

# Exchange Rates and Asset Prices: heterogeneous agents at work

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## Abstract

This paper merges two branches of the literature. On the one hand we have a heterogeneous agents framework, which for the first time is used to model exchange rates as well as stock prices. On the other hand we model the macroeconomic relationship between these two series through a complete open economy DSGE model.

From the behavioral finance literature, investors may choose one of two rules to make their expectations. They maximize profits in both markets by solving a mean-variance problem that allows them to choose between a fundamental rule and a chartist one. An open economy general equilibrium model creates the fundamental expectations by reacting to the shocks coming from the financial markets. Through this macroeconomic structure, exchange rate shocks may influence stocks and viceversa. As a result, agents choose between different combinations of rules in the home equity market and in the foreign exchange market.

The biggest new finding is that bubbles in one market may create values in the other that are similar to chartists bubbles, while following purely fundamentalist expectations. This dynamics is studied for different levels of pass-through between financial shocks and macroeconomic ones.

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## 1 Introduction

The subject of asset prices and exchange rates has recently received a lot of attention by some of the most renowned economic experts. This is due to two main factors. On the one hand the two time series show similarities typical of financial markets. This makes it possible to use similar statistical tools to study both the markets. On the other hand a full understanding of the economy is essential in order to understand how these markets work. This second factor is what makes the subject attractive for theoretical and macroeconomic contributions as well as for empirical and strictly financial papers. In both cases, a paper studying the relationship between exchange rates and stock prices has to make important economic assumptions. These define the big picture according to the authors and set the tone throughout the paper.

This paper takes a new approach in that it combines the literature of behavioral finance with the literature on DSGE modeling. Specifically, it takes a DSGE model to capture the fundamental expectations of a heterogeneous agents model. The results are very encouraging and add to both branches. The behavioral finance literature is enriched by the complete macroeconomic framework that makes the fundamental expectations truly reflective of the situations created by the financial markets. The DSGE literature is used to model financial series, in a way that was not seen before. This opens a wide variety of new applications for DSGE models to be integrated with financial markets.

In order to place this paper within the modern literature, it is useful to make a short survey of papers in two separate branches. The first one, in chronological order, is the behavioral finance literature. The second one is the relatively restricted group of contributions that study specifically the relationship between the stock prices and the exchange rates.

Within the wide field of behavioral finance we will go over the heterogeneous agents models, as this is the main assumption in the following paragraphs. The econometric process that is key to the switch between rules has been introduced by Brock and Hommes in 1997 and 1998. Thanks to their econometric paper, the tools were developed to study systems with alternating rules. Frankel and Froot wrote in 1998 a ground breaking paper in this framework.

Finally, De Grauwe and Grimaldi, in 2006 published a book summarizing most of the findings to date and showing new evolutions for the model. Since then, several other contributions have been given, each showing a different as-

pect of the structure. An example of this, where several fundamental beliefs alternate as well, is the one of De Grauwe and Rovira-Kaltwasser, in 2007.

In the area of stock prices and exchange rates, there are several empirical papers. However we would like to just illustrate 3. From the macroeconomic perspective, Kollmann in 2001 develops and solves a DSGE model that includes the capital market. Here for the first time in such a context, the equity flows are studied in conjunction with the exchange rate created by an open economy two country market of the style of Obstfeld and Rogoff 1998. The model is solved and the dynamics is studied, however the premises of the DSGE are still too wide to study the day to day financial series using this framework alone. Nonetheless this work definitely launches some solid foundations to analyze the relationship between stock prices and exchange rates.

The second paper is by Rigobon and Pavlova (2003). They make a general equilibrium model based on productive and demand shocks that influence both the stocks and the exchange rate. Their model is very practical, and the empirical applications all show a good consistency of the data with the model.

Finally, Rey and Hau in 2006 use the exchange rate microstructure intuition, as introduced in (16), to develop a partial equilibrium model with a very wide financial theory section and a strongly supportive empirical application to over 20 countries. All their main findings are found in the data, and to this day it remains the model with the most supportive empirical evidence. They study the exchange rate as the result of order flows for foreign currency from equity trade in an international two country setting. This model, among all the others, catches the importance of flows to and from the two markets.

The rest of the paper is structured in the following way. The next section introduces the theoretical framework of the paper. Here we focus on the two financial series and describe the heterogeneous agents model in detail. Section 3 analyzes the fundamental expectations in detail. Here we see the assumptions and basic equations of the DSGE model, and we solve it to a canonical log-linearized solution of the type traditionally seen in the literature. Section 4 focusses on the connection between these two approaches. A clear mechanism is outlined that binds the two models in a consistent non-linear system. At the end of this section the theoretical framework should appear outlined in all its aspects.

Section 5 uses some simulations to describe the properties of this system. The emphasis is on the differences between this model and the past ones. In

this way we only gaze upon the usual properties of heterogeneous agents models, to focus on the new results offered by the present structure. We also add an empirical analysis, that shows the parallelism between our simulated series and the real financial markets. A short conclusion adds some further points for research.

## 2 Theory

In our model we focus on one country. Specifically, we work in a small open economy. This country has private equity stocks and a floating exchange rate. There are no transaction costs. Within this setting we lay the basic assumptions for the two time series to be simulated. These are the home stock prices and the effective exchange rate between the home currency and the rest of the world currencies. In order not to make ad hoc assumptions on the evolution of these time series, the framework chosen here is of the most general kind.

The heterogeneous agents model is based on two alternative rules agents may use to create expectations. The first rule is referred to as chartist. This is a backward looking formula that only considers past returns. In our case we take a traditional autoregressive process with decreasing memory. The second rule is a mean-reverting, forward looking one. This is usually called a fundamentalist rule, as agents formulate it by looking at the distance between the realization of the price and the fundamental value they believe that price should take.

In our model the fundamentalist rule is given by a fully developed DSGE model, entailing that the fundamentalist agents believe that a DSGE model best describes the connection between the two markets. In other words, every time these agents see an unexpected change in the market, they create a shock in their model, and use the impulse response functions as a forecast of the two series in the following periods.

Each rule is continuously checked according to its profitability. If it is profitable agents keep it for the next period, otherwise they switch to the other one. While the process is relatively simple, we will see that the dynamics created is fairly complex and it is source of non-linearities as well as heteroscedasticity and time varying behavior.

Most importantly, this paper adds for the first time a macroeconomic dimension to the fundamentalist expectation. We show that this results into fundamentalist expectations that may be as variable as the chartists, and create a bubble-like behavior completely supported by fundamentalist beliefs. This

shows the potential of mixing macroeconomics and financial tools. In the end we show that our model simulates financial markets with very realistic features.

## 2.1 Stock Prices

The stock price series is made by a weighted average of the different expectations, plus a white noise. This makes expectations self realizing on average, and therefore important to describe. There are two types of rules. One is what is usually called a fundamental rule. This is a mean reverting rule that brings the system closer and closer every period to what the agents believe to be the fundamental price. In this paper we assume that there is no disagreement on this fundamental price<sup>1</sup>. The second expectation rule is of a chartist kind. This means that all that is looked at is the past information on the time series, without taking any reference value in consideration. Since these two rules use different sets of information, the agents choose from time to time what is relevant. They do this by continuously checking the profitability of their rule, and switching to the other one if they find it more profitable.

