

# Is the Volatility of the Market Price of Risk due to Intermittent Portfolio Re-balancing?

Preliminary and Incomplete

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## **Abstract**

Our paper examines the impact of heterogeneous trading technologies for households on the volatility of asset prices and in particular on the volatility of the pricing of risk. TO BE CONTINUED ....

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# 1 Introduction

- There is an emerging empirical consensus that the conditional risk premia are counter cyclical and highly volatile. The extent of this volatility is seen as a challenge for conventional models of asset prices since this cyclical, as measured by the consumption Sharpe ratio, is negatively correlated with the conditional Sharpe ratio for the stockmarket. cite Sidney. We explore two potential sources of this volatility.
- The first potential source of this time variation is suggested by the empirical literature on household finance which documents both the apparent suboptimality and passivity of many household's portfolio choices. For example, a majority of households does not invest directly in equity, in spite of the sizeable historical equity premium. And, even among those who participate in equity markets, sophisticated investors invest a larger share of their wealth in equity and realize higher returns, while less sophisticated investors take a more cautious and passive approach. This passivity shows up in their apparently infrequent adjustments to their overall portfolio composition despite shocks that lead to it changing substantially over time. Zeldes he examines the frequency of portfolio flow shifts in TIAA-CREF retirement accounts over a ten year period. 47% of the members of his panel made no change during the period, and 21% made only one change. (See page 31).
- The second potential source suggested by recent work on liquidity cycles, cite Geanakoplos. We examine this second source by including aggressive traders who find it optimal in equilibrium to stay on or close to their borrowing constraints. These traders behave this way because they are able to trade away their idiosyncratic risk and hence do not have a precautionary motive to save.
- In this paper we examine the ability of these two potential sources of increased volatility to account for the high degree of conditional volatility in the market price of risk.

# 2 Model

This is an endowment economy in which households sequentially trade assets and consume. For simplicity, we assume that all households are ex ante identical, except for the restrictions which their face on their asset trading. These limitations differ across households and are imposed by assumption. We refer to the set of restrictions that a household faces as a household trading technology. The goal of these restrictions is to capture the observed limitations in some households portfolio by limiting the set of asset holding technologies available to them. We will refer to

households as being *passive traders*, if they take their portfolio composition as given and simply choose how much to save or dissave in each period. We will also allow for households that are behaving optimally with respect to their portfolio choices. We will refer to these traders as *active traders* since they actively manage the composition of their portfolio each period. We extend the basic model of Chien, Cole and Lustig (2008) (hereafter CCL) to allow for passive trader who only intermittently adjusts their portfolio. In this section we describe the environment, and we describe the household problem for each of different asset trading technologies. We also define an equilibrium for this economy.

## 2.1 Environment

This is an endowment economy with a unit measure of households who are subject to both aggregate and idiosyncratic income shocks. Households are ex ante identical, except for the trading technology they are endowed with. Ex post, these households differ in terms of their idiosyncratic income shock realizations. All of the households face the same stochastic process for idiosyncratic income shocks, and all households start with the same present value of tradeable wealth.

In the model time is discrete, infinite, and indexed by  $t = 0, 1, 2, \dots$ . The first period,  $t = 0$ , is a planning period in which financial contracting takes place. We use  $z_t \in Z$  to denote the aggregate shock in period  $t$  and  $\eta_t \in N$  to denote the idiosyncratic shock in period  $t$ .  $z^t$  denotes the history of aggregate shocks, and, similarly,  $\eta^t$ , denotes the history of idiosyncratic shocks for a household. The idiosyncratic events  $\eta$  are i.i.d. across households. We use  $\pi(z^t, \eta^t)$  to denote the unconditional probability of state  $(z^t, \eta^t)$  being realized. The events are first-order Markov, and we assume that

$$\pi(z^{t+1}, \eta^{t+1} | z^t, \eta^t) = \pi(z_{t+1} | z_t) \pi(\eta_{t+1} | z_{t+1}, \eta_t).$$

Since we can appeal to a law of large number,  $\pi(z^t, \eta^t) / \pi(z^t)$  also denotes the fraction of agents in state  $z^t$  that have drawn a history  $\eta^t$ . We use  $\pi(\eta^t | z^t)$  to denote that fraction. We introduce some additional notation:  $z^{t+1} \succ z^t$  or  $y^{t+1} \succ y^t$  means that the left hand side node is a successor node to the right hand side node. We denote by  $\{z^\tau \succ z^t\}$  the set of successor aggregate histories for  $z^t$  including those many periods in the future; ditto for  $\{\eta^\tau \succ \eta^t\}$ . When we use  $\succeq$ , we include the current nodes  $z^t$  or  $\eta^t$  in the summation.

There is a single final good in each period, and the amount of it is given by  $Y(z^t)$ , which evolves according to

$$Y(z^t) = \exp\{z_t\} Y(z^{t-1}), \tag{2.1}$$

with  $Y(z^1) = \exp\{z_1\}$ . This endowment good comes in two forms. The first form is non-

diversifiable income which is subject to idiosyncratic risk and is given by  $\gamma Y(z^t)\eta_t$ ; hence  $\gamma$  is the share of income that is non-diversifiable. The second form is diversifiable income, which is not subject to the idiosyncratic shock, and is given by  $(1 - \gamma)Y(z^t)$ .

All households are infinitely lived and rank stochastic consumption streams  $\{c(z^t, \eta^t)\}$  according to the following criterion

$$U(c) = \sum_{t \geq 1}^{\infty} \beta^t \pi(z^t, \eta^t) \frac{c(z^t, \eta^t)^{1-\alpha}}{1-\alpha}, \quad (2.2)$$

where  $\alpha > 0$  denotes the coefficient of relative risk aversion, and  $c(z^t, \eta^t)$  denotes the household's consumption in state  $(z^t, \eta^t)$ .

## 2.2 Asset Trading Technologies

Households trade assets in spot securities markets that re-open in every period. Agents are endowed with an asset trading technology which can be thought of as rule specifying which assets the agent can hold and which actions he can take to effect their composition. We assume households cannot switch between technologies. These assets are claims on diversifiable income, and the set of potentially tradeable assets will include one-period Arrow securities as well as debt and equity claims. We follow (?) in defining equity as a leveraged claim to diversifiable income. Households cannot directly trade their claim to non-diversifiable risk. Households trade assets in securities markets and they trade the final good in spot markets that re-open in every period.

To construct the debt and the equity claim, we will assume that diversifiable income is divided into payments to equity and debt (which for simplicity we take to be a sequence of one-period bonds). Since we will assume a constant leverage ration  $\phi$  and denote the price of a claim to diversifiable income by  $\varpi(z^t)$ , then the stream of one period bonds must satisfy

$$b(z^t) = \phi [\varpi(z^t) - b(z^t)],$$

and the bond payouts are given by  $R_t^f(z^{t-1})b(z^{t-1}) - b(z^t)$ . The dividend payouts,  $d_t(z^t)$ , are then determined residually

$$d_t(z^t) = (1 - \gamma)Y_t - R_t^f(z^{t-1})b(z^{t-1}) + b(z^t).$$

We denote by  $R^e$  the return on equities. A trader who invests a fraction  $\phi/(1 + \phi)$  in equities and the rest in debt is holding the market portfolio.

We will denote the price of a unit claim to the final good in aggregate state  $z^{t+1}$  acquired

in aggregate state  $z^t$  by  $q(z^{t+1}, z^t)$ . If there is a group of agents who can trade upon their idiosyncratic shocks, then the absence of arbitrage would imply that the price of a claim to output in state  $(z^{t+1}, \eta^{t+1})$  acquired in state  $(z^{t+1}, \eta^{t+1})$  would be equal to  $\pi(\eta^{t+1}|z^{t+1}, \eta^t)q(z_{t+1}, z^t)$ . Even for agent's who cannot trade upon their idiosyncratic shocks, we can take this as the market's pricing convention.

From the aggregate contingent claim prices, we can back out the present-value state prices recursively as follows:

$$\pi(z^t, \eta^t)P(z^t, \eta^t) = q(z_t, z^{t-1})q(z_{t-1}, z^{t-2}) \cdots q(z_1, z^0)q(z_0).$$

We use  $\tilde{P}(z^t, \eta^t)$  to denote the Arrow-Debreu prices  $P(z^t)\pi(z^t, \eta^t)$ . Let  $m(z^{t+1}|z^t) = P(z^{t+1})/P(z^t)$  denote the stochastic discount factor that prices any random payoffs.

