferences is satisfied and 25 pairs for which the null is violated.

Table 3: Simulation Results for the Test of Efficiency with Non-labor Income

Null Hypothesis	Efficiency with Non-labor Income		
	Average N. of False Hypotheses Rejected	Average N. of True Hypotheses Rejected	Empirical Familywise Error Rate
Order of Polynomial in $\rho^{ij} = 3$			
$\sigma_m = 0.01, k=1$	34.5/35.0	0.13/10.0	0.04
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	28.2/35.0	0.0/10.0	0.0
$\sigma_m = 0.01, k=3$	35.0/35.0	0.41/10.0	0.05
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	34.9/35.0	0.01/10.0	0.0
$\sigma_m = 0.01, k=5$	35.0/35.0	0.56/10.0	0.04
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	35.0/35.0	0.05/10.0	0.0

Note: The results are obtained by simulating an economy with ten households. Five of them share risk efficiently, whereas the remaining five are in autarky. There are therefore 10 pairs for which the null of efficiency is satisfied and 35 pairs for which the null is violated.

Table 4: Simulation Results for the Test of Efficiency with Increasing Expenditure Functions

Null Hypothesis	Efficiency with Increasing Functions		
	Average N. of False Hypotheses Rejected	Average N. of True Hypotheses Rejected	Empirical Familywise Error Rate
Order of Polynomial in $\rho^{ij} = 3$			
$\sigma_m = 0.01, k=1$	1.0/35.0	0.01/10.0	0.01
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	0.4/35.0	0.0/10.0	0.0
$\sigma_m = 0.01, k=3$	6.4/35.0	0.0/10.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	5.5/35.0	0.0/10.0	0.01
$\sigma_m = 0.01, k=5$	10.4/35.0	0.0/10.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	12.2/35.0	0.03/10.0	0.0
Order of Polynomial in $\rho^{ij} = 5$			
$\sigma_m = 0.01, k=1$	1.4/35.0	0.0/10.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=1$	1.2/35.0	0.0/10.0	0.0
$\sigma_m = 0.01, k=3$	8.4/35.0	0.0/10.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=3$	9.3/35.0	0.0/10.0	0.0
$\sigma_m = 0.01, k=5$	20.3/35.0	0.01/10.0	0.0
$\sigma_m = 0.5 * \sigma_{exp}, k=5$	21.5/35.0	0.39/10.0	0.02

See note in Table 3.

Table 5: Test Of Homogeneity In Risk Preferences.

	N. of Hypotheses/Pairs	N. of Rejections 1 <sup>st</sup> Order Polyn. in $\rho^{ij}$	N. of Rejections 2 <sup>nd</sup> Order Polyn. in $\rho^{ij}$
<b>Aurepalle</b>			
<b>All Households</b>	424	104/424	113/424
<b>By caste</b>			
Caste Score=7.5	10	2/10	0/10
Caste Score=18.75	1	0/1	0/1
Caste Score=30	10	0/10	0/10
Caste Score=55	26	0/26	4/26
Caste Score=86.25	10	0/10	0/10
Caste Score=97.5	1	0/1	1/1
<b>Shirapur</b>			
<b>All Households</b>	396	62/396	69/396
<b>By caste</b>			
Caste Score=5	6	0/6	1/6
Caste Score=23.75	26	1/26	1/26
Caste Score=72.5	78	7/78	6/78
<b>Kanzara</b>			
<b>All Households</b>	428	148/428	107/428
<b>By caste</b>			
Caste Score=11.25	8	0/8	0/8
Caste Score=55	78	11/78	8/78
Caste Score=76.25	1	0/1	0/1
Caste Score=91.25	6	2/6	1/6

Note: The sample period corresponds to 1975-1985 for Aurepalle and to 1975-1984 for Shirapur and Kanzara. The results are obtained using a  $k$  in the k-FWE rate that corresponds to 3/45% of total hypotheses. The significance level is 0.05. Household expenditure is the sum of expenditure on non-durable consumption and expenditure on leisure. The caste ranking corresponds to the one considered in Behrman (1988).