Formally, the home stocks are defined by the following specification:

$$\Delta S_{t+1} = m_{f,t}^s [Rule1Forecast] + m_{c,t}^s [Rule2Forecast] + \varepsilon_{t+1} \quad (1)$$

where  $\Delta S_{t+1}$ , the change in tomorrow's stock price, is given by the first rule's forecast weighted by the agents that believe in this rule plus the forecast from the second rule, weighted by the agents believing in this second rule.  $\varepsilon$  represents a white noise shock, or news, that is not incorporated in either rule.

Equation 1 makes it clear that expectations are crucial in the evolution of the stock prices. This shows the self realizing nature of expectations. It is now important to formalize the two rules. We make assumptions similar to the literature on heterogeneous agents, by assuming that one of the rules will be forward looking, and the other will be backward looking.

The backward looking or chartist rule is, still in tune with the majority of the literature in this field, where the relevant information is given by all the past returns, weighted by a discount factor. Formally:

$$E_{c,t}(\Delta S_{t+1}) = \beta \sum_{k=1}^T \eta(1-\eta)^k \Delta(S_{t-k}) \quad (2)$$

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<sup>1</sup>This assumption could be relaxed, and then we would have a dynamics similar to the one described in De Grauwe and Rovira Kaltwasser, 2007.

The discount factor  $\eta$  shows the memory of the system, and its resiliency to past shocks. The higher this parameter, the longer is the memory of the system.

The fundamentalist rule, or forward looking, calculates the distance to some value of the stocks believed to be the “fundamental” value, and adjusts this distance period after period. Formally, we could write it as:

$$E_{f,t}(\Delta S_{t+1}) = -\alpha(S_t - \bar{S}_t) \quad (3)$$

Where  $\alpha$  is smaller than one and allows a smoothed progression of the stock price towards its fundamental price. However in our paper this equation is substituted by impulse response functions from the DSGE model mentioned above. These functions act exactly as the equation above, in that they are mean-reverting around the steady state, and that the distance to the steady state diminishes every period. We will analyze the fundamental expectation in depth in the next paragraph.

## 2.2 Exchange Rate

The exchange rate in this model has similar assumptions to the stock prices. Also in this case, the exchange rate value is going to be given by the weighted average of the expectations, plus a noise. This market is also characterized by a chartist and a fundamentalist rule. It is important to remember that this last rule is going to be affected by also changes in the stock prices. In the same way, changes in this market are going to provoke a reaction in the stock market. The structure of this relation will be shown shortly. For clarification, we redefine the mechanism of the chartist expectation given a steady state value.

Obviously whoever believes in the chartist rule, will take the exchange rate series to be its own auto-regressive process, with no pegging onto a theoretical pre-set value. So the equations for the fundamental expectation of the exchange rate will be the following:

The chartist expectation is given by:

$$E_{c,t}(\Delta e_{t+1}) = \beta_e \sum_{k=1}^T \eta_e (1 - \eta_e)^k \Delta(e_{t-k}) \quad (4)$$

Where  $\eta$  may be, but is not necessarily, equal to the one in the stock market.

Now a closer look at the fundamentalist rule for exchange rates. Also in this case we use this formula to convey the the intuition mechanism, since the fundamental expectation in this paper is given by the impulse response functions

from the DSGE model.

$$E_{f,t}(\Delta e_{t+1}) = -\alpha_e(e_t - \bar{e}_t) \quad (5)$$

Where all remains similar to the formalization for the stock market.

The series of the exchange rate evolves from the expectations as we have seen for the stock prices. Formally:

$$\Delta e_{t+1} = m_{f,t}^e [Rule1Forecast] + m_{c,t}^e [Rule2Forecast] + \varepsilon_{t+1}^e \quad (6)$$

Also in this case the profits and the switching rule dynamics are similar to the stocks, and are described below.

### 2.3 Switching between the rules

A key feature of this model is that the agents are not static, but keep evaluating their returns and check if their rule is still the most profitable one. So if the rule is good they can keep it, otherwise they will switch over to the other rule, in the hope that it will yield them higher returns. This happens in both markets and independently from one another. Following this idea and applying the fitness criterion based on discrete choice theory, as shown by Brock and Hommes ((7) (8)), we can calculate the fraction of population that uses each of the rules as a variable of the (risk adjusted) profitability of the same rule. Formally:

$$m_{f,t} = \frac{\exp \gamma \pi'_{f,t}}{\exp \gamma \pi'_{f,t} + \exp \gamma \pi'_{c,t}} \quad ; \quad m_{c,t} = \frac{\exp \gamma \pi'_{c,t}}{\exp \gamma \pi'_{f,t} + \exp \gamma \pi'_{c,t}} \quad (7)$$

Obviously in this case  $m_{f,t}$  and  $m_{c,t}$  are the fractions of population that at a given time are following the two rules (for each of the markets), and therefore they add up to 1 at all times. The variables  $\pi'_{f,t}$  and  $\pi'_{c,t}$  are the risk adjusted profits realized through the use of each of the rules. For example we could think of them as  $\pi'_{f,t} = \pi_{f,t} - \rho \sigma_{f,t}^2$  and  $\pi'_{c,t} = \pi_{c,t} - \rho \sigma_{c,t}^2$  where the symbol  $\pi$  represents the profits made by each of the rules, and the  $\sigma^2$  are a measure of the risk that the rules involve. For this purpose forecast errors could solve the information quest. Finally  $\rho$  is a coefficient of risk aversion.

This system of equations shows how the agents revise their decision making rules as one of the two rules becomes more attractive (read profitable). An important factor in this picture is how quick the agents are to revise and eventually correct their rules, concept that is inserted in the model through the parameter  $\theta$ . For an increasing  $\theta$  all agents revise their rules quickly and therefore a higher amount of population follows the most profitable rule at every time  $t$ . When

the agents are insensitive to the profitability of their rules a good starting point could be a 0.5 fraction of the population using each rule.

All the parameters outlined above may take different values in the exchange rate market as opposed to the stock market. Similarly, it is also possible that the same agent in the same period has a fundamental expectation in one market and a chartist one in the other.

The model's frame is built, and the main concept left to formalize is how to calculate the profits  $\pi'_{i,t}$  for every rule  $i$ . To begin with, we should define the profits non adjusted for the risk, as the single period returns from 1 unit of local currency invested into a foreign asset:

$$\pi_{i,t} = \Delta S_t \operatorname{sgn}[E_{t-1}^i(S_t) - S_{t-1}] \quad (8)$$

$$\text{with } \operatorname{sgn}(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases} \quad \text{and } i = c, f$$

Therefore the profit for every rule goes up whenever the forecast is right, and goes down whenever the forecast is far from the realized price. In order to calculate the risk associated with every rule we are going to look at the most general case in which it is the forecast error of the previous periods weighted by a discount factor. Therefore, formally:

$$\sigma_{i,t}^2 = \sum_{k=1}^T \rho(1 - rho)^k (E_{t-1-k}^i(S_{t-k}) - S_{t-k})^2 \quad (9)$$

Having defined all the variables of the system it is possible to describe the market forecast as a weighted average of the two groups' forecasts, as we did in Equation 1. Only, this time Rules 1 and 2 will be written out as in the following equation:

$$E_t \Delta S_{t+1} = m_{f,t}[-\eta(S_t - S_t^*)] + m_{c,t}[\beta \Delta S_t] \quad (10)$$

This being the forecast, the actual change of the stock price, given the described model, is formally written as:

$$\Delta S_{t+1} = m_{f,t}[-\eta(S_t - S_t^*)] + m_{c,t}[\beta \Delta S_t] + \varepsilon_{t+1} \quad (11)$$

Therefore the market is governed by this dynamics and by a white noise shock occurring in time  $t + 1$ . This concludes the description of the variables affecting the stock price market. Everything else will come into this market through its

impact on chartist or fundamental expectations. This lack of overall structure is very crucial in this literature, as it allows to just focus on the expectations and their impact on the market.