The general form of the flow budget constraint for our households is

$$\begin{aligned} & \gamma Y(z^t)\eta_t + \hat{a}(z^t, \eta^t) \\ = & \sum_{\substack{z^{t+1} \succ z^t \\ \eta^{t+1} \succ \eta^t}} q(z_{t+1}, z^t)\pi(\eta_{t+1}|z_{t+1}, \eta_t)a_t([z^{t+1}, \eta^{t+1}], [z^t, \eta^t]) \\ & + e(z^t, \eta^t) \forall(z^t, \eta^t) + b(z^t) + c(z^t), \text{ for all } z^t, \eta^t, \end{aligned} \quad (2.3)$$

where  $a_t([z^{t+1}, \eta^{t+1}], [z^t, \eta^t])$  denotes the number of one period ahead Arrow securities acquired in state  $[z^t, \eta^t]$  that pay one unit in state  $[z^{t+1}, \eta^{t+1}]$ ,  $e(z^t, \eta^t)$  and  $b(z^t)$  are the value of the equity and debt claims acquired in state  $[z^t, \eta^t]$ . Finally,  $\hat{a}(z^t, \eta^t)$  denotes the agent's net financial wealth in state  $(z^t, \eta^t)$ , and is given by

$$\hat{a}(z^t, \eta^t) = a_{t-1}([z^t, \eta^t], [z^{t-1}, \eta^{t-1}]) + e(z^{t-1}, \eta^{t-1}) [d(z^t) + \varpi^e(z^t)] + R_t^f(z^{t-1})b(z^{t-1}). \quad (2.4)$$

If the agent can only hold aggregate-contingent securities then, in a slight abuse of notation we express his aggregate state contingent bond holds as  $a_t(z^{t+1}, [z^t, \eta^t])$ , and express the cost of his purchase of these bonds as  $\sum_{z^{t+1} \succ z^t} q(z_{t+1}, z^t)a_t([z^{t+1}, \eta^{t+1}], [z^t, \eta^t])$ .

A trading technology is a restriction on the allowable that the agent can make as to the assets in his flow budget constraint. The set of asset trading technologies that we consider can be divided into two main classes: active trading technologies and passive trading technologies. Agents with an active trading technology optimally choose their portfolio composition given the set of assets that they are allowed to trade. For *complete traders* this set consists of Arrow securities, aggregate state-contingent securities, debt and equity. For *aggregate-complete traders* this set consists of aggregate state-contingent securities, debt and equity. Finally, for all passive

trading technologies, this set consists of only debt and equity claims.

The composition of a passive trader's portfolio is determined by the portfolio rule in his technology in conjunction with his savings behavior. We will assume that the composition of a passive trader's portfolio is determined by a rule which specifies a bond-stock ratio target and that their portfolio's are rebalanced at fixed intervals of time. For example, a passive trader's rule might specify that he holds the fraction  $x$  of equity every period. This trader is a *constant-rebalancer* in that he adjusts his holdings of bonds and stocks in every period in response to the realized returns on debt and equity and his savings choice to maintain this portfolio composition. An passive trader who only adjusts his equity holdings at fixed frequencies to the target value  $x$  is called an *intermittent-rebalancer* since the composition of his portfolio in the in the periods in which he does not rebalance will adjust depending upon the returns to debt and equity and his savings choice, which runs up or down his bond holdings in these periods. One special type of passive trader is the *nonparticipant*, who holds no equity and therefore sets  $x = 0$ .<sup>1</sup>

We allow for the possibility that there could be multiple types of both active and passive traders. All households are initially endowed with a claim to their per capita share of both diversifiable and non-diversifiable income. Since in the quantitative analysis we only look at the equilibrium of the model once the economy has "settled down", the assumption about initial wealth does matter. During the initial trading period, we assume that households with portfolio restriction sell their claim to diversifiable income in exchange for their type appropriate fixed weighted portfolio of bonds and equities.

The households face exogenous limits on their net asset positions, or solvency constraints,

$$\hat{a}(z^t, \eta^t) \geq \underline{M}_t(z^t, \eta^t). \quad (2.5)$$

In determining the solvency constraint, we assume that the value of the household's net assets must always be greater than  $-\psi$  times the value of their non-diversifiable income, where  $\psi \in (0, 1)$ . We allow households to trade away or borrow up to 100% of the value of their claims to diversifiable capital. We also allow for the possibility that this borrowing constraint may itself be a function of the aggregate history of shocks. Given this,

$$\underline{M}_t(z^t, \eta^t) = -\psi(z^t) \sum_{\tau \geq t} \sum_{\{z^\tau \succeq z^t, \eta^\tau \succeq \eta^t\}} \gamma Y(z^\tau) \eta_\tau \frac{\pi(z^\tau, \eta^\tau) P(z^\tau, \eta^\tau)}{\pi(z^t, \eta^t) P(z^t, \eta^t)}$$

Finally, each agent's period 1 financial wealth is constrained by the value of their claim to

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<sup>1</sup>It is straightforward to extend the methodology we develop to allow for exogenous transitions between trading technologies. The probability of these transitions could even be contingent on the household's realized shocks. We could also allow the equity share  $x$  to evolve deterministically in response to the history of the agent's shocks.

tradeable wealth in the period 0 planning period, which is given by

$$\varpi(z^0) \geq \sum_{z_1} q(z_1, z^0) \hat{a}_0(z^1, \eta^0), \quad (2.6)$$

where both  $z^0$  and  $\eta^0$  simply indicate the degenerate starting values for the stochastic income process.

### 2.3 Measurability Restrictions

To capture the portfolio restrictions, we use measurability constraints. For example, if we denote the net financial wealth position of a trader by  $\hat{a}(z^t, \eta^t)$  - where  $z^t$  is the history of aggregate shocks and  $\eta^t$  is his history of idiosyncratic shocks - then for an active trader who can only trade securities contingent on the aggregate state his net wealth position is

$$\begin{aligned} \hat{a}(z^t, \eta^t) \equiv & a(z^t | z^{t-1}, \eta^{t-1}) + e(z^{t-1}, \eta^{t-1}) [d(z^t) + \varpi(z^t)] \\ & + R_t^f(z^{t-1}) b(z^{t-1}), \end{aligned}$$

where  $a(z^t | z^{t-1}, \eta^{t-1})$  are the number of pure discount real bonds purchased in the prior state  $(z^{t-1}, \eta^{t-1})$  which payoff today in aggregate state  $z_{t+1}$ ,  $e(z^{t-1}, \eta^{t-1})$  is the agent's holdings of equity claims and  $b(z^{t-1})$  of debt claims. Since idiosyncratic shocks are not spanned for this agents, this implies that his net wealth satisfies:

$$\hat{a}_t(z^t, [\eta_t, \eta^{t-1}]) = \hat{a}_t(z^t, [\tilde{\eta}_t, \eta^{t-1}]), \quad (2.7)$$

for all  $t$  and  $\eta_t, \tilde{\eta}_t \in N$ .

A passive trader who constantly rebalances his portfolio to a fixed fraction  $x$  of equity claims and  $1 - x$  of debt claims will have financial wealth given by

$$\hat{a}(z^t, \eta^t) \equiv R^p(z^t, x) \sigma(z^{t-1}, \eta^{t-1}),$$

where  $\sigma(z^{t-1}, \eta^{t-1})$  his overall investment level and his return is given by

$$R^p(z^t, x) = x \frac{[d(z^t) + \varpi(z^t)]}{\varpi(z^{t-1})} + (1 - x) R_t^f(z^{t-1}).$$

Hence his net wealth position will satisfy the measurability constraint

$$\frac{\hat{a}_t(z_t, \eta_t)}{R^p([z^{t-1}, z_t], x)} = \frac{\hat{a}_t(\tilde{z}_t, \tilde{\eta}_t)}{R^p([z^{t-1}, \tilde{z}_t], x)}, \quad (2.8)$$

for all  $t$ ,  $z_t, \tilde{z}_t \in Z$ , and  $\eta_t, \tilde{\eta}_t \in N$ . If  $x = \phi/(1 + \phi)$ , then this trader holds the market in each period and earns the return on claims to tradeable income, or  $[(1 - \gamma)Y(z^t) + \varpi(z^t)]/\varpi(z^{t-1})$ . We will refer to this type of passive trader as a *diversified trader*.

Next we characterize the constraints on a passive trader's type is specified by his portfolio target  $x$  and the periods in which he rebalances  $T$ . We will assume that his rebalancing takes place at fixed intervals. For example if he rebalances every other period, then  $T = \{1, 3, 5, \dots\}$  or  $T = \{2, 4, 6, \dots\}$ . Since this agent cannot hold any type of state-contingent bond, his flow budget constraint reduces to

$$\begin{aligned} & \gamma Y(z^t) \eta_t + e(z^{t-1}, \eta^{t-1}) R^e(z^t) + b(z^{t-1}, \eta^{t-1}) R^f(z^{t-1}) \\ & \geq c(z^t, \eta^t) + e(z^t, \eta^t) + b(z^t, \eta^t) \quad \forall (z^t, \eta^t). \end{aligned}$$

In rebalancing periods, the trader faces the portfolio constraint that

$$\frac{b(z^t, \eta^t)}{e(z^t, \eta^t)} = x, \quad (2.9)$$

for each  $t \in T$ . In nonrebalancing periods, he faces the constraint that

$$e(z^t, \eta^t) = e(z^{t-1}, \eta^{t-1}) R^e(z^t) \quad (2.10)$$

for each  $t \notin T$ . Since setting  $T = \{1, 2, 3, \dots\}$  generates the constant-rebalancer's measurability constraint, he can simply be thought of as a degenerate case of the intermittent-rebalancer.