Table 6: Standard Test of Efficient Risk Sharing

	Coefficient on Non-labor Income	Standard Error	N. Observations
<b>Aurepalle</b>			
<b>All Households</b>	0.014**	0.005	2822
<b>By caste</b>			
Caste Score=7.5	0.075	0.062	582
Caste Score=18.75	0.032	0.032	217
Caste Score=30	0.038	0.030	493
Caste Score=55	0.038**	0.019	763
Caste Score=86.25	0.012*	0.007	541
Caste Score=97.5	-0.004	0.011	170
<b>Shirapur</b>			
<b>All Households</b>	0.011**	0.003	2537
<b>By caste</b>			
Caste Score=5	-0.019	0.034	378
Caste Score=23.75	0.012**	0.005	733
Caste Score=72.5	0.011**	0.004	1426
<b>Kanzara</b>			
<b>All Households</b>	0.020**	0.004	2565
Caste Score=11.25	0.009	0.017	558
Caste Score=55	0.023**	0.010	1240
Caste Score=76.25	0.030**	0.008	258
Caste Score=91.25	0.002	0.007	479

See note in Table 5. (\*\*) and (\*) indicate that the coefficient is significant, respectively, at the 5 and 10 percent level.

Table 7: Tests of Efficient Risk-Sharing.

	N. of Hypotheses/Pairs	N. of Rejections With Non-labor Income	N. of Rejections With Increasing Functions
<b>Aurepalle</b>			
<b>All Households</b>	424	12/424	12/424
<b>By caste</b>			
Caste Score=7.5	10	0/10	0/10
Caste Score=18.75	1	0/1	0/1
Caste Score=30	10	0/10	0/10
Caste Score=55	26	0/26	0/26
Caste Score=86.25	10	0/10	0/10
Caste Score=97.5	1	0/1	0/1
<b>Shirapur</b>			
<b>All Households</b>	396	18/396	22/396
<b>By caste</b>			
Caste Score=5	6	0/6	0/6
Caste Score=23.75	26	0/26	0/26
Caste Score=72.5	78	0/78	0/78
<b>Kanzara</b>			
<b>All Households</b>	428	10/428	27/428
<b>By caste</b>			
Caste Score=11.25	8	0/8	0/8
Caste Score=55	78	1/78	5/78
Caste Score=76.25	1	0/1	0/1
Caste Score=91.25	6	0/6	0/6

See note in Table 5. The order of the polynomial in  $\rho^{i,j}$  is 3 for the efficiency test with non-labor income and 5 for the efficiency test with increasing functions. The  $k$  in the k-FWE rate is set to 3/45% of total hypotheses for the efficiency test with non-labor income and to 5/45% of total hypotheses for the efficiency test with increasing functions.

Table 8: Average Real Per-capita Transfers and Loans in Rural Villages.

Variable	Aurepalle		Shirapur		Kanzara	
	Mean	% of Expen.	Mean	% of Expen.	Mean	% of Expen.
Real Per-capita Non-durable Expen.	33.1	–	47.3	–	50.0	–
<b>Real Per-capita Transfers Given</b>						
Total Transfers	4.8	14.5%	4.5	9.5%	2.4	4.8%
Total Transfers within Village	2.3	7.0%	1.5	3.2%	1.5	3%
Tr. to Relatives/Caste Fellows	4.0	12.1%	3.9	8.3%	2.1	4.2%
Tr. to Relat./Caste Fellows w. Village	2.2	6.7%	1.1	2.3%	1.4	2.8%
<b>Real Per-capita Transfers Received</b>						
Total Transfers	4.3	13.0%	7.1	15%	5.8	11.6%
Total Transfers within Village	1.2	3.6%	1.9	4%	4.7	9.4%
Tr. to Relatives/Caste Fellows	3.0	9.1%	5.5	11.6%	3.3	6.6%
Tr. to Relat./Caste Fellows w. Village	0.5	1.5%	0.9	1.9%	3.2	6.4%
<b>Real Per-capita Loans</b>						
Loans Given	0.0	0%	0.30	0.6%	0.04	0.1%
Loans Given to Relatives/Caste Fellows	0.0	0%	0.30	0.6%	0.04	0.1%
Loans Received	13.9	42.0%	20.0	42.0%	8.3	17.0%
Loans received from Relat./Caste Fellows	0.29	0.9%	0.74	1.6%	0.19	0.4%
<b>Real Per-capita Loans Conditional on a Positive Amount</b>						
Loans Given	0.0	–	42.0	–	20.7	–
Loans given to Relatives/Caste Fellows	0.0	–	42.0	–	20.7	–
Loans Received	38.2	–	57.8	–	45.4	–
Loans received from Relat./Caste Fellows	0.8	–	2.1	–	1.0	–
<b>Exchanged Laborers and Bullocks</b>						
Exchanged Laborers	0.1	0.3%	0.2	0.4%	0.1	0.2%
Exchanged Bullocks	0.32	1%	0.9	1.9%	0.4	0.8%

