

# Dissipative quantum phase transition in a quantum dot

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(Dated: May 23, 2006)

We study the transport properties of a quantum dot (QD) with highly resistive gate electrodes, and show that the QD displays a quantum phase transition analogous to the famous dissipative phase transition first identified by S. Chakravarty [Phys. Rev. Lett. **49**, 681-684 (1982)]; for a review see [A. J. Leggett *et al.*, Rev. Mod. Phys. **59**, 1 (1987)]. At temperature  $T = 0$ , the charge on the central island of a conventional QD changes smoothly as a function of gate voltage, due to quantum fluctuations. However, for sufficiently large gate resistance charge fluctuations on the island can freeze out even at the degeneracy point, causing the charge on the island to change in sharp steps as a function of gate voltage. For  $R_g < R_C$  the steps remain smeared out by quantum fluctuations. The Coulomb blockade peaks in conductance display anomalous scaling at intermediate temperatures, and at very low temperatures a sharp step develops in the QD conductance.

The single electron transistor (SET) is one of the most basic mesoscopic devices: A conducting island or quantum dot is attached by tunnel barriers to two leads and a capacitively-coupled gate electrode sets the number of electrons on the dot. For low enough temperatures,  $T \ll E_C$ , charge fluctuations of the dot are suppressed except when the gate is tuned to make two charge states nearly degenerate. At these “charge degeneracy points” the charge on the dot strongly fluctuates. For typical metallic SETs with a very large number of tunneling modes quantum fluctuations of the charge turn out to be suppressed at low temperatures [1, 2, 3]. For semiconducting SETs with *single mode junctions*, however, quantum fluctuations of the charge are important and broaden out the charging steps at low  $T$ : In the limit of vanishing level spacing,  $\delta\epsilon \rightarrow 0$  charge fluctuations are described by the two-channel Kondo model [4], while for  $T \ll \delta\epsilon$  one recovers the so-called “mixed valence” regime of the Anderson model [5].

In the above discussion we neglected the effect of Ohmic dissipation in the lead electrodes. While this has been extensively studied for SETs with a very large number of tunneling modes [2], there is much less known about the effects of dissipation in the Kondo regime: In a recent paper Le Hur showed that, assuming a continuum of quantum levels on the SET and a single tunnel mode, coupling to a dissipative bath drastically modifies the results of Ref. [4]: large enough dissipation drives a Kosterlitz-Thouless-type *phase transition* and leads to a complete suppression of charge fluctuations even at the degeneracy point [6, 7]. However, for most semiconducting devices the level spacing  $\delta\epsilon$  *cannot* be neglected in comparison to temperature, and spin fluctuations must also be considered. Here we shall therefore investigate the effects of dissipation at temperatures *far below* the level spacing on the dot,  $T \ll \delta\epsilon$ , a more realistic low-temperature limit for typical semiconductor SETs. As we

show below, a dissipation-induced quantum phase transition takes place for  $T \ll \delta\epsilon$  as well, although with different and more complicated properties due to the interplay of charge and spin fluctuations, and at a larger dissipation strength (gate resistance) than that needed for  $\delta\epsilon \rightarrow 0$  [6, 7, 8].

In this  $T \ll \delta\epsilon$  regime, the coupling of the quantum dot to the gate voltage is usually described by the Hamiltonian,

$$H_{\text{dot}} = E_C \left( \sum_{\sigma} d_{\sigma}^{\dagger} d_{\sigma} - n_g \right)^2, \quad (1)$$

where  $E_C \equiv e^2/2C_{\Sigma}$  denotes the charging energy, with  $C_{\Sigma}$  the total capacitance of the dot, and  $e$  the electron charge. We retain only one single-particle level  $d$ , and we assume that it is empty or singly occupied, depending on the dimensionless gate voltage,  $n_g$  [9]. Assuming weak coupling between the dot and the source and drain electrodes, charge transfer can be described within the tunneling approximation,

$$H_{\text{tun}} = V \sum_{\sigma} \int d\epsilon \left( \sum_{\sigma} d_{\sigma}^{\dagger} \psi_{\sigma}(\epsilon) + \text{h.c.} \right), \quad (2)$$

where  $\psi_{\sigma}(\epsilon)$  annihilates an appropriate linear combination of left and right lead electrons of energy  $\epsilon$  that hybridize with the dot state  $d_{\sigma}$ , and satisfies the anticommutation relation  $\{\psi_{\sigma}(\epsilon), \psi_{\sigma'}(\epsilon')\} = \delta_{\sigma\sigma'} \delta(\epsilon - \epsilon')$  [10]. Throughout this paper we assume that the quantum dot is close to symmetrical but our analysis carries over easily to asymmetrical dots as well.