## 2.4 Trader's Problems

We find it useful to write agent's problems in terms of their equivalent time-zero trading problem in which they select the optimal policy sequence given a complete set of Arrow-Debreu securities, subject to a sequence of measurability and debt constraints. (See Chien, Cole and Lustig 2008 for more on this equivalence.) This section reformulates the household's problem in terms of a present-value budget constraint, and sequences of measurability constraints and solvency constraints. These measurability constraints capture the restrictions imposed by the different trading technologies of households. We will show how to use the cumulative multipliers on these

constraints to fully characterize equilibrium allocations and prices.

**Active Traders:** For our active trader types, there are two types. The complete trader's problem is to choose  $\{c(z^t, \eta^t), a(z^{t+1}, \eta^{t+1}), e(z^t, \eta^t), b(z^t, \eta^t)\}$ , so as to maximize (2.2) subject the flow budget constraint (2.3), and the solvency constraint (2.5). The aggregate-complete trader's problem is the same as the complete-trader's problem except that we now include his measurability constraint (2.7). Since the complete-trader's problem is simply a simplification of the aggregate-complete, we focus on the aggregate-complete trader in our discussion.

Let  $\chi$  denote the multiplier on the present-value budget constraint, let  $\nu(z^t, \eta^t)$  denote the multiplier on the measurability constraint in node  $(z^t, \eta^t)$ , and, finally, let  $\varphi(z^t, \eta^t)$  denote the multiplier on the debt constraint. The saddle point problem of an aggregate-complete trader can be stated as:

$$\begin{aligned}
L = & \min_{\{\chi, \nu, \varphi\}} \max_{\{c, \hat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t, \eta^t)} u(c(z^t, \eta^t)) \pi(z^t, \eta^t) \\
& + \chi \left\{ \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \tilde{P}(z^t, \eta^t) [\gamma Y(z^t) \eta_t - c(z^t, \eta^t)] + \varpi(z^0) \right\} \\
& + \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \nu(z^t, \eta^t) \left\{ \sum_{\tau \geq t} \sum_{(z^\tau, \eta^\tau) \succeq (z^t, \eta^t)} \tilde{P}(z^\tau, \eta^\tau) [\gamma Y(z^\tau) \eta_\tau - c(z^\tau, \eta^\tau)] + \tilde{P}(z^t, \eta^t) \hat{a}_{t-1}(z^t, \eta^{t-1}) \right\} \\
& + \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \varphi(z^t, \eta^t) \left\{ -\underline{M}_t(z^t, \eta^t) \tilde{P}(z^t, \eta^t) - \sum_{\tau \geq t} \sum_{(z^\tau, \eta^\tau) \succeq (z^t, \eta^t)} \tilde{P}(z^\tau, \eta^\tau) [\gamma Y(z^\tau) \eta_\tau - c(z^\tau, \eta^\tau)] \right\},
\end{aligned}$$

where  $\tilde{P}(z^t, \eta^t) = \pi(z^t, \eta^t) P(z^t, \eta^t)$ . This is a standard convex programming problem –the constraint set is still convex, even with the measurability conditions and the solvency constraints. The first order conditions are necessary and sufficient. The complete-trader's problem is simply this problem where with net financial wealth allowed to depend on the full idiosyncratic history, or  $\hat{a}_{t-1}(z^t, \eta^t)$ , and hence this measurability constraint is degenerate.

The first-order condition for consumption is given by

$$\beta^t u'(c(z^t, \eta^t)) \pi(z^t, \eta^t) = \chi + \sum_{(z^\tau, \eta^\tau) \succeq (z^t, \eta^t)} [\nu(z^\tau, \eta^\tau) - \varphi(z^\tau, \eta^\tau)] \tilde{P}(z^t, \eta^t),$$

Following (?), we can construct new weights for this Lagrangian as follows. First, we define the initial cumulative multiplier to be equal to the multiplier on the budget constraint:  $\zeta_0 = \chi$ .

Second, the multiplier evolves over time as follows for all  $t \geq 1$ :

$$\zeta(z^t, \eta^t) = \zeta(z^{t-1}, \eta^{t-1}) + \nu(z^t, \eta^t) - \varphi(z^t, \eta^t). \quad (2.11)$$

If we restate the first order condition for consumption in terms of our cumulative multiplier, we get that

$$\frac{\beta^t u'(c(z^t, \eta^t))}{P(z^t)} = \zeta(z^t, \eta^t). \quad (2.12)$$

This condition is common to all of our traders irrespective of their trading technology because differences in their trading technology does not effect the way in which  $c(z^t, \eta^t)$  enters the objective function or the constraint. This implies that the marginal utility of households is proportional to their cumulative multiplier, regardless of their trading technology.

The first order condition with respect to net wealth  $\widehat{a}_t(z^{t+1}, \eta^t)$  is given by:

$$\sum_{\eta^{t+1} \succ \eta^t} \nu(z^{t+1}, \eta^{t+1}) \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0. \quad (2.13)$$

We refer to this as the martingale condition. This condition is specific to the trading technology. For the aggregate-complete trader, it implies that the average measurability multiplier across idiosyncratic states  $\eta^{t+1}$  is zero since  $P(z^{t+1})$  is independent of  $\eta^{t+1}$ . In each aggregate node  $z^{t+1}$ , the household's marginal utility innovations not driven by the solvency constraints  $\nu_{t+1}$  have to be white noise. The trader has high marginal utility growth in low  $\eta$  states and low marginal utility growth in high  $\eta$  states, but these innovations to marginal utility growth average out to zero in each node  $(z^t, z_{t+1})$ . If the solvency constraints do bind, then the cumulative multipliers decrease on average for any given aggregate-complete trader:

$$E\{\zeta(z^{t+1}, \eta^{t+1}) | z^{t+1}\} \leq \zeta(z^t, \eta^t),$$

which we obtained by substituting (2.11) into the first-order condition (2.13).

For the complete trader, the first-order condition for to net wealth  $\widehat{a}_t(z^{t+1}, \eta^{t+1})$  is given by:

$$\nu(z^{t+1}, \eta^{t+1}) \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0, \quad (2.14)$$

and this implies that if the solvency constraints do not bind,

$$\zeta(z^{t+1}, \eta^{t+1}) | z^{t+1} = \zeta(z^t, \eta^t).$$

**Passive Traders:** The passive traders' equity position can be expressed as

$$e(z^{t-1}, \eta^{t-1}) = \begin{cases} \frac{1}{\phi^*} b(z^{t-1}, \eta^{t-1}) & \text{if } t-1 \in T \\ e(z^{t-1}, \eta^{t-1}) = R^e(z^{t-1})e(z^{t-2}, \eta^{t-2}) & \text{o.w.} \end{cases} \quad (2.15)$$

Note that the agent's equity position is being determined in rebalancing periods by his current debt position, and in nonrebalancing periods by his past equity position. Thus, it is completely determined by the bond position he took in rebalancing periods and the returns on equity.

Here again, we will work with the present-value problem. As before, let  $\chi$  denote the multiplier on the present-value budget constraint, let  $\nu(z^t, \eta^t)$  denote the multiplier on the measurability constraint in node  $(z^t, \eta^t)$ , let  $\varphi(z^t, \eta^t)$  denote the multiplier on the debt constraint. In addition, let  $\kappa(z^t, \eta^t)$  denote the multiplier on the equity transition condition. The saddle point problem of a passive trader of type  $(\phi^*, T)$  can be stated as:

$$\begin{aligned} L = & \min_{\{\chi, \nu, \varphi\}} \max_{\{c, b, e\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t, \eta^t)} u(c(z^t, \eta^t)) \pi(z^t, \eta^t) \\ & + \chi \left\{ \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \tilde{P}(z^t, \eta^t) [\gamma Y(z^t) \eta_t - c(z^t, \eta^t)] + \varpi(z^0) \right\} \\ & + \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \nu(z^t, \eta^t) \left\{ \begin{array}{l} \sum_{\tau \geq t} \sum_{(z^\tau, \eta^\tau) \succeq (z^t, \eta^t)} \tilde{P}(z^\tau, \eta^\tau) [\gamma Y(z^\tau) \eta_\tau - c(z^\tau, \eta^\tau)] \\ + \tilde{P}(z^t, \eta^t) [b(z^{t-1}, \eta^{t-1}) R^f(z^{t-1}) + e(z^{t-1}, \eta^{t-1}) R^e(z^{t+1})] \end{array} \right\} \\ & + \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \varphi(z^t, \eta^t) \left\{ \begin{array}{l} -\underline{M}_t(z^t, \eta^t) \tilde{P}(z^t, \eta^t) \\ - \sum_{\tau \geq t} \sum_{(z^\tau, \eta^\tau) \succeq (z^t, \eta^t)} \tilde{P}(z^\tau, \eta^\tau) [\gamma Y(z^\tau) \eta_\tau - c(z^\tau, \eta^\tau)] \end{array} \right\} \\ & + \sum_{t \geq 1} \sum_{(z^t, \eta^t)} \kappa(z^t, \eta^t) \left\{ \begin{array}{l} I_{\{t \in T\}} [e(z^t, \eta^t) - \frac{1}{\phi^*} b(z^t, \eta^t)] \\ + I_{\{t \notin T\}} [e(z^t, \eta^t) - R^e(z^t) e(z^{t-1}, \eta^{t-1})] \end{array} \right\}. \end{aligned}$$

where  $\tilde{P}(z^t, \eta^t) = \pi(z^t, \eta^t) P(z^t, \eta^t)$ .