This concludes the description of the model used in order to replicate the dynamics of the stock prices and the exchange rates. As shown, this theory is based on very simple and straightforward assumptions. These cause a very complex dynamics, that will be analyzed in a few of its realizations in the remainder of the paper. Indeed the major trait of this model is that it creates a non-linear dynamics, that is impossible to solve into a general solution, except for making more constrictive assumptions, that would restrict the model. However we feel that this is exactly its most valuable asset. The flexibility of this partial equilibrium model, and its adaptability to include and generate most statistical characteristics found in real time series from the major stocks and exchange rates of the world. Our next section is our specific contribution to the literature. For the first time, we trace a connection between traditional macroeconomics, and these financial models. We hope to show that the two models are not so opposite and incompatible as some people might believe. This is the focus of the next section.

### **3 The macroeconomics in finance: fundamental expectations**

The most interesting part of any paper on exchange rates and stock prices is always to study how the connection between stocks and exchange rates is modelled. The study of this particular relationship forces many assumptions that are to date not shared by all. This leaves a niche in the literature for some research to find a way to unify most of the intuitions in one organized economic theory.

The assumptions taken within this paper are taking from the macroeconomic literature. Specifically, we will include within our heterogeneous agents framework, expectations drawn from a typical textbook model featuring a small open economy, of the type of Gali 2008<sup>2</sup>.

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<sup>2</sup>This model was also included in the book by Gali 2008. It models a small open economy from a purely DSGE approach.

### 3.1 General equilibrium model

We proceed now to the description of the small open economy model used for fundamental expectations.

#### 3.1.1 Households

Households maximize a utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (12)$$

with  $N_t$  are the hours of work in period  $t$  and  $C_t$  is a consumption index defining the composition of the consumption basket for every level of income:

$$C_t \equiv [(1 - \alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \quad (13)$$

with  $C_{H,t}$  being an index of consumption of domestic goods that has a constant elasticity of substitution and  $C_{F,t}$  is the sum of goods produced in all the foreign countries and consumed by the home country.  $\alpha$  is a parameter that shows the openness of the home country.

Moreover, the family is subject to a budget constraint, that may be written as:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t \quad (14)$$

with  $P_{H,t}(j)$  being the price of good  $j$  produced at home,  $P_{i,t}(j)$  is good  $j$  that is produced in country  $i$  and consumed at home.  $Q_{t,t+1}$  is the stochastic discount factor for buying a one period ahead bond,  $W_t$  is the salary per hour,  $D_{t+1}$  is the nominal payoff on the portfolio held during  $t$ . The markets are complete because we assume that we can trade Arrow securities for all risks. The system may be solved to get demand functions of the different types of goods. This is when all expenditure levels are optimized by spending in the different allocations. These demand functions are:

$$C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{i,t} \quad ; \quad C_{i,t}(j) = \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t} \quad (15)$$

and the optimal allocations between home produced and imported goods are:

$$C_{H,t}(j) = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (16)$$

This allows optimization of consumption expenditures for  $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$ . Overall, now we can simplify the budget constraint to be equal to:

$$P_t C_t + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t + T_t \quad (17)$$

We may specify the Utility function to be equal to the following one:

$$U(C, N) \equiv \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \quad (18)$$

where we see the utility coming from consumption and the disutility arising from hours of labor.

### 3.1.2 Terms of trade, Exchange rate, Inflation

Having defined most of the features of our representative household choice domain, we may look at the international side of the economy. Following is a form for the effective terms of trade:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \quad (19)$$

This may be linearized around a symmetric steady state where  $S_t = 1$  for all foreign countries.

$$s_t = \int_0^1 s_{i,t} di \quad (20)$$

On a different stand, inflation is composed of domestic inflation and changes to terms of trade:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \quad (21)$$

Domestic inflation, indicated by  $\pi_{H,t}$ , is the inflation on home produced goods.

Furthermore the law of one price holds for all goods. The following is an expression connecting foreign prices to the domestic price index for the different foreign countries and to the nominal exchange rate.

$$p_{F,t} = e_t + p_t^* \quad (22)$$

We also have a form of uncovered interest rate parity, as shown below:

$$i_t = i_t^* + E_t\{\Delta e_{t+1}\} \quad (23)$$

### 3.1.3 Firms

The representative firm produces a differentiated good and has a production function of the following type:

$$Y_t(j) = A_t N_t(j) \quad (24)$$

where  $A_t$  is a linear technology autoregressive process, with coefficient  $\rho$  and innovation  $\epsilon_t$ . This leads to a marginal cost that is common to all firms and of the form:

$$mc_t = -v + w_t - p_{H,t} - a_t \quad (25)$$

Overall this model assumes a la Calvo pricing, where each period a fraction  $\theta$  of the firms lets its price fixed, and a fraction equal to  $1 - \theta$  fixes a new price. This leads to a pricing dynamics that may be described by:

$$\bar{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{mc_{t+k} + p_{H,t+k}\} \quad (26)$$

where  $\bar{p}_{H,t}$  is the log of the prices that are set new in period  $t$ , and  $\mu$  is the log of the gross mark up in the steady state.

### 3.1.4 Monetary policy and equilibrium

The model laid out in this paragraph may be solved around a steady state and log linearized in order to get a closed form solution<sup>3</sup>. When this is done we can solve for a more typical form, with a Dynamic IS curve, a New Keynesian Phillips curve, and a monetary policy rule.

So we get the following New Keynesian Phillips curve:

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + k_\alpha \tilde{y}_t \quad (27)$$

where  $\tilde{y}_t$  is the output gap and for  $\alpha = 0$  we get a closed economy version of the same relationship. We can also get a dynamic IS curve, defined in the following manner:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma_\alpha} (i_t - E_t \{\pi_{H,t+1}\} - r_t^n) \quad (28)$$

with the natural interest rate being:

$$r_t^n \equiv \rho - \sigma_\alpha \Gamma_a (1 - \rho_a) a_t + \frac{\alpha \Theta \sigma_\alpha \varphi}{\sigma_\alpha + \varphi} E_t \{\Delta \tilde{y}_{t+1}^*\} \quad (29)$$

Finally we can choose monetary policy rule. For the purpose of this paper we have CPI inflation targeting, however the model would work equally well with other types of monetary policies. For the CPI inflation targeting we have a Taylor rule of the type:

$$i_t = \rho + \phi_\pi \pi_t \quad (30)$$

<sup>3</sup>We do not explicitly show this part, since it would not add to the understanding of our model. However for further reading refer to Galí, 2008 or any similar model in the literature.

