

Diverse Beliefs and Time Variability of Asset Risk Premia

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- ▶ Realized Premium

$$\pi_{t+1} = \frac{p_{t+1} + D_{t+1} - R_t p_t}{p_t}$$

Assume: Much past data available to compute empirical moments.

No one knows true probability distributions.

- ▶ m = probability implied by the empirical distribution
- ▶ m is a unique and stationary probability.
- ▶ All agree on m
- ▶ *The Premium* is the conditional expectations under m

$$E_t^m [\pi_{t+1} | I_t] = \frac{1}{p_t} E_t^m [p_{t+1} + D_{t+1} - R_t p_t | I_t]$$

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The questions we aim to answer are:

What are the factors determining the Risk Premium?

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The questions we aim to answer are:

What are the factors determining the Risk Premium?

Why does it fluctuate over time?

Bond Market: macroeconomic Variables

Fama and Bliss (1987)

Internal dynamics of
past yields

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Federal Reserve policy shocks

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Fama and Bliss (1987)	Internal dynamics of past yields
Campbell and Shiller (1991)	Unexplained shocks to the bond market
Bernanke and Kuttner (2003)	Federal Reserve policy shocks
Cochrane and Piazzesi (2005)	Past yields and Business Cycles

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Piazzesi and Swanson (2004)	Recessions forecasts, i.e. Non-Farm Payroll

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Piazzesi and Swanson (2004)	Recessions forecasts, i.e. Non-Farm Payroll
Kurz and Motolese (2008)	The importance of Market Beliefs

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Stock Market: framed as a problem of forecasting returns.

Using Belief Variables some studies focus on earning forecasts dispersion:

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Stock Market: framed as a problem of forecasting returns.

Using Belief Variables some studies focus on earning forecasts dispersion:

Miller (1977),

Diether et al. (2002),

Lee et al. (2002),

Park (2005),

Baker and Wurgler (2006),

Campbell and Diebold (2007).

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Miller (1977),

Diether et al. (2002),

Lee et al. (2002),

Park (2005),

Baker and Wurgler (2006),

Campbell and Diebold (2007).

- ▶ Virtually no theory: most start from *Noise Trading* conception
- ▶ Hence no real hypotheses to test

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Conclusions

- ▶ Effects of diverse beliefs on the risk premium
- ▶ Restrictions diverse beliefs rationality imposes
- ▶ Measuring market belief and testing hypotheses
- ▶ Conclusions complementary to Kurz and Motolese (2008) on the Bond Market

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The Approach:

1. Outline basic model of belief
2. Formulate an Asset Pricing model
3. Deduce conclusion about risk premium
4. Test empirically using a reference model from literature

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- ▶ Risky asset payoff $\{D_t, t = 1, 2, \dots\}$ with true probability Π .
- ▶ Non stationary with structural breaks. Π is unknown.
- ▶ True process is Stable:
 - Relative frequencies converge
 - Has empirical distribution
- ▶ Substantial past data.
- ▶ All compute empirical probability m : *Common knowledge*.

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- ▶ All compute empirical probability m : *Common knowledge*.

Assume: under m process is Markov with mean μ and transition F

$$d_{t+1} = \lambda_d d_t + \rho_{t+1}^d, \quad \rho_{t+1}^d \sim N(0, \sigma_d^2) \quad \text{where } d_t = D_t - \mu$$

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- ▶ The truth is [I tell you, agents do not know it, and it does not matter]

$$d_{t+1} = \lambda_d d_t + b_t + \xi_{t+1}^d, \quad \xi_{t+1}^d \sim N(0, \sigma_\xi^2)$$

b_t sequence of regimes.

Note $\Pi \neq m$.

Theorem: m is stationary, unique and expressed by F .

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Note $\Pi \neq m$.

Theorem: m is stationary, unique and expressed by F .

- ▶ Q is a Rational Belief if data generated under it reproduces m
- ▶ We assume agents' beliefs Q^i are Markov
- ▶ Disagreement persists:
 - Data shows diverse forecasts
 - Must hold diverse transitions F_t^i

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Rationality implies (as a minimum):

- (A) A Rational agent cannot hold a constant $F^i \neq F$
- (B) F_t^i fluctuate over time with the restriction
- (C) $1/N \sum_{t=1}^N (F_t^i - F) \implies 0$ for all i : Rational Agents are right on average

Rationality \implies Dynamics

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Definition: A **Belief State** g_t^i pins down i 's perceived transition of all state variables. In the case of dividend, it takes the form

$$d_{t+1}^i = \lambda_d d_t + \lambda_d^g g_t^i + \rho_{t+1}^{id}, \quad \rho_{t+1}^{id} \sim N(0, \hat{\sigma}_d^2)$$