The first-order conditions for this problem are

$$\begin{aligned} \beta^t u'(c(z^t, \eta^t)) \pi(z^t, \eta^t) &= \left\{ \chi + \sum_{(z^\tau, \eta^\tau) \succeq (z^t, \eta^t)} [\nu(z^\tau, \eta^\tau) - \varphi(z^\tau, \eta^\tau)] \right\} \tilde{P}(z^t, \eta^t), \\ \sum_{(z^{t+1}, \eta^{t+1})} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^f(z^t) &- I_{\{t \in T\}} \kappa(z^t, \eta^t) \frac{1}{\phi^*} = 0, \end{aligned}$$

and

$$\sum_{(z^{t+1}, \eta^{t+1})} \left\{ \begin{array}{l} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^e(z^{t+1}) \\ -\kappa(z^{t+1}, \eta^{t+1}) I_{\{t+1 \notin T\}} R^e(z^{t+1}) \end{array} \right\} + \kappa(z^t, \eta^t) = 0.$$

There are four cases with respect to the last two first-order conditions depending upon whether  $t$  and/or  $t + 1$  is an element in  $T$ . They are

(i) If  $t \in T$  and  $t + 1 \in T$  then the last two conditions reduce to

$$\sum_{(z^{t+1}, \eta^{t+1})} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) [\phi^* R^f(z^t) + R^e(z^{t+1})] = 0,$$

where  $\phi^* R^f(z^t) + R^e(z^{t+1})$  is the simply overall return on the agent's portfolio conditional on the transition from  $z^t$  to  $z^{t+1}$ . This is the condition for the constant rebalancing agent.

(ii) If  $t \in T$  and  $t + 1 \notin T$  then the last two conditions become

$$\phi^* \sum_{(z^{t+1}, \eta^{t+1})} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^f(z^t) = \kappa(z^t, \eta^t),$$

and

$$\sum_{(z^{t+1}, \eta^{t+1})} \left\{ \begin{array}{l} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^e(z^{t+1}) \\ -\kappa(z^{t+1}, \eta^{t+1}) R^e(z^{t+1}) \end{array} \right\} = -\kappa(z^t, \eta^t).$$

(iii) If  $t \notin T$  and  $t + 1 \in T$  then the last two conditions become

$$\phi^* \sum_{(z^{t+1}, \eta^{t+1})} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^f(z^t) = 0,$$

and

$$\sum_{(z^{t+1}, \eta^{t+1})} \left\{ \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^e(z^{t+1}) \right\} = -\kappa(z^t, \eta^t).$$

(iv) If  $t \notin T$  and  $t + 1 \notin T$  then the last two conditions become

$$\phi^* \sum_{(z^{t+1}, \eta^{t+1})} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^f(z^t) = 0,$$

and

$$\sum_{(z^{t+1}, \eta^{t+1})} \left\{ \begin{array}{l} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^e(z^{t+1}) \\ -\kappa(z^{t+1}, \eta^{t+1}) R^e(z^{t+1}) \end{array} \right\} = -\kappa(z^t, \eta^t).$$

In the simple case in which the rebalancing takes place every other period, then these condi-

tions boil down to

$$0 = \sum_{(z^{t+1}, \eta^{t+1})} \left\{ \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) [\phi^* R^f(z^t) + R^e(z^{t+1})] \right\} \\ + \sum_{(z^{t+2}, \eta^{t+2})} \left\{ \nu(z^{t+2}, \eta^{t+2}) \tilde{P}(z^{t+2}, \eta^{t+2}) \left[ \begin{array}{c} \phi^* R^f(z^t) R^f(z^{t+1}) \\ + R^e(z^{t+2}) R^e(z^{t+1}(z^{t+2})) \end{array} \right] \right\}$$

in the rebalancing periods, and

$$\phi^* \sum_{(z^{t+1}, \eta^{t+1})} \nu(z^{t+1}, \eta^{t+1}) \tilde{P}(z^{t+1}, \eta^{t+1}) R^f(z^t) = 0.$$

in the nonrebalancing periods.

## 2.5 Equilibrium

We allow for the possibility that there may be a positive measure of multiple types of active and passive traders. We assume there is always a non-zero measure of either complete or aggregate-complete traders to guarantee the uniqueness of the stochastic discount factor. For our active traders, let  $\mu_C$  denote the measure of complete traders and  $\mu_{AC}$  denote the measure of aggregate-complete traders. For our passive traders, we will assume for simplicity that there are only two types participating passive traders with measure  $\mu_{PP}$  and portfolio target  $x > 0$ , and nonparticipants with measure  $\mu_{NP}$  and portfolio target equal to zero. The non-state-contingent bond market condition is given by

$$\sum_{\eta^t} \left[ \begin{array}{c} \mu_C b_t^C(z^t, \eta^t) + \mu_{AC} a_t^{AC}(z^t, \eta^t) \\ + \mu_{PP} b_{t-1}^{PP}(z^t, \eta^t) + \mu_{NP} a_{t-1}^{NP}(z^t, \eta^t) \end{array} \right] \pi(\eta^t | z^t) = b(z^t),$$

and the equity market condition is given by

$$\sum_{\eta^t} \left[ \begin{array}{c} \mu_C e_t^C(z^t, \eta^t) + \mu_{AC} e_t^{AC}(z^t, \eta^t) \\ + \mu_{PP} e_{t-1}^{PP}(z^t, \eta^t) + \mu_{NP} e_{t-1}^{NP}(z^t, \eta^t) \end{array} \right] \pi(\eta^t | z^t) = e(z^t)$$

where we index the holdings by  $\{C, AC, PP, NP\}$  of the complete-markets, aggregate-complete, passive-participants, and passive-bonds-only traders respectively. In an abuse of notation, we use  $b(z^t)$  and  $e(z^t)$  to denote the value of the aggregate supply of bonds and equity claims respectively. For the sake of clarity, we use (e.g.)  $\eta^{t-1}(\eta^t)$  to denote the history from zero to  $t-1$  contained in  $\eta^t$ . We use the same convention for the aggregate histories. Using this notation,

the market clearing condition in the state contingent bond market is given by:

$$\sum_{\eta^t} [\mu_C a_{t-1}^C(z^t, \eta^t) + \mu_{AC} a_{t-1}^{AC}(z^t, \eta^{t-1}(\eta^t))] \pi(\eta^t | z^t) = 0.$$

Note that unlike bonds and equity, state-contingent bonds are in zero net supply.

An equilibrium for this economy is defined in the standard way. It consists of a list of bond and output claim holdings, a consumption allocation and a list of bond and tradeable output claim prices such that: (i) given these prices, a trader's asset and consumption choices maximize her expected utility subject to the budget constraints, the solvency constraints and the measurability constraints, and (ii) the asset markets clear.

As in CCL, we can characterize equilibrium prices and allocations using the household's multipliers and the aggregate multipliers.

**Proposition 2.1.** *The household consumption share, for all traders is given by*

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t). \quad (2.16)$$

The SDF is given by the Breeden-Lucas SDF and a multiplicative adjustment:

$$m(z^{t+1} | z^t) \equiv \beta \left( \frac{C(z^{t+1})}{C(z^t)} \right)^{-\alpha} \left( \frac{h(z^{t+1})}{h(z^t)} \right)^{\alpha}. \quad (2.17)$$

The consumption sharing rule follows directly from the ratio of the first order conditions and the market clearing condition. Condition (2.12) implies that

$$c(z^t, \eta^t) = u'^{-1} \left[ \frac{\zeta(z^t, \eta^t) P(z^t)}{\beta^t} \right].$$

In addition, the sum of individual consumptions aggregate up to aggregate consumption:

$$C(z^t) = \sum_{\eta^t} c(z^t, \eta^t) \pi(\eta^t | z^t).$$

This implies that the consumption share of the individual with history  $(z^t, \eta^t)$  is

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{u'^{-1} \left[ \frac{\zeta(z^t, \eta^t) P(z^t)}{\beta^t} \right]}{\sum_{\eta^t} u'^{-1} \left[ \frac{\zeta(z^t, \eta^t) P(z^t)}{\beta^t} \right] \pi(\eta^t | z^t)}.$$

With CRRA preferences, this implies that the consumption share is given by

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t).$$

Hence, the  $-1/\alpha^{\text{th}}$  moment of the multipliers summarizes risk sharing within this economy. We refer to this moment of the multipliers simply as **the aggregate multiplier**. The equilibrium SDF is the standard Breeden-Lucas SDF times the growth rate of the aggregate multiplier. This aggregate multiplier reflects the aggregate shadow cost of the measurability and the borrowing constraints faced by households. The expression for the SDF can be recovered directly by substituting for the consumption sharing rule in the household's first order condition for consumption (2.12).