Eqs.(1) and (2) are thought to provide a satisfactory description of the SET for  $T \ll \delta\epsilon$  for most experimental situations studied so far, including the Kondo regime [5]. However, Eq. (1) does not account for the *relaxation of electrostatic charges* in the nearby electrodes: in reality, when an electron tunnels into the dot, an electrostatic charge  $\delta Q = eC_g/C_{\Sigma}$  is also generated on the

gate. Transferring this charge from the outside world to the gate electrode through a shunt resistor requires time, and creates dissipation [11]. Consequently, tunneling between dot and leads will be suppressed by Anderson's orthogonality catastrophe. The simplest way to account for this shunt resistance is to add a term [1]

$$H_{\text{diss}} = \lambda \left( \hat{n} - \frac{2}{3} \right) \varphi, \quad (3)$$

where the operator  $\hat{n} \equiv \sum_{\sigma} d_{\sigma}^{\dagger} d_{\sigma}$  measures the number of electrons on the dot, and the bosonic field  $\varphi$  describes charge density excitations in the gate electrode participating in the screening process, with their imaginary time correlation function defined as  $\langle T_{\tau} \varphi(\tau) \varphi(0) \rangle = 1/\tau^2$  [12]. The dissipation strength  $\alpha \equiv \lambda^2/2$  can be estimated following the procedure of Ref. [13], and we find

$$\alpha = \frac{C_g^2 R_g}{4 C_{\Sigma}^2 R_Q}, \quad (4)$$

with  $R_g$  the low frequency resistance of the gate electrode, and  $R_Q = h/2e^2$  the resistance quantum. In the absence of dissipation, Eqs.(1) and (2) would predict that the average charge on the island changes smoothly from  $\langle \hat{n} \rangle \approx 0$  to  $\langle \hat{n} \rangle \approx 1$  around  $n_g \approx 1/2$  due to the presence of strong quantum fluctuations at  $n_g \approx 1/2$  [5]. As we shall see below, the properties of this transition may be dramatically modified for  $\alpha > 1/2$ , producing a *sharp step* in  $\langle \hat{n} \rangle$  at  $T = 0$ .

Since we shall focus on the vicinity of the charging step,  $n_g \approx 1/2$ , we restrict the Hilbert space to the empty state  $|0\rangle$  and the two singly-occupied states  $|\sigma\rangle \equiv d_{\sigma}^{\dagger}|0\rangle$  of the dot, and approximate the sum of Eqs. (1), (2) and (3) as

$$\tilde{H} = V \sum_{\sigma} (|\sigma\rangle\langle 0| \psi_{\sigma} + \text{h.c.}) - \Delta \hat{Q} + \lambda \hat{Q} \varphi, \quad (5)$$

where  $\Delta \equiv E_C(1 - 2n_g)$  measures the difference between the classical energy of the two charge states of the dot, the operator  $\hat{Q} \equiv (|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| - 2|0\rangle\langle 0|)/3$  measures the difference between the charge on the dot and its value at the charge degeneracy point, and  $\psi_{\sigma} = \int d\epsilon \psi_{\sigma}(\epsilon)$ .

To map out the complete phase diagram of the model, we bosonize the above Hamiltonian [14], and perform a renormalization group analysis using an operator product expansion method [15]. Although we use different methods, the derivation of the scaling equations is similar to that in Ref. [16]. In fact, a mapping between the present model and the one studied in Ref. [16] enables us to carry over much of the analysis as well. Intriguingly, a new exchange coupling  $j$  is also generated under scaling[16]:

$$H_K = \frac{j}{2} \sum_{i,\alpha,\beta} S^i (\psi_{\alpha}^{\dagger} \sigma_{\alpha\beta}^i \psi_{\beta}), \quad (6)$$

where the spin operators denote  $S^i = \frac{1}{2} \sum_{\alpha,\beta} |\alpha\rangle \sigma_{\alpha\beta}^i \langle\beta|$ , with  $\sigma^i$  the Pauli matrices. To lowest order, the scaling

equations read [17]

$$\frac{dv}{dl} = \left( \frac{1}{2} - \alpha \right) v + \frac{3j}{4} v + \dots, \quad (7)$$

$$\frac{dj}{dl} = j^2 + 2v^2 + \dots, \quad (8)$$

$$\frac{d\alpha}{dl} = -3\left(\frac{1}{2} + \alpha\right) v^2 + \dots, \quad (9)$$

$$\frac{d\tilde{\Delta}}{dl} = \tilde{\Delta} - \frac{3}{8}j^2 + v^2 + \dots, \quad (10)$$

where  $l = \ln a$  is the scaling variable, and  $v = Va^{1/2}$  is the dimensionless tunneling rate with  $a$  a microscopic time scale initially of the order of  $a_0 \sim 1/\delta\epsilon$ . The coupling  $\tilde{\Delta} \equiv a\Delta$  is the dimensionless splitting of the charge states, slightly renormalized by the dissipative coupling. Note that these equations are exact in the dissipative coupling  $\alpha$ , but they contain contributions in the small couplings  $v$  and  $j$  only up to second order.