Note: In the ICRISAT questionnaire, a transfer is defined as a transaction in which resources or money change ownership without compensation. A loan is a similar transaction with a compensation.

## F Figures

Figure 1: Efficiency Condition with Heterogeneous HARA Preferences and  $\mu_1 = \mu_2$ .

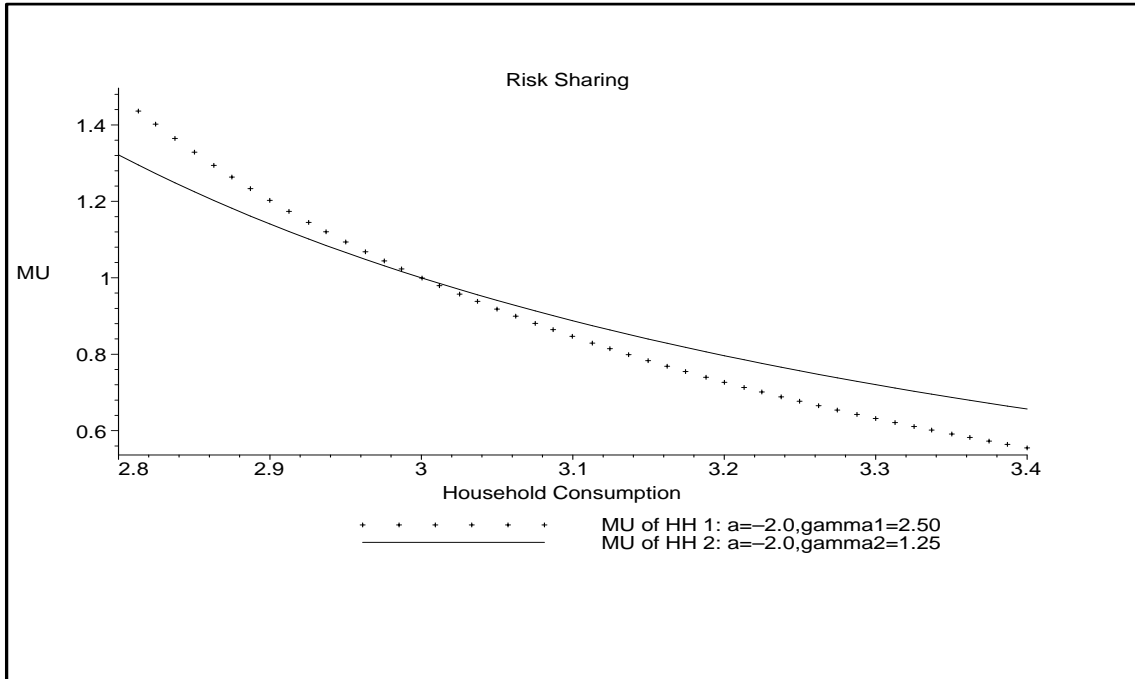


Figure 2: Efficient Consumption with Heterogeneous HARA Preferences and  $\mu_1 = \mu_2$ .