This proposition directly implies that an equilibrium for this class of incomplete market economies can be completely characterized by a process for these cumulative multipliers  $\{\zeta(\eta^t, z^t)\}$ , and by the associated aggregate multiplier process  $\{h_t(z^t)\}$ .

## 2.6 Wealth Accumulation and Aggregate Risk

By aggregating household wealth across all households in a trading segment  $j$ , we can define the aggregate wealth for each group of traders  $j \in \{C, AC, PP, NP\}$ :

$$A^j(z^t) = \left[ \frac{h^j(z^t)}{h(z^t)} - \gamma \mu^j \right] C(z^t) + \sum_{z^{t+1}} \frac{\pi(z^{t+1}) P(z^{t+1})}{\pi(z^t) P(z^t)} A^j(z^{t+1}),$$

where we use the linearity of the pricing functional. Finally, total aggregate wealth equals the market portfolio:

$$\sum_j A^j(z^t) = [\varpi(z^t) + (1 - \gamma)Y(z^t)]$$

This follows directly from market clearing.

For passive traders, the return on their savings between period  $t$  and  $t + 1$  is given by

$$R_{t+1}^j(z^{t+1}, \eta^t) = \frac{e(z^t, \eta^t) R^e(z^{t+1}) + b(z^t, \eta^t) R^f(z^t)}{e(z^t, \eta^t) + b(z^t, \eta^t)}.$$

If  $e(z^t, \eta^t) = 0$ , then  $R_{t+1}^j(z^{t+1}, \eta^t) = R^f(z^t)$ , and if

$$\frac{b(z^t, \eta^t)}{e(z^t, \eta^t)} = \phi \Rightarrow R_{t+1}^j(z^{t+1}, \eta^t) = \frac{[\varpi(z^{t+1}) + (1 - \gamma)Y(z^{t+1})]}{[\varpi(z^t) + (1 - \gamma)Y(z^t)]} \equiv R(z^{t+1}),$$

since these traders are holding the market and hence simply earn the market return  $R(z^t)$ . Moreover, in states in which  $R(z^{t+1}) > R^f(z^t)$ , a traders return is strictly increasing in the share of equities.

If we compare outcomes across two future states  $z'_{t+1}$  and  $z''_{t+1}$  in which

$$R([z'_{t+1}, z^t]) > R^f(z^{t-1}) > R([z''_{t+1}, z^t]),$$

the ratio of his financial wealth in period  $t + 1$  will be higher in state  $z'_{t+1}$  than in state  $z''_{t+1}$  if  $b(z^t, \eta^t) < \phi e(z^t, \eta^t)$ , and lower if the reverse is true. Moreover, the extent of the difference depends upon the extent to which his portfolio differs from the market ratio  $\phi$ . Since a continuously rebalancing trader has a constant portfolio share, the cyclicalities of their wealth share's response to gap between the market return and the risk-free rate is not time varying. However, an intermittent-rebalancer will have a time varying portfolio share since past high returns on equity will, *ceteris paribus*, lead to his having a higher equity share in nonrebalancing periods and hence a higher procyclicality (or at least a lower counter-cyclicality) of his wealth share across high vs. low market return states.

In CCL, the primary mechanism for generating a market price of risk was having a low share of active traders and a high share of passive traders with a low overall ratio of equities-to-debt in their portfolio. Since these passive traders had a wealth share that negatively covaried with the excess return on the market portfolio, it meant that the active traders had to have a wealth share that positively covaried. As a result the active traders were absorbing market risk, and because of their relative small size, it was individually large. In CCL, time variation volatility of the pricing of risk came largely from the relative wealth of the active and the passive traders. With intermittent-rebalancers, there is an additional dimension by which the market price of risk may vary. This additional channel is coming from time variation in the portfolio share of equities in the portfolios of these traders. In the next section, we quantitatively explore the impact of this additional channel on the volatility of the market price of risk.

### 3 Quantitative Results

This section evaluates a calibrated version of the model to examine the extent to which our model can account the empirically observed pattern of asset prices, and in particular their cyclical volatility. The first subsection discusses the calibration of the parameters and the endowment processes. We follow the algorithm described in Chien, Cole and Lustig (2008) for computing the equilibrium of this economy. We then examine asset prices, consumption growth, portfolio

returns and the distribution of financial wealth respond to changes in the frequency of rebalancing by passive equity holders, the level of their equity target, and the composition of the active trader traders between aggregate-complete and complete traders.

The model is calibrated to annual data. We choose a coefficient of relative risk aversion  $\alpha$  of five and a time discount factor  $\beta$  of .95. These preference parameters allow us to match the collateralizable wealth to income ratio in the data when the tradeable or collateralizable income share  $1 - \gamma$  is 10%, as discussed below. Non-diversifiable income includes both labor income and entrepreneurial income, among other forms. With respect to the stochastic process for aggregate output, our model is calibrated to match the aggregate consumption statistics taken from Alvarez and Jermann, which is fairly similar to (?). The average consumption growth rate is 1.8%. The standard deviation is 3.15%. Recessions are less frequent: 27% of realizations are low aggregate consumption growth states. For idiosyncratic risk we calibrate to (?), who find that the variance of idiosyncratic risk is negatively correlated with the growth rate of consumption: The Markov process for  $\log \eta(y, z)$  is has standard deviation is .60, and the autocorrelation is 0.89. We use a 4-state discretization for both aggregate and idiosyncratic risk. The elements of the process for  $\log \eta$  are  $\{0.38, 1.61\}$ . In addition, we also report the results for a second calibration of the Markov process for  $\log \eta(y, z)$  in which we calibrated to (?), except that we force the idiosyncratic shocks to be independent of the aggregate shocks to consumption growth.

The average ratio of household wealth to aggregate income in the US is 4.30 between 1950 and 2005. The wealth measure is total net wealth of households and non-profit organizations (Flow of Funds Tables). We choose a collateralizable income ratio  $\alpha$  of 10%. The implied ratio of wealth to consumption is 4.88 in the model's benchmark calibration.<sup>2</sup>

In the most recent Survey of Consumer Finance, the share of responders holding no equities was **X**, so we set the share of passive traders who hold no equities equal to 50%. We set the share of passive traders who hold equities equal to 45%, and the overall share of active traders to 5%. We consider two types of passive equity holders: (1) those who rebalance every period and those who rebalance every 3 periods. We also consider three different rebalancing targets for our passive equity holders: 30%, 35% and 40%. We will assume that our traders cannot borrow against their labor income,  $\psi(z^t) = 0$ .

Equity in our model is simply a leveraged claim to diversifiable income. In the Flow of Funds, the ratio of corporate debt-to-net worth is around 0.65, suggesting a leverage parameter  $\psi$  of 2. However, (?) report that standard deviation of the growth rate of dividends is at least 3.6 times that of aggregate consumption, suggesting that the appropriate leverage level is over 3.

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<sup>2</sup>As is standard in this literature, we compare the ratio of total outside wealth to aggregate non-durable consumption in our endowment economy to the ratio of total tradeable wealth to aggregate income in the data. Aggregate income exceeds aggregate non-durable consumption because of durable consumption and investment.

Following (?) and (?) , we choose to set the leverage parameter  $\psi$  to 3. The returns on this security are denoted  $R_E$ .

### Asset Prices

- Several systematic patterns emerge if we start from the baseline model of CCL in which the active traders are aggregate-complete and the passive equity holders continuously rebalance. To discuss these patterns we focus for the moment on the results in Table 2 which reports asset pricing results for the midrange target of 35% for our passive equity traders.
  - Changing the composition of active traders from aggregate-complete to complete traders raises the market price of risk and the equity premium substantially.
    - \* For example, when we switch from 5% aggregate-complete traders to 5% complete traders, in the case of continuously rebalancing in the market price of risk increases by almost .1, which is a 25% increase. This change also raises the equity premium from 7.44 to 8.61 which is also a substantial increase.
    - \* At the same time, this switch doubles the volatility of the market price of risk, which the conditional standard deviation goes from 0.04 to 0.08.
    - \* Even at the mid-range composition of the active traders in which there are equal numbers of complete and aggregate-complete traders, the increases in the price of risk and the volatility of the price of risk are smaller but still substantial.
    - \* This composition change in the active traders has very little impact on either the level or the volatility of the risk-free rate. It stays in the slightly below 2% with a standard deviation slightly below 3.
  - Changing the type of the passive equity traders from continuous to intermittent rebalancers roughly doubles the volatility of the market price of risk. At the same time it has small but opposing effects on the market price of risk and the equity premium. The market price of risk increases, but the equity premium falls. The impact on the risk-free rate is also small, with the change raising the level of the rate but lowering its volatility.
- If we compare our asset pricing results across Tables 1-3 in which we change the equity target for our passive equity holders from 30% to 35% and then 40% several other patterns emerge. For reference purposes, note that the share of equities in the market is 25%.
  - Increases in the equity target lowers the market price of risk and the risk premium, and raises the risk-free rate.