This ends the description of the model that fundamentalist agents use to make their forecasts. While we have overseen some of the technical steps to solve the model, the basic assumptions as well as the main implications of this model should be evident. This allows us to move forward to a solution to our complete model, that includes this structure as a rule of expectations. This step is explained in the next paragraph.

#### 4 Merging two literatures

The core characteristic of the whole DSGE literature is that a great number of economic variables is made endogenous. These models become so complex that have no closed form solution. In order to study the dynamics of the economy, the most common adjustment is to log linearize all the relationships around some steady state values of the macroeconomics variables. This is what we have seen in the previous section. This allows the researcher to study reactions of the whole system to shocks in some exogenous variables. The most common type of shocks may be productivity (or technology) shocks. After a shock of this type the complex system of equations brings the economy back to its initial equilibrium.

It is precisely this pressure to bring the system back to its steady state, or fundamental value, that makes this models so attractive in the calculation of fundamental expectations within the framework discussed above. Through this model's dynamics, it is possible to show how shocks in one variable may affect the others.

To make the story more practical, it is useful to imagine how would an agent adopting a fundamentalist rule think. After a movement in say, exchange rates, a typical reaction would be to start questioning which exogenous (and therefore unexpected) shock could have created such change. Since everything in these models is endogenous, only a shock to the exogenous process of productivity can create such unexpected changes. So now the choice is simply to understand which kind of shock to productivity could have caused the unexpected change in my financial variables.

There are four types of shocks that we will study, following this literature. These are a positive and negative shock on Home productivity, and a positive and negative shock to the Rest of the World productivity. For the case of this model we will assume that the agent will consider changes in home variables (like the stock prices) to come from its Home productivity process, and changes

H shock	RoW shock	$\Delta Y_{gap}$	$\Delta E$
H=0	F=-1	+	-
H=0	F=1	-	+
H=-1	F=0	-	+
H=1	F=0	+	-
H=-1	F=-1	-	+
H=1	F=1	-	+
H=-1	F=1	-	+
H=1	F=-1	+	-

**Table 1: Responses from shocks of size 1 on Home and RoW productivity**

affecting the country coming from the exchange rate market will be interpreted as a reaction to shocks coming from the outside, or rest of the world productivity.

#### 4.1 Macroeconomists at work

Before proceeding further it is useful to start outlining the relationship between our financial variables and the variables found in this DSGE world. Exchange rates are in our financial market as well as in the macro model in nominal terms. This concordance in terms allows us to interpret changes in our exchange rates as directly caused by foreign technology. We can therefore look for the kind of rest of the world productivity shock that would create a positive or negative reaction in the nominal exchange rates within the DSGE. The mapping in Table 1 shows that a positive foreign tech shock will cause a rise in exchange rates. Viceversa a negative foreign tech shock will provoke a negative change in exchange rates.

However this model is still not complete with stock prices. Therefore it is important to locate the variables in our DSGE model that would be able to affect home stock prices. This is a very well outlined relationship in the traditional economic literature that shows stock prices as a function of GDP growth. The relationship is that outlined in the Gordon model that defines stock prices in terms of its future dividends. Formally:

$$S = \sum_{k=1}^T \frac{D_{t+k}}{R^k} \quad (31)$$

with  $R$  a discount rate used to discount future dividends into a current value.

Since future dividends are dependent on future economic growth, the relationship to GDP growth is established. Indeed DSGE models consider the variable of output gap, which is the distance of output from its steady state.

Having univocally connected positive changes in stock prices to positive changes in output gap as expressed in the DSGE model referenced above, we can go back to the agent beliefs. An agent with fundamentalist expectations will look at changes in stock prices as caused by exogenous shocks in home technology. He will consider that a positive unexpected home stock price rise is due to a positive technology shock, and viceversa.

Having studied the single markets, it is useful to underline how the fundamentalist expectation on both the markets is derived by the combined effect of the past changes in both markets. This is possible through the macroeconomic model used. In practice each period we will calculate the unexpected changes (from a fundamentalist point of view) in the two markets. These changes will allow us to plug proportional changes in the exogenous variables. Then we will take the impulse response functions on our variables of interest.

In order to have our fundamentalist expectation we add these impulse response functions from the last unexpected shocks to the ones of the past few periods that are still on the way to absorbing past shocks. The value that we get is the proportional distance from the steady state value of our financial variable.

The next period, we will take care to consider only the distance between the new value and the old sum of impulse responses to the old shocks. That distance will be the one to plug back into the DSGE model.

Finally this shows how the fundamentalist expectation as described in the first paragraph was a simplification of this dynamics. In our case the parameter  $\alpha$  shown there is not actually part of the heterogeneous agents framework, but is rather imported from the macroeconomic model, which gives us an impulse response function (implying different  $\alpha$  levels).

This new connection between the markets creates a very rich dynamics, not only in the actual financial variables, but also, and more strikingly in the fundamental expectations. Now the agents may have an expectation that is at times even far from the steady state fundamental value, and still be completely in line with the fundamentalist rule.

The following paragraphs show how another important parameter will be the degree of pass-through between the financial changes and the shocks inserted into the model. This means that, while the shocks into the DSGE model are

always proportional to the ones in the stock market, a larger correspondence between them will make the fundamental expectation considerably more variable. Most importantly, it is possible to study these changes within a stable model. This means that the system has quite wide ranges of parameters that allow stability.

As a final remark, the past few paragraphs have shown that, although stable, the model is highly non-linear. This does not allow a study of a closed form solution without making considerably stronger assumptions. While this complexity is a strength of these models, it also means that we will mainly proceed by numerical simulations. This will nonetheless lead us to some conclusions on the model dynamics, time series characteristics, and will allow us to make comparisons with real like financial markets.

## 5 Model specification and simulation dynamics

This section shows the practical fine tuning of the model above to simulate the financial series described. We also describe in depth the mechanism to transfer imbalances within the system.

In what follows, we set the initial values of the stock prices to be equal to 20. The steady state of the stock prices is also set to 20. The exchange rate is equal to 1, both in the initial value, and in the steady state fundamental value. Other parameters are:  $\beta$  and  $\beta_e$ , describing the autoregressive coefficient of the chartist rule, are set both equal to 0.9. This is fairly similar if compared to some real life stock time series.  $\eta$  and  $\eta_e$ , setting the memory of the chartist rule, is equal to 0.5. This parameter allows the chartists to consider in their forecast also past returns on stocks, with decreasing weight. The values of  $\gamma$ ,  $\gamma_e$  describe the quickness of adjustment to more profitable rules. For low values we see systems in which the fundamental expectation is usually right, without bubbles. This is because, even though some noise may bring the system temporary further from the fundamental value, people do not adjust to that difference quickly enough, and so they stay on the fundamentalist view. These parameters are also weighted as a function of the initial values. So the values of  $\gamma = 10$  and  $\gamma_e = 700$  let the two financial markets exposed to bubbles, while allow for system stability.