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- ▶ g_t^i is observable

$$E_t^i [d_{t+1}^i | I_t, g_t^i] - E_t^m [d_{t+1} | I_t] = \lambda_d^g g_t^i$$

- ▶ Persistent Diversity $\implies g_t^i$ are different across i

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In Sum: if rational, g_t^i must fluctuate and have a zero mean

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In Sum: if rational, g_t^i must fluctuate and have a zero mean

We represent state of belief with:

$$g_{t+1}^i = \lambda z g_t^i + \rho_{t+1}^{ig}, \quad \rho_{t+1}^{ig} \sim N(0, \sigma_g^2) \quad (1)$$

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$$g_{t+1}^i = \lambda z g_t^i + \rho_{t+1}^{ig}, \quad \rho_{t+1}^{ig} \sim N(0, \sigma_g^2) \quad (1)$$

Why:

- (I) It is Compatible with empirical evidence in survey data
- (II) Can give an analytic-Bayesian justification. See Kurz (2006) and will be discussed later in the conference.

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Definition: Market belief is the distribution $(g_t^1, g_t^2, \dots, g_t^N)$ observed by sampling hence with known moments.

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Definition: Market belief is the distribution $(g_t^1, g_t^2, \dots, g_t^N)$ observed by sampling hence with known moments.

▶ Define mean market state of belief by $Z_t = \frac{1}{N} \sum_{i=1}^N g_t^i$

▶ Average (1) $\frac{1}{N} \sum_{i=1}^N g_{t+1}^i = \lambda_Z \frac{1}{N} \sum_{i=1}^N g_t^i + \frac{1}{N} \sum_{i=1}^N \rho_{t+1}^{ig}$

▶ Key condition: ρ_t^{ig} are correlated hence

$$\frac{1}{N} \sum_{i=1}^N \rho_{t+1}^{ig} = \rho_{t+1}^Z \neq 0$$

▶ Z_t is a state variable with empirical distribution

$$Z_{t+1} = \lambda_Z Z_t + \rho_{t+1}^Z, \quad \rho_{t+1}^Z \sim N(0, \sigma_Z^2)$$

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$$\frac{1}{N} \sum_{i=1}^N \rho_{t+1}^{ig} = \rho_{t+1}^Z \neq 0$$

▶ Z_t is a state variable with empirical distribution

$$Z_{t+1} = \lambda_Z Z_t + \rho_{t+1}^Z, \quad \rho_{t+1}^Z \sim N(0, \sigma_Z^2)$$

In all models: this correlation is the crucial factor

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We thus *expand the empirical distribution* to

$$\{(d_{t+1}, Z_{t+1}), t = 1, 2, \dots\}$$

$$\begin{aligned} d_{t+1} &= \lambda_d d_t + \rho_{t+1}^d \\ Z_{t+1} &= \lambda_Z Z_t + \rho_{t+1}^Z \end{aligned} \quad \begin{pmatrix} \rho_{t+1}^d \\ \rho_{t+1}^Z \end{pmatrix} \sim N(0, \tilde{\Sigma}), \text{ i.i.d.}$$

where

$$\tilde{\Sigma} = \begin{bmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_Z^2 \end{bmatrix}.$$

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Individual i 's *perception model* (together with (1)) takes the form:

$$\begin{aligned}d_{t+1}^i &= \lambda_d d_t + \lambda_d^g g_t^i + \rho_{t+1}^{id} \\ Z_{t+1}^i &= \lambda_Z Z_t + \lambda_Z^g g_t^i + \rho_{t+1}^{iZ}\end{aligned} \quad \begin{pmatrix} \rho_{t+1}^{id} \\ \rho_{t+1}^{iZ} \end{pmatrix} \sim N(0, \Sigma^i) \quad (2)$$

where

$$\Sigma^i = \begin{bmatrix} \hat{\sigma}_d^2 & \hat{\sigma}_{Zd} \\ \hat{\sigma}_{Zd} & \hat{\sigma}_Z^2 \end{bmatrix}.$$

Parameter sign $\lambda_d^g \geq 0$ and $\lambda_Z^g \geq 0$ orient the model:

When $g_t^i > 0$, agent i believes $t+1$ dividend and market belief will persist above normal.