- The impact of the equity target on the volatility of the market price of risk depends upon the composition of the active traders, though the predominate pattern is that it increases it.
  - \* For the standard configuration in which the active traders are aggregate-complete and the passive equity holders continuously rebalance, the volatility of the market price of risk is increasing in the equity target and roughly doubles as it goes from 30% to 40%. This same pattern emerges when the passive equity traders are intermittent rebalancers, though the level of volatility is substantially higher. When the active traders include both aggregate-complete and complete, the same pattern still emerges though including complete traders also increase the volatility of the pricing of risk.
  - \* The equity premium was more sensitive to the equity target when the passive equity holders were continuous rebalancers than when they were intermittent rebalancers. At an equity target of 30%, the cases with continuous rebalancers had substantially higher equity premia, while at a 40% equity target, the equity premia were essentially equal across the two types of passive equity holders, and this occurred because the continuous rebalancing cases had larger falls in the equity premium.
  - \* For the volatility of the market price of risk the results were not completely monotonic in the equity target. For example, in the case in which the active traders consist of aggregate complete traders only, and the passive equity holders are intermittent rebalancers, the highest volatility of the pricing of risk is at the intermediate equity target of 35%, though the increase in volatility that comes from switching the target from 30% to 35 % is large and the drop from going from 35% to 40% is small.
- With respect to both level and the volatility of the market price of risk, some of our configurations deliver values in the neighborhood of those reported in the literature.

**Consumption:**

- The pricing of risk is driven by the consumption risk of the agents doing the pricing. Focusing for the moment on Table 5 which reports results for the mid-range equity target 35%, several patterns emerge.
  - First, just as in CCL, the volatility of consumption growth is decreasing in the sophistication of a traders portfolio type for the baseline case in which the active traders are

aggregate-complete and the passive equity holders are continuous rebalancers. Also, just as in CCL, the standard deviation of the average growth rate in consumption is increasing in the trader's sophistication. These results indicate that more sophisticated traders' consumption is more sensitive to aggregate risk but less sensitive to idiosyncratic risk.

- In the cases in which we have both types of active traders, we see that the aggregate-complete traders' consumption growth is more sensitive to both idiosyncratic and aggregate risk than the complete traders. However, when we compare the cases in which we have only one type of active trader, the aggregate risk sensitivity of the complete traders is higher than the aggregate-complete traders though the overall volatility of consumption is still lower for the complete traders.
- When the passive equity holders are intermittent rebalancers, the volatility of their consumption growth is now roughly equal to that of the passive debt holders. If we compare cases in which the passive equity holders are intermittent as opposed to continuous, we see that the intermittent rebalancers are more exposed to aggregate risk, and this appears to account for the fact that the volatility of their consumption is higher.
- When we increase the portfolio target of the passive equity holders, the following changes occur in the volatility of consumption. The volatility of the passive equity holder's consumption growth increases and in particular its sensitivity to aggregate risk as measured by the standard deviation of the average consumption growth for this group. However, the standard deviation of the average growth rate of consumption for the active traders falls. These results suggest that this change is leading to the passive equity holders holding a greater share of aggregate risk and the active trader's holding less.

## Portfolio Returns

- Tables 7-9 report some statistics on financial investment returns by trader type for our different configurations. The overall pattern is fairly similar across the different cases. More sophisticated traders earn higher average returns. The intermittent rebalancers earn lower returns and have lower Sharpe ratios than the continuous rebalancers. The complete traders have low Sharpe ratios but this is because they are hedging their idiosyncratic nontradable income risk and this raises the volatility of their financial returns.
- Average excess returns for both the active traders and the passive fall as the equity target of the passive traders increases.

- In the 40% target case, our continuous rebalancers and the aggregate-complete traders (who are really just optimizing bond-stock portfolio traders since we have only two aggregate states), earn virtually identical returns on their portfolio and have very similar Sharpe ratios in the cases in which both types are included in the trader composition. This suggests that a 40% equity target is very close to the optimum fixed target and that the aggregate-complete traders aren't earning much from market timing.

### **Wealth Distribution Statistics**

- The average level of wealth was very similar across all of our economies, however the volatility of the level of financial wealth was not.
- The wealth statistics indicate that aggregate-complete traders accumulate the most financial wealth on average and that complete traders accumulate the least among all of the traders. This validates the original conjecture that the ability to hedge their idiosyncratic risk was going to reduce the precautionary savings motive of the complete traders and lead to their being close to their borrowing constraints. Among passive traders, the passive equity holders accumulate more than the passive debt-only traders.
- The standard deviation of the relative wealth of the active traders, as well as the correlation between their relative wealth and the return on an overall claim to financial wealth, was very sensitive to whether the passive equity-holders were continuous or intermittent rebalancers, as well as their equity target. In the case of continuous rebalancers, the standard deviation falls sharply with the equity target. This leads to a reversal in the relative size of the standard deviation, with it being larger for the continuous rebalancing case than for the intermittent case when the equity share is low, and the reverse being true when it is high.
- The standard deviation of the average equity share of the passive traders is higher when the passive traders are intermittent rebalancers rather than continuous rebalancers. The correlation between the average equity share and the realized return is positive in all our cases. But, what's striking is that the correlation between this share and the realized return is much higher for the intermittent rebalancers when the equity target is 30%, but roughly the same when it is either 35% or 40%.

### **Overall Comments:**

- These quantitative experiments were done to evaluate the following two conjectures: (i) having more aggressive active traders who do not end up close to their borrowing constraints could raise the average price of risk, and (ii) having passive equity holders who

intermittently rebalance their portfolios would raise the volatility of the pricing of risk. The results not only validate those conjectures, but indicate that both of these effects are quantitatively large. Focusing on Table 2, which had the intermediate equity target of 35% for our passive traders, we see that switching from aggregate-complete active traders to complete traders increases the equity premium by 120 basis points. At the same time, switching from continuous rebalancers to intermittent rebalancers tripled the volatility of the market price of risk. The quantitative experiments also revealed that switching from aggregate-complete to complete also doubled the volatility of the price of risk, while switching from continuous to intermittent rebalancers lowered the average price of risk. If we compare an economy with only complete active traders and intermittent rebalancing passive equity holders to one with aggregate-complete active traders and continuously rebalancing passive equity holders, we see that the equity premium is almost identical at 7.45, but that the standard deviation of the market price of risk by a factor 7.

- In CCL, the primary channel for the high average price of risk was the large amount of aggregate risk being absorbed by active traders. That channel is clearly still active and explains part of the results. In Table 5, which reports the consumption results for the 35% target, we see that switching from aggregate-complete to complete active traders and from continuous to intermittent rebalancers increases the standard deviation of average consumption for both of these groups.
- In CCL, the primary channel for time variation in the price of risk was time variation in the relative wealth of active traders. Surprisingly, this channel seems to sharply attenuated in this same switch. Looking at Table 11, which reports the wealth statistics for the 35% target case, we see that standard deviation of relative wealth falls roughly in half for the active traders from 0.42 to 0.24. The standard deviation of the relative wealth for the passive equity holders does increase from does increase from 0.09 to 0.13, but the impact of this is strongly reenforced by the time variation in the average equity share of the passive traders as a whole. This goes from 0.06 to 0.10. Since this share is also highly correlated with the realized return on tradable wealth, this is leading to a strongly counter-cyclical price of risk.

Table 1: Asset Pricing Results: 30% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(m)$	0.98547	0.98597	0.98464	0.98583	0.98407	0.98561
$\sigma(m)$	0.51860	0.50879	0.45138	0.46321	0.41397	0.43252
$\frac{\sigma(m)}{E(m)}$	0.52625	0.51603	0.45842	0.46987	0.42067	0.43883
$Std(\frac{\sigma_t(m)}{E_t(m)})$	0.18426	0.065209	0.11712	0.041914	0.090672	0.031373
$E(R_{Lc} - R_f)(L = 3)$	9.2915	10.149	8.5664	9.3208	8.1462	8.9186
$\sigma(R_{Lc} - R_f)(L = 3)$	20.356	20.164	20.243	20.153	20.567	20.563
$\frac{E(R_{Lc} - R_f)}{\sigma(R_{Lc} - R_f)}(L = 3)$	0.45645	0.50331	0.42317	0.46250	0.39608	0.43372
$E(R_f)$	1.8089	1.6036	1.7600	1.5766	1.7824	1.5817
$\sigma(R_f)$	2.4652	2.9442	2.5957	2.9825	2.6891	3.0255

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 2: Asset Pricing Results: 35% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(m)$	0.98270	0.98367	0.98182	0.98290	0.98118	0.98213
$\sigma(m)$	0.47838	0.45162	0.39933	0.39477	0.35765	0.36308
$\frac{\sigma(m)}{E(m)}$	0.48680	0.45912	0.40672	0.40163	0.36451	0.36969
$Std(\frac{\sigma_t(m)}{E_t(m)})$	0.28794	0.084495	0.19433	0.047555	0.14798	0.038991
$E(R_{Lc} - R_f)(L = 3)$	7.4900	8.6100	7.1153	7.9419	6.7645	7.4352
$\sigma(R_{Lc} - R_f)(L = 3)$	23.484	19.519	22.400	20.182	21.974	20.452
$\frac{E(R_{Lc} - R_f)}{\sigma(R_{Lc} - R_f)}(L = 3)$	0.31894	0.44111	0.31765	0.39352	0.30784	0.36355
$E(R_f)$	2.1627	1.8445	2.0985	1.8785	2.1152	1.9391
$\sigma(R_f)$	2.2165	2.8017	2.4595	2.9588	2.5980	2.9782