Another parameter in the model is the traditional risk aversion. This is seen when we calculate the way of discounting absolute profits of a rule by the variance of those profits. The higher the variance, the lower the final, risk

weighted profits will be considered. The weight that sets how important the variance is, is the parameter  $\rho$ , which we set at 0.5. In this setting we do not need to define the parameter  $\alpha$  since the fundamental expectation is actually given by the macroeconomic model, as stated above.

Before continuing to the simulation, it is useful to stress the fact that each of these parameters may vary and cause smaller or bigger changes in the simulation that will come from the model. We have already mentioned, for example, how for certain values of  $\gamma$  we will have solely a fundamentalist regime take over during the whole simulation. However these properties have been studied already for number of systems similar to ours, therefore we refer the interested reader to several interesting papers and books for a detailed study of the nonlinear properties of this system (12; 2; 11; 10).

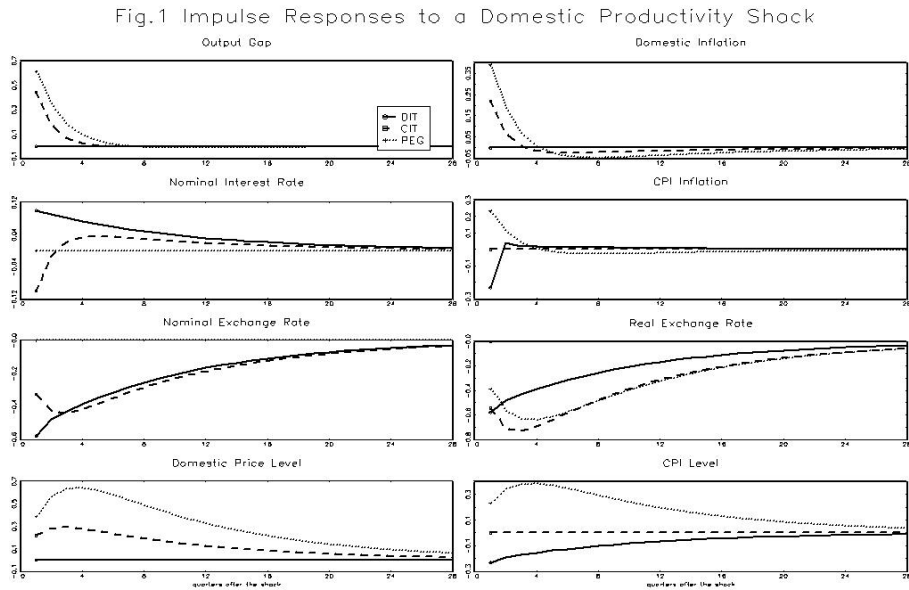
For this text it will suffice to know that for certain parameters we have a linear system, with one stable, fundamental equilibrium (as mentioned above) and then we proceed towards systems with more and more equilibria until reaching a chaotic (or aperiodic) system.

On the other hand, within this paper we find it interesting to study the changes in the system due to changes in the connection between macroeconomic expectations and heterogeneous agents. This is also the novice part of the paper, so we can start by looking at some simulations under different parameters, and we can study the first and second moments of the data.

Finally, within the DSGE model we will assume for the moment a steady state of no growth. Furthermore, all changes will be transferred from the DSGE to the financial markets as percentages, with a pass-through coefficient to regulate exactly how much of these changes eventually become shocks. This mechanism allows us to create shocks in the model that are proportional to the ones occurring in the market. In this way not only the direction of the shock is carried, but also the relevance of its size (big bubbles, big shocks). This will become important when we study the overall effect of the shocks on the fundamental expectations. Lastly, we use the regime of monetary policy of CPI inflation targeting. This model could be also applied for other regimes of monetary policy with the exception, of course, of the pegged exchange rate.

## 5.1 Some Simulations

Summarizing all that we have seen until now, the core connection between the financial and the macroeconomic model is through the fundamental expectation

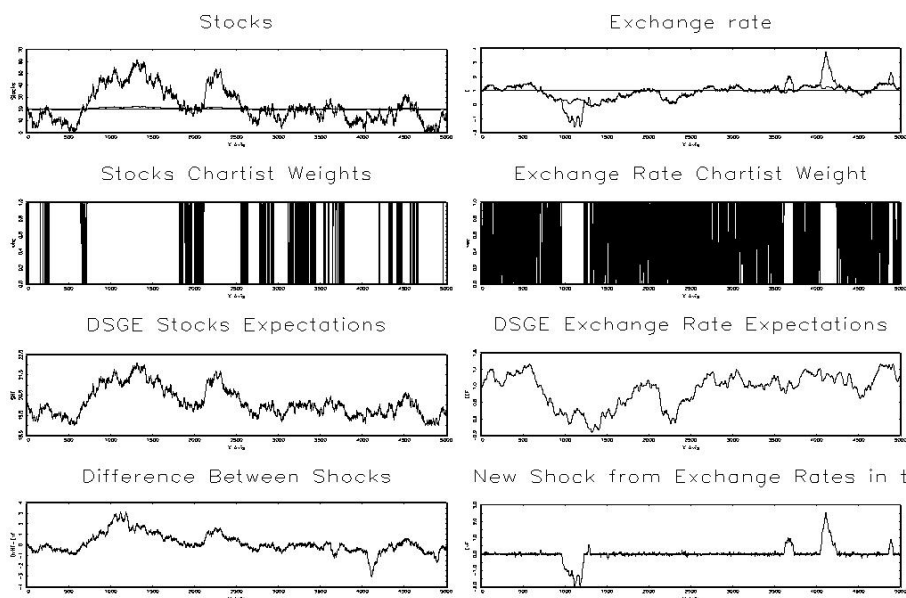


**Figure 1: Impulse responses from a positive shock on H productivity of size 1**

channel. Agents working in the financial markets look at the unexpected changes in the series, and realize that they were unable to expect these changes because they are caused by exogenous shocks. Specifically, they interpret unexpected variations in stock prices as due to a home productivity shock, and they interpret exchange rates non predicted variations as due to a foreign productivity shock.

Once the agents have decided which shocks have occurred in the precedent period, they focus on making expectations for the following period. Here it is important to stress that they may decide (and indeed this is the majority of the cases) to shock their model for both types of shocks. Thanks to the DSGE framework, they can do that at the same time. This obviously creates a very intricate dynamics of fundamental expectations, much different than what we usually see in the literature.

In this section we will show three different types of simulations. First we will see what happens when agents only use changes from the stock market to shock the model and form expectations. Then we will only use shocks from the exchange rate market, and finally we will look at shocks from both sources may work together. Figure 1 shows an impulse response function of the DSGE model to a shock on the Home productivity process. This is in accordance



**Figure 2: Stochastic simulation from shocks coming only from H stocks**

with Table 1, which shows how a positive shock of this type would provoke a temporary raise of the GDP gap, and a temporary decrease (or appreciation) of the exchange rate.

Figure 2 shows what happens when the unexpected changes in the Home stock market are transferred to the DSGE model, and agents with a fundamentalist rule form expectations accordingly. The first panel on the left side shows the realization of the stock price series of 5000 iterations. The second panel on the left shows the weight of chartists during each of the 5000 periods. Obviously the weight of chartists and fundamentalists always adds up to 1. What we usually see when the series follows a fundamentalist value is that the weight is alternating quickly between the two rules. This is because they are both at about the same level of profitability.