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Assumptions:

- ▶ Large number of agents.
- ▶ A single commodity – “consumption”.
- ▶ Riskless technology producing $R > 1$ at $t + 1$ with 1 unit of input at t .
- ▶ A single aggregate risky asset with supply $S=1$.
- ▶ The sequence of dividends $\{D_t, t = 1, 2, \dots\}$ with *unknown* distribution Π .
- ▶ Under m $\{D_t, t = 1, 2, \dots\}$ is Markov with transition

$$d_{t+1} = \lambda_d d_t + \rho_{t+1}^d, \quad \rho_{t+1}^d \sim N(0, \sigma_d^2)$$

where $d_t = D_t - \mu$

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Some Notation:

- ▶ θ_t^i = date t stock purchases of agent i .
- ▶ B_t^i = amount invested in the riskless asset.
- ▶ p_t = price of the stock. Think of it as the S&P500.

Agent i optimization problem is:

$$\max_{(\theta^i, B^i)} E_t^i \left[\sum_{k=t}^{\infty} -\beta^{k-t} e^{-\left(\frac{c_k^i}{\tau}\right)} \middle| I_t \right]$$

Subject to:

$$c_t^i = \theta_{t-1}^i (p_t + d_t + \mu) + B_{t-1}^i R - \theta_t^i p_t - B_t^i$$

Initial values (θ_0^i, B_0^i)

i 's belief as specified in (2)

Advantage: only mean market belief matters.

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The Exponential utility model is common in studies of asset pricing. See for example

Singleton (1987)

Brown and Jennings (1989)

Grundy and McNichols (1989)

Wang (1994)

He and Wang (1995)

Duffie (2002)

Dai and Singleton (2002)

Allen, Morris and Shin (2005) and many others

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Conclusions

- ▶ Assume for a moment:
agents believe $p_{t+1} + d_{t+1}$ is conditionally normal.
- ▶ Define:
Excess return per share $V_{t+1} = p_{t+1} + (d_{t+1} + \mu) - Rp_t$.
- ▶ Define the state variables in the optimization:

$$\psi_t^i = (1, d_t, Z_t, g_t^i)$$

$$u = (u_0, u_1, u_2, u_3)$$

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- ▶ Define the state variables in the optimization:

$$\psi_t^i = (1, d_t, Z_t, g_t^i)$$

$$u = (u_0, u_1, u_2, u_3)$$

We show that the demand function of agent i is:

$$\theta_t^i(p_t, \psi_t^i) = \frac{R\tau}{r\hat{\sigma}_V^2} [E_t^i(V_{t+1} | I_t, g_t^i) + u\psi_t^i]$$

where

$\hat{\sigma}_V^2 = \text{adjusted variance of } V_t \text{ assumed constant for all } i$.

Stability Conditions:

$$R = 1 + r > 1, \quad 0 < \lambda_d < 1, \quad \lambda_Z < 1, \quad 0 < \lambda_Z + \lambda_Z^g < 1.$$

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Theorem: There is a unique equilibrium price function which takes the form $p_t = a_d d_t + a_z Z_t + P_0$ with parameters

$$a_d = \frac{\lambda_d + u_1}{R - \lambda_d} > 0 \quad a_z = \frac{(a_d + 1) \lambda_d^g + (u_2 + u_3)}{R - (\lambda_z + \lambda_z^g)} > 0$$

$$P_0 = \frac{(\mu + u_0)}{r} - \frac{\hat{\sigma}_V^2}{R\tau}.$$

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- ▶ The Theorem above confirms equilibrium price p_t is conditionally normal
- ▶ Price exhibits “excess” volatility due to beliefs

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- ▶ The Theorem above confirms equilibrium price p_t is conditionally normal
- ▶ Price exhibits “excess” volatility due to beliefs
- ▶ Diverse perceived premia:

$$E_t^i [\pi_{t+1} | I_t] = \frac{1}{\rho_t} E_t^i [p_{t+1} + d_{t+1} + \mu - R p_t | I_t]$$

- ▶ Compute the Premium:

$$E_t^m [\pi_{t+1} | I_t] = \frac{1}{\rho_t} E_t^m [p_{t+1} + D_{t+1} - R p_t | I_t]$$

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We compute:

$$E_t^m [\pi_{t+1} | I_t] = \frac{1}{\rho_t} E_t^m \left[\left(\frac{r \hat{\sigma}_V^2}{R_T} - u_1 d_t - u_0 \right) - a_z (R - \lambda_z) Z_t \right]$$

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Main Theorem: Since $a_z (R - \lambda_z) > 0$, the Risk Premium increases with $\hat{\sigma}_V^2$ and declines with market belief Z_t

Also, $\hat{\sigma}_V^2 \approx (a_d + 1)^2 \hat{\sigma}_d^2 + a_z^2 \hat{\sigma}_Z^2$

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(I) Mean Premium increases with variance of d and Z : belief state volatility increases mean premium

(II) Time Changes in Market Belief reflected in

$$-a_z (R - \lambda_z) > 0.$$

The risk premium on a long position

- ▶ lower when market belief about the future is favorable
- ▶ higher when market belief about the future is unfavorable

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What It Does Not Say:

- ▶ Agents are on their demand functions
- ▶ Not “optimal” to “choose” to be a contrarian; may be short when wants to be long and long when wants to be short
- ▶ Analogous to why we do not adopt a log utility
- ▶ Note: you don't change forecast of d_{t+1} when you find $Z_t < 0$!