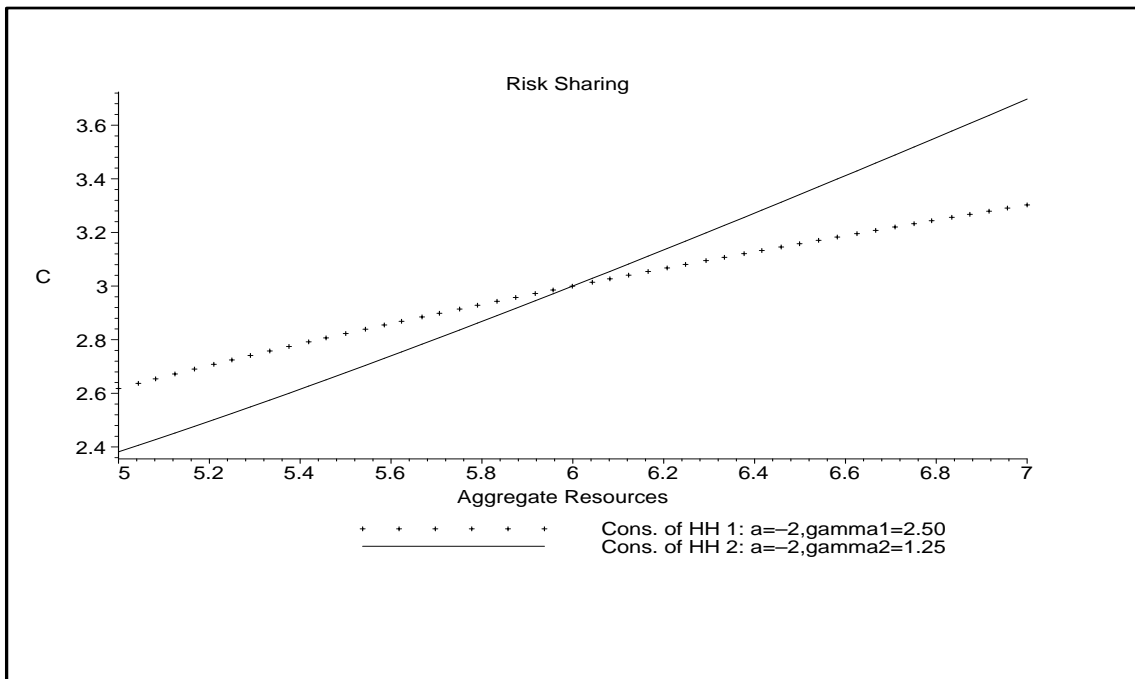


Figure 3: Efficiency Condition with Identical HARA Preferences and  $\mu_1 > \mu_2$ .