The difference arises whenever the realization gets away from the fundamentalist value. What happens in this case is that the chartist rule becomes progressively more profitable, and therefore a bigger and bigger number of people switches to the chartist rule. When we have bubbles, these are only held by a great majority of chartists. In the second panel on the left side, we can see this exact behavior. In the bubbles, the weight of chartists increases to very

close to 1.

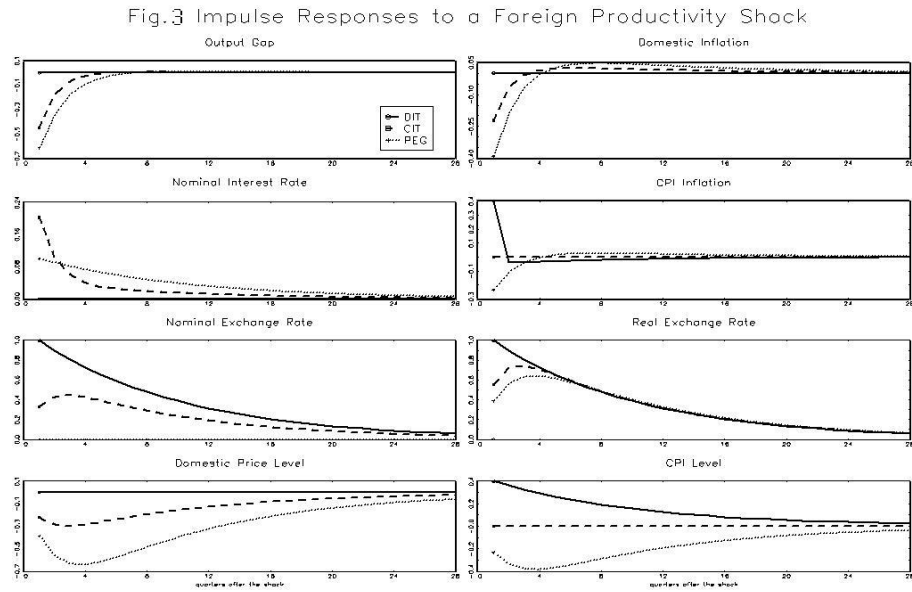
On the other hand the first panel on the right shows the realizations of the exchange rate for the same 5000 iterations. The fundamental value here, like in the stock prices, is set by the impulse response of the DSGE model, being shocked for stock price changes. This is the reason why it is possible to see a fairly active fundamental value, that is most of the times able to capture the market noise on its own. Obviously also here we see cases in which the chartists take over the market, creating variance to the exchange rate that is not justified by the fundamental value. This is shown by the second panel on the right side, where the weight of chartist is plotted in an analogous manner to the weight in the stock prices.

The third row of panels shows the fundamental expectation levels. This levels are also shown in the first row, together with the actual realizations, but here they are shown from closer up. This allows to see that the fundamental expectations of the stock prices and exchange rates are pretty much opposite to each other in returns. The reason for this is that the DSGE moves the exchange rate in exactly the opposite way than the output gap, when it comes down to shocks coming from home productivity. The intuition behind the model is that a temporary shock on productivity allows a stronger GDP. This leads to a growing economy with a temporarily stronger currency.

The last panel on the left shows the difference between the shocks coming from the stocks market and from the exchange rate market. In this case, the agents only shock the model with variations from the stock market, so the difference is equal to the percentage changes in the fundamental expectation. We will show later how this indicator will gain importance when the model is shocked from both sources.

Finally the last panel on the right shows the detachments of the exchange rate from the fundamental value. It is evident when the bubbles arise.

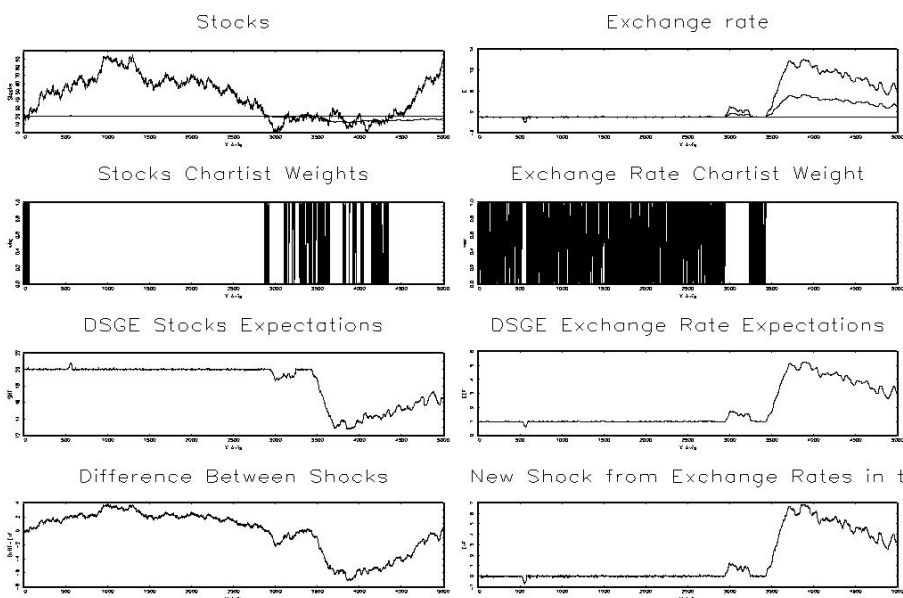
An important comment to this first case is how the system is very variable even though changes are only coming from one of the two markets. These changes shape a very defined dynamics also for the other market. The movements in the exchange rate fundamental value are the direct effect of the macroeconomic dynamics described by the DSGE. As such, they are desirable to our model. It is already easy to notice how the variance of the exchange rate is fairly high even when the exchange rate is just following its fundamental expectation value.



**Figure 3: Impulse responses from a positive shock on F productivity of size 1**

The second case that we are going to show is when the shocks reported to the DSGE model are solely coming from the exchange rate market. As such, agents construct a foreign productivity shock when their expected exchange rate is far from the realized value. As above, this creates complex expectations for both the financial markets studied.

First of all it is useful to check what happens to our DSGE framework when it is shocked accordingly. Figure 3 shows the impulse response function for a shock to foreign technology. Once again the findings are in line with Table 1. Figure 4 shows a simulation where exchange rate shocks are leading the system. It is possible to see that the movements in the stock fundamental value are exactly opposite of the shocks coming from the exchange rate market. A peculiarity of this set up is that we may often get bubbles in the stock prices that are going in the opposite direction than the fundamental value. This is part of the structure of the model. Obviously in these cases how exchange rate shocks may not be a good indicator for the home stocks. This allows us to see that the system functions normally even with shocks coming from only one of the two markets.