What It Does Say:

- ▶ A formal theorem about market overshooting: today's price adjusts to Z_t more than expected tomorrow's price
- ▶ Rational investing agents must form an opinion about future beliefs of “others” - the market.
- ▶ Diverse beliefs are central to this results (topic of conference)

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Definition: An equilibrium exhibits “Endogenous Uncertainty” if its price map depends upon market belief.

- ▶ With Non-Exponential Utility: entire distribution (g_t^1, \dots, g_t^N) matters but Main Result holds if slope of Z_t satisfies $a_Z > 0$.

- ▶ Mean market belief $Z_t = \frac{1}{N} \sum_{i=1}^N g_t^i$.

$$\text{Diversity } \sigma_t^Z = \sqrt{\frac{1}{N} \sum_{i=1}^N (g_t^i - Z_t)^2}$$

- ▶ Empirical work considers first two moments of (Z_t, σ_t^Z) distribution
- ▶ Hypothesis: effect of cross sectional diversity σ_t^Z on risk premium is negative (the “Narrow Door Hypothesis”)

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Excess Stock Returns

$R_{t,t+6}$

six-month return on the CRSP value weighted index net of the return on a 90 day T-bill

$R_{t,t+12}$

1 year return on the CRSP value weighted index net of the return on a 90 day T-bill

Livingston Six-Month Growth Forecasts

$F_{t+6,t+12}^g$

Mean forecast of six-month real GDP growth rate from 6 months to 12 month after t computed from individual Livingston survey responses about the level of nominal GDP and the CPI 6 and 12 months after date t

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Financial Predictors

DP_t dividend yield on the CRSP value-weighted portfolio

DEF_t the yield spread between a broad corporate bond portfolio and the AAA yield spread

$TERM_t$ the yield spread between a 10 year U.S. Treasury bond and a one-month Treasury bill

Macroeconomic Predictors

CAY_t Lettau and Ludvigson (2001) log consumption-wealth ratio

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Beliefs Variables

$Z_{t+6,t+12}^g$

Belief Index of Real GDP Growth Rate between the end of period t+6 and t+12

σZ^g

Cross-sectional standard deviation of individual Livingston forecasts of 6-month growth rate in real GDP between the end of period t+6 and t+12

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	μ	σ	$R_{t,t+6}$	$R_{t,t+12}$
$R_{t,t+6}$	6.646	23.376	1.000	
$R_{t,t+12}$	5.098	16.668	0.692	1.000

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Descriptive Statistics of Predictors (1971:S1-2007:S2)

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	μ	σ	$F_{t+1,t+2}^g$	DP_t	DEF_t	$TERM_t$	CAY_t
$F_{t+6,t+12}^g$	3.090	1.125	1.000				
DP_t	3.104	1.258	0.143	1.000			
DEF_t	1.028	0.376	0.451	0.609	1.000		
$TERM_t$	1.011	1.182	0.297	-0.236	0.056	1.000	
CAY_t	0.000	0.016	-0.327	0.119	-0.172	0.158	1.000

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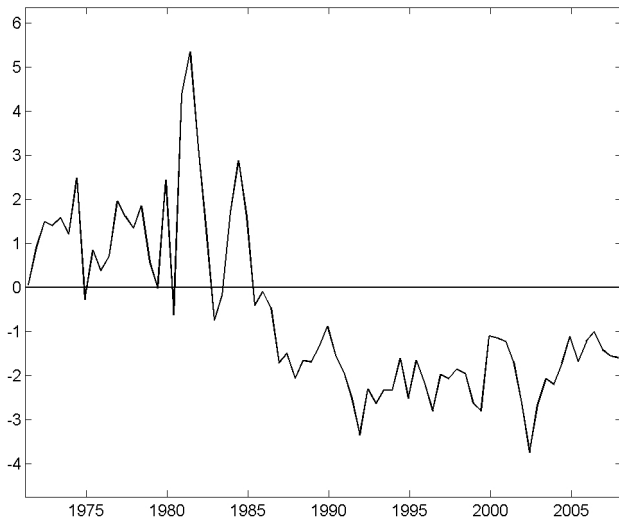
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	μ	σ	$Z_{t+6,t+12}^g$	σZ^g
$Z_{t+6,t+12}^g$	-0.603	1.890	1.000	
σZ^g	2.319	1.012	0.576	1.000