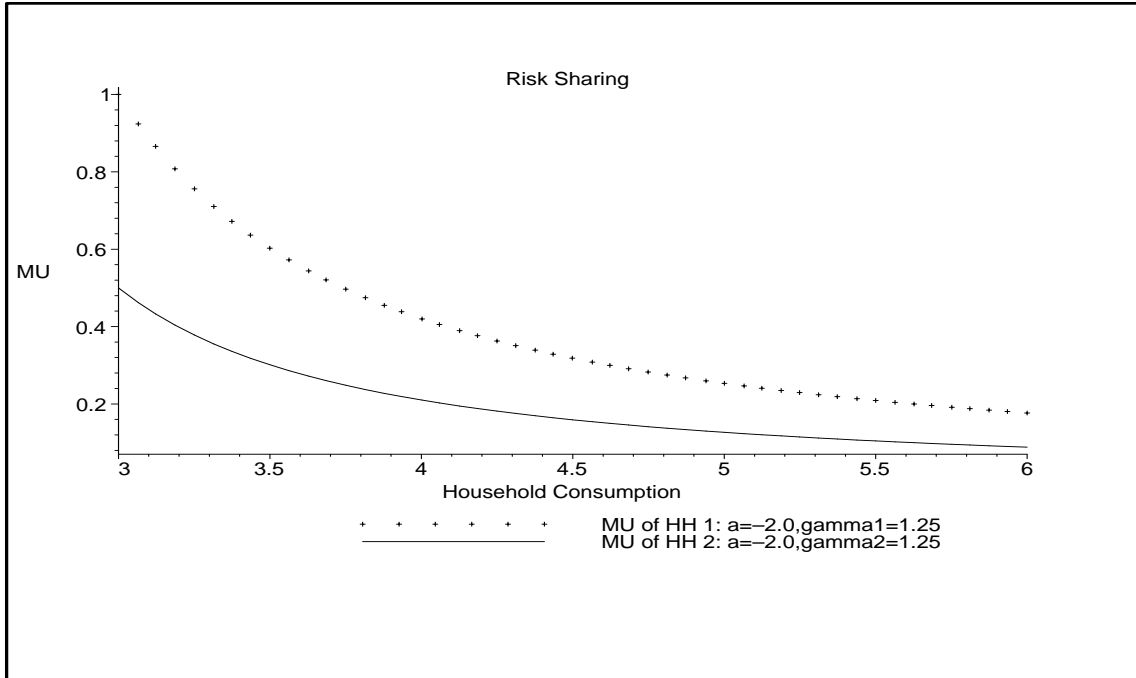


Figure 4: Efficient Consumption with Identical HARA Preferences and  $\mu_1 > \mu_2$ .

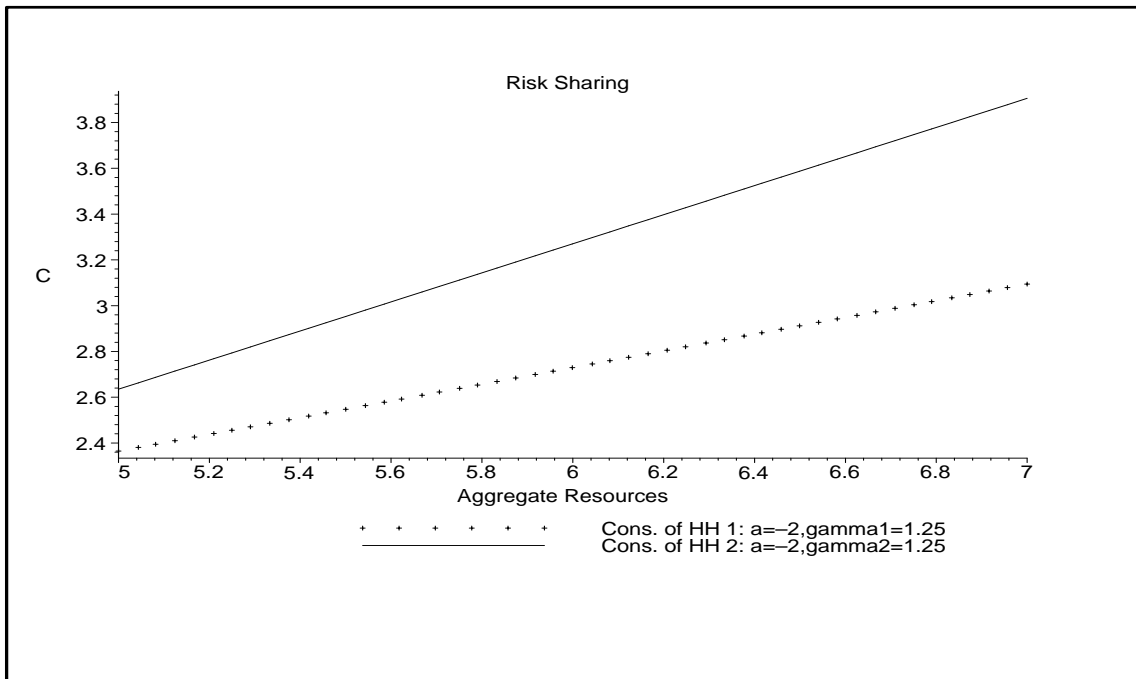


Figure 5: Expenditures as a Function of Aggregate Resources.