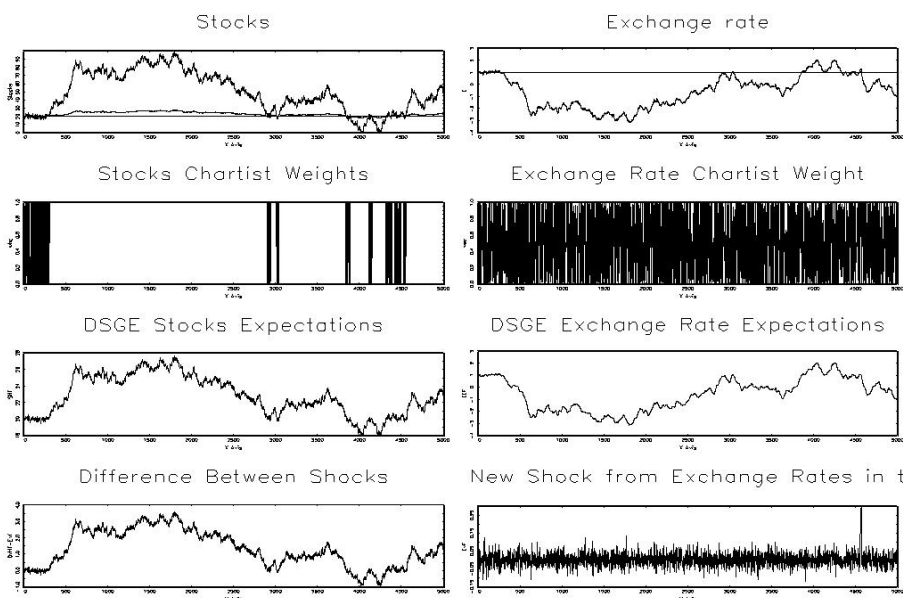


**Figure 4: Stochastic simulation from shocks coming only from Exchange Rates**

## 5.2 New heterogeneous agents dynamics: “fundamental bubbles”

In this section we study the implications of the new theoretical contribution of this paper. That is that now we can have markets following a solely fundamental expectation and still exhibit a bubble-like behavior. This is shown in figure 5. The exchange rate market is clearly always following a fundamental value. However this fundamental expectation is constructed to have a strong macroeconomic structure. In other words it will always reflect changes in any financial market that may have impact on the macroeconomic variables of the system. Then it is logical how if the stock market is in a chartist equilibrium, this will be balanced by a bubble-like behavior in the exchange rate market. However this is not necessarily the result of chartist behavior in the exchange rate market.

On a side note, figure 5 shows a market in which the value of  $\gamma_e$  is very low on purpose to allow the exchange rate to follow mainly the fundamental realization. This is only done to emphasize the concept of the “fundamental bubbles”. Relaxing this choice, and making a more traditional value of  $\gamma_e$ , is obviously possible, and produces a system in which “fundamental bubbles”



**Figure 5: Stochastic simulation from shocks coming from both markets with a large pass-through and no bubble in exchange rate**

will be present in the system as well as normal bubbles, governed by a chartist behavior. This is seen in the exchange rate series of figure 6.

### 5.3 Other changes in parameters

Another original feature of this model is to control over the pass-through of shocks from the financial markets to the fundamental expectations. On one hand, whenever a large part of the variability of these markets is transferred on to the fundamental expectations, we see these fundamental expectations to be really variable. On the other hand, when there are more frictions, the agents only use a fraction of the financial market unexpected change to make their expectations. In this case the difference between the expectations and the time series variability is much larger. This concept may be noticed best by comparing figure 7 with figure 8. Figure 8 has a larger fraction of shocks reflected in the expectations. This allows the markets to recreate bubble-like dynamics that are completely supported by fundamental expectations.

This concludes our description of simulation dynamics. Due to the nature of the model, the system has infinite realizations. However it is possible to summarize the main characteristics common to all simulations with similar parameters.

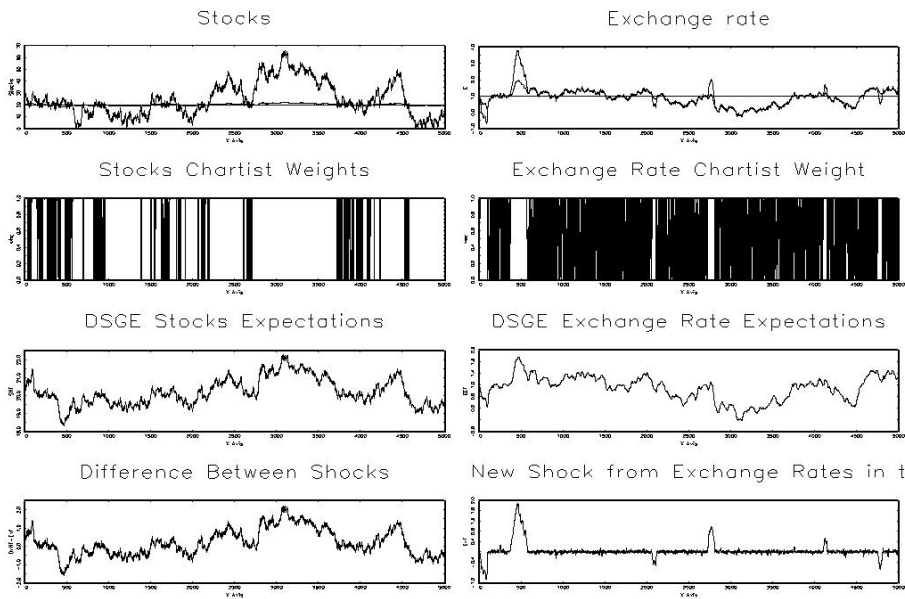


Figure 6: Stochastic simulation from shocks coming from both markets and bubbles of both kinds

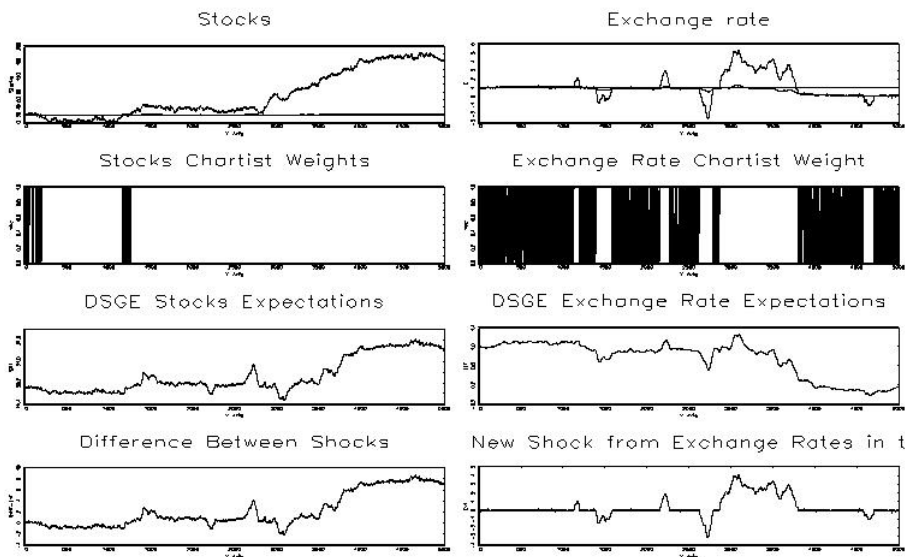
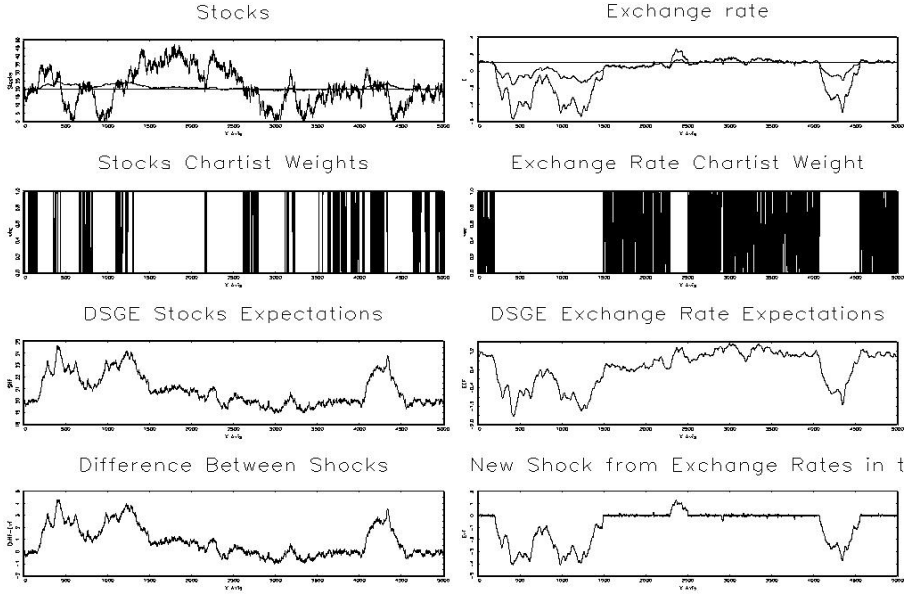