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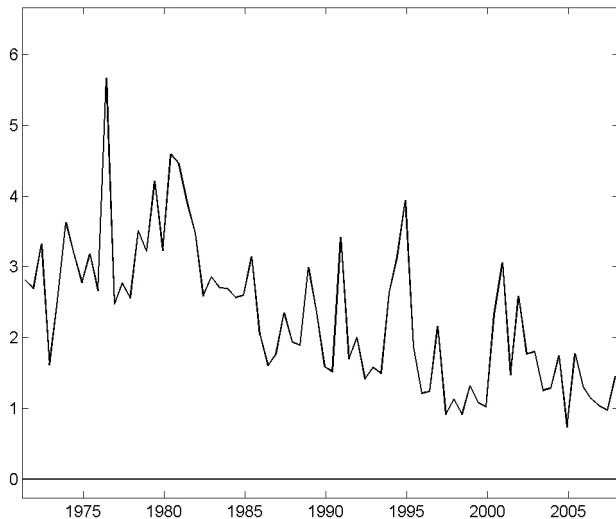
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The Reference Model (1971:S1-2007:S2)

	$R_{t,t+6}$		$R_{t,t+12}$	
	(A)	(B)	(A)	(B)
$F_{t+6,t+12}^g$	–	-0.254 (0.032)	–	-0.170 (0.270)
DP_t	0.086 (0.563)	0.100 (0.474)	0.197 (0.244)	0.206 (0.185)
DEF_t	0.138 (0.417)	0.225 (0.149)	0.000 (1.000)	0.058 (0.719)
$TERM_t$	0.084 (0.489)	0.171 (0.145)	0.172 (0.246)	0.230 (0.126)
CAY_t	0.244 (0.026)	0.161 (0.188)	0.264 (0.074)	0.209 (0.240)
R^2	0.067	0.097	0.112	0.119
S.E.	0.966	0.951	0.942	0.939

P-values in parenthesis. All R^2 are adjusted

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Test of Theorem for 6-month horizon (1971:S1-2007:S2)

	$R_{t,t+6}$			
	(A)	(B)	(1)	(2)
$F_{t+6,t+12}^g$	–	-0.254 (0.032)	–	–
$Z_{t+6,t+12}^g$	–	–	-0.587 (0.000)	-0.555 (0.000)
σ_{Z^g}	–	–	–	-0.082 (0.506)
DP_t	0.086 (0.563)	0.100 (0.474)	0.469 (0.007)	0.499 (0.009)
DEF_t	0.138 (0.417)	0.225 (0.149)	0.143 (0.213)	0.142 (0.203)
$TERM_t$	0.084 (0.489)	0.171 (0.145)	-0.078 (0.397)	-0.085 (0.378)
CAY_t	0.244 (0.026)	0.161 (0.188)	0.000 (0.998)	0.010 (0.926)
R^2	0.067	0.097	0.215	0.208
S.E.	0.966	0.951	0.886	0.890

P-values in parenthesis. All R^2 are adjusted

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Test of Theorem for 12-month horizon (1971:S1-2007:S2)

	$R_{t,t+12}$			
	(A)	(B)	(1)	(2)
$F_{t+6,t+12}^g$	–	-0.170 (0.270)	–	–
$Z_{t+6,t+12}^g$	–	–	-0.422 (0.006)	-0.363 (0.016)
σ_{Z^g}	–	–	–	-0.150 (0.279)
DP_t	0.197 (0.244)	0.206 (0.185)	0.445 (0.006)	0.498 (0.004)
DEF_t	0.000 (1.000)	0.058 (0.719)	0.016 (0.902)	0.016 (0.903)
$TERM_t$	0.172 (0.246)	0.230 (0.126)	0.053 (0.642)	0.041 (0.721)
CAY_t	0.264 (0.074)	0.209 (0.240)	0.101 (0.517)	0.119 (0.446)
R^2	0.112	0.119	0.176	0.177
S.E.	0.942	0.939	0.908	0.907

P-values in parenthesis. All R^2 are adjusted

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- ▶ Data supports theoretical conclusion
- ▶ Standard variables DEF_t , $TERM_t$ and CAY_t predominantly reflect market beliefs not exogenous conditions
- ▶ More empirical proof that markets overshoot

- ▶ More evidence the risk of future market belief (i.e. belief of “others”) is a dominant market risk

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THANKS

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