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 3: Asset Pricing Results: 40% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(m)$	0.97709	0.97800	0.97744	0.97773	0.97741	0.97731
$\sigma(m)$	0.44656	0.30736	0.37023	0.27676	0.31504	0.26043
$\frac{\sigma(m)}{E(m)}$	0.45703	0.31428	0.37877	0.28306	0.32232	0.26648
$Std(\frac{\sigma_t(m)}{E_t(m)})$	0.26293	0.12788	0.21081	0.081652	0.17711	0.061883
$E(R_{Lc} - R_f)(L = 3)$	5.2566	5.6704	5.2073	5.4638	5.0598	5.2019
$\sigma(R_{Lc} - R_f)(L = 3)$	27.782	21.097	25.397	20.877	23.790	20.532
$\frac{E(R_{Lc} - R_f)}{\sigma(R_{Lc} - R_f)}(L = 3)$	0.18921	0.26878	0.20503	0.26172	0.21268	0.25336
$E(R_f)$	2.7334	2.4405	2.5651	2.4290	2.5207	2.4510
$\sigma(R_f)$	2.1210	2.8064	2.3229	2.9059	2.5061	2.8908

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 4: Consumption Growth Results: 30% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$\frac{\sigma(\Delta \log(c_{com}))}{\sigma(\Delta \log C)}$	2.3891	2.4330	2.3351	2.3491	NA	NA
$\frac{\sigma(\Delta \log(c_z))}{\sigma(\Delta \log C)}$	NA	NA	2.8161	2.7874	2.9332	2.8896
$\frac{\sigma(\Delta \log(c_{cap}))}{\sigma(\Delta \log C)}$	NA	3.1741	NA	3.2314	NA	3.2592
$\frac{\sigma(\Delta \log(c_{prb}))}{\sigma(\Delta \log C)}$	3.5321	NA	3.5942	NA	3.6245	NA
$\frac{\sigma(\Delta \log(c_{bond}))}{\sigma(\Delta \log C)}$	3.6523	3.6881	3.6614	3.6925	3.6597	3.6925
$\sigma\left(\frac{C'_{com}}{C_{com}}\right)$	7.8353	8.1462	6.5814	7.1127	NA	NA
$\sigma\left(\frac{C'_z}{C_z}\right)$	NA	NA	7.4541	7.8253	6.9277	7.3470
$\sigma\left(\frac{C'_{cap}}{C_{cap}}\right)$	NA	3.9948	NA	4.0093	NA	4.0445
$\sigma\left(\frac{C'_{prb}}{C_{prb}}\right)$	4.1476	NA	4.1697	NA	4.2016	NA
$\sigma\left(\frac{C'_{bond}}{C_{bond}}\right)$	2.4576	2.4515	2.4345	2.4375	2.4113	2.4120

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 5: Consumption Growth Results: 35% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$\frac{\sigma(\Delta \log(c_{com}))}{\sigma(\Delta \log C)}$	2.4325	2.3098	2.4925	2.3672	NA	NA
$\frac{\sigma(\Delta \log(c_z))}{\sigma(\Delta \log C)}$	NA	NA	2.9763	2.9717	3.0900	3.0699
$\frac{\sigma(\Delta \log(c_{cap}))}{\sigma(\Delta \log C)}$	NA	3.1502	NA	3.1999	NA	3.2273
$\frac{\sigma(\Delta \log(c_{prb}))}{\sigma(\Delta \log C)}$	3.5832	NA	3.6051	NA	3.6309	NA
$\frac{\sigma(\Delta \log(c_{bond}))}{\sigma(\Delta \log C)}$	3.5857	3.6442	3.6002	3.6410	3.5986	3.6307
$\sigma\left(\frac{C'_{com}}{C_{com}}\right)$	6.8166	6.7309	5.5351	5.6372	NA	NA
$\sigma\left(\frac{C'_z}{C_z}\right)$	NA	NA	6.4823	6.7227	5.9371	6.2329
$\sigma\left(\frac{C'_{cap}}{C_{cap}}\right)$	NA	4.2883	NA	4.3645	NA	4.3920
$\sigma\left(\frac{C'_{prb}}{C_{prb}}\right)$	4.4846	NA	4.5042	NA	4.5168	NA
$\sigma\left(\frac{C'_{bond}}{C_{bond}}\right)$	2.3962	2.4435	2.3813	2.3933	2.3641	2.3646

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 6: Consumption Growth Results: 40% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$\frac{\sigma(\Delta \log(c_{com}))}{\sigma(\Delta \log C)}$	2.4673	2.6949	2.6034	2.7815	NA	NA
$\frac{\sigma(\Delta \log(c_z))}{\sigma(\Delta \log C)}$	NA	NA	2.9889	3.2392	3.1490	3.2706
$\frac{\sigma(\Delta \log(c_{cap}))}{\sigma(\Delta \log C)}$	NA	3.2396	NA	3.2546	NA	3.2661
$\frac{\sigma(\Delta \log(c_{prb}))}{\sigma(\Delta \log C)}$	3.7591	NA	3.7261	NA	3.7050	NA
$\frac{\sigma(\Delta \log(c_{bond}))}{\sigma(\Delta \log C)}$	3.4733	3.5358	3.5089	3.5397	3.5206	3.5356
$\sigma\left(\frac{C'_{com}}{C_{com}}\right)$	6.6311	4.1839	5.1431	3.6810	NA	NA
$\sigma\left(\frac{C'_z}{C_z}\right)$	NA	NA	6.0258	4.8172	5.2258	4.5798
$\sigma\left(\frac{C'_{cap}}{C_{cap}}\right)$	NA	4.7346	NA	4.7556	NA	4.7375
$\sigma\left(\frac{C'_{prb}}{C_{prb}}\right)$	4.7993	NA	4.7980	NA	4.7999	NA
$\sigma\left(\frac{C'_{bond}}{C_{bond}}\right)$	2.3299	2.3604	2.3208	2.3222	2.3078	2.3038

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 7: Portfolio Return Results: 30% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(R_{com}^W - R_f)$	0.14180	0.13621	0.10925	0.12323	NA	NA
$E(R_z^W - R_f)$	NA	NA	0.062752	0.064047	0.055436	0.058654
$E(R_{cap}^W - R_f)$	NA	0.030321	NA	0.027838	NA	0.026625
$E(R_{prb}^W - R_f)$	0.028722	NA	0.026555	NA	0.025266	NA
$E(R_{bond}^W - R_f)$	0	0	0	0	0	0
$\sigma(R_{com}^W - R_f)$	0.94079	0.80173	1.4449	1.2426	NA	NA
$\sigma(R_z^W - R_f)$	NA	NA	0.13400	0.13623	0.12930	0.13375
$\sigma(R_{cap}^W - R_f)$	NA	0.060011	NA	0.060014	NA	0.061195
$\sigma(R_{prb}^W - R_f)$	0.064289	NA	0.063839	NA	0.064708	NA
$\sigma(R_{bond}^W - R_f)$	0	0	0	0	0	0
$\frac{E(R_{com}^W - R_f)}{\sigma(R_{com}^W - R_f)}$	0.15073	0.16989	0.075607	0.099170	NA	NA
$\frac{E(R_z^W - R_f)}{\sigma(R_z^W - R_f)}$	NA	NA	0.46828	0.47014	0.42873	0.43855
$\frac{E(R_{cap}^W - R_f)}{\sigma(R_{cap}^W - R_f)}$	NA	0.50527	NA	0.46385	NA	0.43509
$\frac{E(R_{prb}^W - R_f)}{\sigma(R_{prb}^W - R_f)}$	0.44676	NA	0.41596	NA	0.39047	NA
$\frac{E(R_{bond}^W - R_f)}{\sigma(R_{bond}^W - R_f)}$	-	-	-	-	-	-

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 8: Portfolio Return Results: 35% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(R_{com}^W - R_f)$	0.11255	0.11460	0.077271	0.085464	NA	NA
$E(R_z^W - R_f)$	NA	NA	0.051455	0.050909	0.042591	0.043846
$E(R_{cap}^W - R_f)$	NA	0.030004	NA	0.027657	NA	0.025881
$E(R_{prb}^W - R_f)$	0.025422	NA	0.024616	NA	0.023585	NA
$E(R_{bond}^W - R_f)$	0	0	0	0	0	0
$\sigma(R_{com}^W - R_f)$	1.2546	1.3923	1.7129	1.6286	NA	NA
$\sigma(R_z^W - R_f)$	NA	NA	0.11292	0.12628	0.10694	0.11840
$\sigma(R_{cap}^W - R_f)$	NA	0.067809	NA	0.070138	NA	0.071040
$\sigma(R_{prb}^W - R_f)$	0.083371	NA	0.080332	NA	0.079055	NA
$\sigma(R_{bond}^W - R_f)$	0	0	0	0	0	0
$\frac{E(R_{com}^W - R_f)}{\sigma(R_{com}^W - R_f)}$	0.089712	0.082310	0.045111	0.052477	NA	NA
$\frac{E(R_z^W - R_f)}{\sigma(R_z^W - R_f)}$	NA	NA	0.45567	0.40314	0.39827	0.37032
$\frac{E(R_{cap}^W - R_f)}{\sigma(R_{cap}^W - R_f)}$	NA	0.44247	NA	0.39433	NA	0.36432
$\frac{E(R_{prb}^W - R_f)}{\sigma(R_{prb}^W - R_f)}$	0.30492	NA	0.30643	NA	0.29834	NA
$\frac{E(R_{bond}^W - R_f)}{\sigma(R_{bond}^W - R_f)}$	-	-	-	-	-	-