Figure 7: Stochastic simulation from shocks coming from both markets



**Figure 8: Stochastic simulation from shocks coming from both markets with a large pass-through**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DSH	0.095	0.056	1.73	0.08
C	0.001	0.001	1.23	0.22

**Table 2: Coefficients of the regression on the exchange rate from simulation**

These characteristics are what we have described in the simulations above.

#### 5.4 Empirical comparison

This section will show the similarities (and differences) between the simulated model and the real data. First of all we are going to show the graph of a typical realization. Figure 9 has all the properties identified above.

We test all the series for unit roots. The results are uniform across the series. They are both integrated of order 1. Secondly we regress the stocks on the exchange rate series, to verify whether they are significant and consistent with the real data.

Tables 2 and 3 show the results of the regression for both data. As an

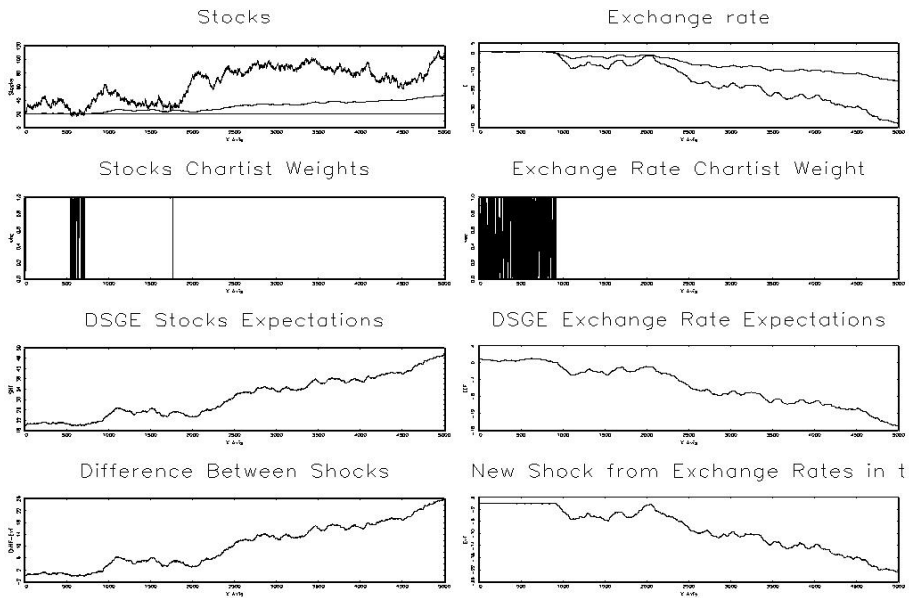


Figure 9: Graph from time series studied

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DFTSE	0.270	0.054	4.957	0.00
C	-0.001	0.001	-0.881	0.38

Table 3: Coefficients of the regression on the dollar-pound exchange rate

example of real data we have chosen one of the most studied markets, namely the dollar pound exchange rate. The stock indexes are the S&P500 for the US and the FTSE100 for the United Kingdom. The period is from 1973 through 2006 on a monthly basis. Figure 8 shows the dynamics of the markets and how they visually look similar in variance and behavior to the simulated data reported above.

We may now compare the two tables. In the real data the returns on stocks are significant in explaining the variance of the exchange rate returns. In our simulated series they are significant at a 10% level. Furthermore the direction of correlation is also similar, since in both case we have a positive relationship between the returns. This means that overall the series are showing a similar dynamics to the one of the real markets. Finally, the residuals of the two regressions reject the normality assumption both in the data and in the model.

We should also compare the markets by looking at the statistical properties of the single series. Since this exercise has already been done in the past, we refer to the papers of De Grauwe and Grimaldi and De Grauwe and Rovira-Kaltwasser for a detailed explanation of the properties. Here we simply want to confirm that we find all the properties shown in this papers such as high kurtosis and heteroskedasticity in both exchange rates and stock prices.

## 6 Conclusion

This paper has described a model of exchange rates and stock prices. While using a financial model to set the dynamics of the two markets, this paper draws a strong connection with the state of the art macroeconomic models. The aim is to analyze the macroeconomic connection between the two financial markets.

With this aim in mind we constructed a model that merges two branches of the economic literature. The first one is the behavioral finance branch, featuring expectations non necessarily rational, and a setting of incomplete information. The second branch is made of the DSGE models. Within this approach we selected a classical textbook example of a small open economy model. This was at the base of the fundamental expectations from our financial model.

This combination has never been tried before. Our paper shows the potential for a new update of traditional behavioral finance models as well as a new point of view for the implementation of DSGE modelling in different branches of finance.

The result is a model that accounts for high and time dependent variance and the phenomena that are typical of the financial markets, while taking into account the macroeconomic pressures coming from a complete economy. We spend some time looking at the new dynamics that is created. We realize that indeed, our financial model has some new dynamics with respect to other papers. In particular, our fundamental expectations are richer and exhibit a well behaved dynamics that arises from past variations on both markets.

Since the same model generates both series we have expectations that are consistent across the financial markets and include past information that is interpreted through a macroeconomic lens. We show that the similarity of the generated series with the real markets is not only by single series, but is also in the relationship between the two financial markets.

During the work on this paper, we have also come across some potential new ideas for the future. We especially like to mention two. On one hand it would be interesting to integrate the stock prices in the DSGE model. We believe that the general results will not change radically from what we find, but this might be nevertheless interesting from a theoretical point of view. On the other hand a gap in the literature should be filled by reversing the logic in this paper. It would be interesting to solve the DSGE model for heterogeneous expectations within this framework.

Until this is done, this paper constitutes the furthest extent to which full DSGE modelling has been combined with behavioral finance. We hope to have made the case for the new potential of more research in this direction.

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