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 9: Portfolio Return Results: 40% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(R_{com}^W - R_f)$	0.10968	0.044619	0.070046	0.035695	NA	NA
$E(R_z^W - R_f)$	NA	NA	0.045107	0.024325	0.033150	0.020743
$E(R_{cap}^W - R_f)$	NA	0.022519	NA	0.021699	NA	0.020653
$E(R_{prb}^W - R_f)$	0.018295	NA	0.019058	NA	0.019076	NA
$E(R_{bond}^W - R_f)$	0	0	0	0	0	0
$\sigma(R_{com}^W - R_f)$	1.1105	1.9285	1.6703	1.9609	NA	NA
$\sigma(R_z^W - R_f)$	NA	NA	0.11233	0.083728	0.099067	0.077294
$\sigma(R_{cap}^W - R_f)$	NA	0.083137	NA	0.082666	NA	0.081461
$\sigma(R_{prb}^W - R_f)$	0.10742	NA	0.10029	NA	0.095267	NA
$\sigma(R_{bond}^W - R_f)$	0	0	0	0	0	0
$\frac{E(R_{com}^W - R_f)}{\sigma(R_{com}^W - R_f)}$	0.098774	0.023137	0.041937	0.018203	NA	NA
$\frac{E(R_z^W - R_f)}{\sigma(R_z^W - R_f)}$	NA	NA	0.40157	0.29053	0.33462	0.26837
$\frac{E(R_{cap}^W - R_f)}{\sigma(R_{cap}^W - R_f)}$	NA	0.27087	NA	0.26249	NA	0.25353
$\frac{E(R_{prb}^W - R_f)}{\sigma(R_{prb}^W - R_f)}$	0.17031	NA	0.19002	NA	0.20024	NA
$\frac{E(R_{bond}^W - R_f)}{\sigma(R_{bond}^W - R_f)}$	-	-	-	-	-	-

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 10: Wealth stats Results: 30% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(W)$	5.6188	5.7495	5.8364	6.0024	5.9613	6.1346
$std(W)$	0.14902	0.15170	0.13263	0.12120	0.12861	0.11435
$E(W_{com}/W)$	0.79356	1.1789	0.52952	0.61036	NA	NA
$E(W_z/W)$	NA	NA	3.4425	4.2365	2.4957	2.9579
$E(W_{cap}/W)$	NA	1.2233	NA	1.1235	NA	1.0808
$E(W_{prb}/W)$	1.2247	NA	1.1313	NA	1.0908	NA
$E(W_{bond}/W)$	0.81844	0.78118	0.78327	0.74655	0.76867	0.73150
$std(W_{com}/W)$	0.27496	0.71759	0.11839	0.20248	NA	NA
$std(W_z/W)$	NA	NA	1.0085	1.6019	0.64013	0.97389
$std(W_{cap}/W)$	NA	0.067476	NA	0.053793	NA	0.049363
$std(W_{prb}/W)$	0.097619	NA	0.082443	NA	0.077939	NA
$std(W_{bond}/W)$	0.10114	0.10324	0.10889	0.10961	0.11370	0.11611
$std(W_{active}/W)$	0.27496	0.71759	0.54329	0.87305	0.64013	0.97389
$corr(\frac{W_{active}}{W}, R_{Lc})$	0.29831	0.19218	0.27253	0.22532	0.27637	0.23410
$std(W_{cap}/W_{bond})$	0.31919	0.28430	0.31471	0.27779	0.31797	0.28438
$std(EqShare_{passive})$	0.086031	0.11077	0.10250	0.091447	0.10064	0.086367
$corr(W_{cap}/W_{bond}, R_{Lc})$	0.25165	0.28229	0.25628	0.28566	0.25666	0.28604
$corr(EqShare_{passive}, R_{Lc})$	0.75242	0.18541	0.80543	0.40772	0.81984	0.45267

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 11: Wealth stats Results: 35% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(W)$	5.7071	5.8043	5.8560	5.9654	5.9612	6.0709
$std(W)$	0.15189	0.12725	0.13785	0.11545	0.13616	0.11223
$E(W_{com}/W)$	0.61948	0.55526	0.45078	0.45762	NA	NA
$E(W_z/W)$	NA	NA	2.2602	2.2874	1.7560	1.8269
$E(W_{cap}/W)$	NA	1.2773	NA	1.2069	NA	1.1653
$E(W_{prb}/W)$	1.2205	NA	1.1696	NA	1.1398	NA
$E(W_{bond}/W)$	0.83962	0.79487	0.81182	0.77658	0.79861	0.76857
$std(W_{com}/W)$	0.24150	0.12380	0.10631	0.091954	NA	NA
$std(W_z/W)$	NA	NA	0.70520	0.61331	0.44688	0.42168
$std(W_{cap}/W)$	NA	0.097079	NA	0.089248	NA	0.085494
$std(W_{prb}/W)$	0.12858	NA	0.12460	NA	0.12165	NA
$std(W_{bond}/W)$	0.11334	0.098196	0.11431	0.11088	0.11584	0.11546
$std(W_{active}/W)$	0.24150	0.12380	0.39170	0.33895	0.44688	0.42168
$corr(\frac{W_{active}}{W}, R_{Lc})$	0.16891	0.41630	0.19130	0.29676	0.20884	0.29516
$std(W_{cap}/W_{bond})$	0.37443	0.32848	0.37618	0.34428	0.37536	0.34687
$std(EqShare_{passive})$	0.10044	0.027869	0.12189	0.054783	0.12071	0.059662
$corr(W_{cap}/W_{bond}, R_{Lc})$	0.24882	0.27027	0.24879	0.27115	0.24899	0.27129
$corr(EqShare_{passive}, R_{Lc})$	0.76092	0.78537	0.78848	0.73315	0.80545	0.77181

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)

Table 12: Wealth stats Results: 40% equity share passive target

<i>Active complete</i>	5%	5%	2.5%	2.5%	0%	0%
<i>Active agg-complete</i>	0%	0%	2.5%	2.5%	5%	5%
<i>Passive continuous-rebalancing</i>	0%	45%	0%	45%	0%	45%
<i>Passive periodic-rebalancing</i>	45%	0%	45%	0%	45%	0%
<i>Passive bond only</i>	50%	50%	50%	50%	50%	50%
$E(W)$	5.7860	5.8826	5.9163	5.9799	5.9964	6.0602
$std(W)$	0.23081	0.11336	0.17499	0.11263	0.15301	0.11055
$E(W_{com}/W)$	0.70908	0.38450	0.44508	0.36263	NA	NA
$E(W_z/W)$	NA	NA	2.1234	1.2556	1.5228	1.1601
$E(W_{cap}/W)$	NA	1.2458	NA	1.2150	NA	1.1859
$E(W_{prb}/W)$	1.1569	NA	1.1359	NA	1.1278	NA
$E(W_{bond}/W)$	0.88792	0.84035	0.84925	0.82559	0.83266	0.81667
$std(W_{com}/W)$	0.39343	0.062635	0.13971	0.057371	NA	NA
$std(W_z/W)$	NA	NA	0.86381	0.20471	0.49132	0.15390
$std(W_{cap}/W)$	NA	0.12376	NA	0.12691	NA	0.12503
$std(W_{prb}/W)$	0.14593	NA	0.14856	NA	0.14980	NA
$std(W_{bond}/W)$	0.12486	0.11162	0.12256	0.11818	0.12307	0.11917
$std(W_{active}/W)$	0.39343	0.062635	0.48635	0.11899	0.49132	0.15390
$corr(\frac{W_{active}}{W}, R_{Lc})$	0.063370	0.19375	0.079344	0.26284	0.089286	0.28663
$std(W_{cap}/W_{bond})$	0.37944	0.35585	0.39494	0.37797	0.40493	0.37756
$std(EqShare_{passive})$	0.13065	0.034207	0.14840	0.050991	0.14376	0.058654
$corr(W_{cap}/W_{bond}, R_{Lc})$	0.26634	0.25840	0.25631	0.25727	0.24947	0.25833
$corr(EqShare_{passive}, R_{Lc})$	0.73905	0.78115	0.74976	0.82740	0.75653	0.85499

Notes: Parameters setting:  $\gamma = 5$ ,  $\beta = 0.95$ , collateralized share of income is 0.1. The simulation results are generated by an economy with 3000 agents and 10000 periods. The calibrations of aggregate shocks are based on Alvarez and Jermann (2001) and the calibration of idiosyncratic are from Storesletten, Telmer and Yaron (2006)