Following Ref. [16], we first neglect the effect of the exchange coupling  $j$  and focus on the vicinity of the degeneracy point, defined through the condition  $\tilde{\Delta}(l \rightarrow \infty) = 0$ . As we shall see, this approximation describes much of the experimentally-relevant regions, because the Kondo temperature associated with spin fluctuations turns out to be usually quite small in the vicinity of the phase transition,  $\alpha \approx \alpha_c$ , even for single occupancy. Within this approximation we can solve the scaling equations analytically [18], and for large enough  $\alpha$  we find that as  $T \rightarrow 0$  the effective tunneling  $v$  scales to zero, while  $\alpha$  scales to a finite value  $\alpha_{\infty}$  larger than  $1/2$ . Thus, quantum fluctuations of the dot charge *vanish* as  $T \rightarrow 0$ . In the above approximation, the relation between the corresponding critical value of the parameter  $\alpha = \alpha_c^{(0)}$  and the level width  $\Gamma \approx 2\pi v^2 \delta\epsilon$  is determined by the equation

$$\frac{\Gamma}{2\pi \delta\epsilon} \approx v^2 = \frac{2}{3} \left( \alpha_c^{(0)} - \frac{1}{2} - \ln(\alpha_c^{(0)} + \frac{1}{2}) \right), \quad (11)$$

and is plotted in Fig. 1. For small values of  $v$  we find  $\alpha_c^{(0)}(v) = 1/2 + \sqrt{3}v + v^2 + \dots$ , and the transition is of Kosterlitz-Thouless type [19]: On the localized side,  $\alpha > \alpha_c^{(0)}$  (or  $\Gamma < \Gamma_c^{(0)}$ ), at the degeneracy point the height  $\delta G$  of Coulomb blockade peaks scales to zero as a power law [18],

$$\frac{\delta G(T)}{G_Q} \sim v^2 \sim \left( \frac{T}{\delta\epsilon} \right)^{2\alpha_{\infty}-1} \quad (12)$$

with  $G_Q$  the quantum conductance. On the metallic side, on the other hand, quantum fluctuations always dominate and preserve conductance even at  $T = 0$ , though near the transition the conductance shows a non-monotonic behavior:  $\delta G(T)$  first slowly decays and then starts to increase below a temperature  $T^*$  that vanishes exponentially as one approaches the phase transition,

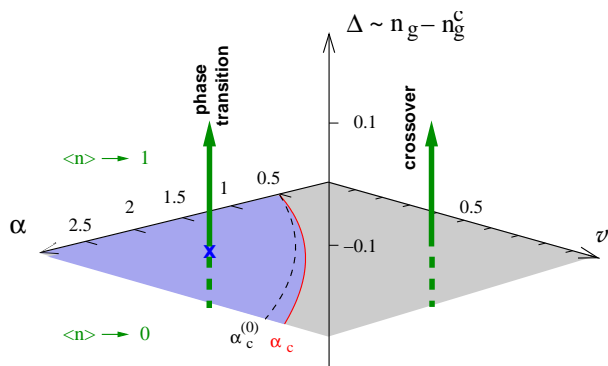


FIG. 1: Schematic phase diagram of the SET in the presence of dissipative coupling.  $\alpha_c$  denotes the critical value of  $\alpha$ , while  $\alpha_c^{(0)}$  is its value obtained by neglecting the generated exchange coupling  $j$ . For  $\alpha > \alpha_c$  there is a phase transition from  $\langle n \rangle = 0$  to  $\langle n \rangle = 1$ , while in the more familiar situation of weak dissipation there is a crossover.

$T^* \approx \delta\epsilon \exp\{-\pi/2(\alpha_c^{(0)2} - \alpha^2)^{1/2}\}$ , until finally a mixed valence state with a large conductance is formed at a temperature  $T^{**} \sim T^{*2}/\delta\epsilon \ll T^*$ . For the critical value of  $\alpha$ ,  $\delta G$  decays to zero logarithmically,

$$\delta G(T, \alpha = \alpha_c^{(0)}) \sim G_Q \frac{1}{\ln^2(\delta\epsilon/T)}. \quad (13)$$

The gate-voltage dependence of the expectation value of the charge on the dot,  $\langle \hat{Q} \rangle$ , can be directly detected by another SET or point contact electrostatically coupled to the dot. So far, we only considered the degeneracy point,  $\hat{\Delta}(\infty) = 0$ . However, in the localized phase,  $\Delta$  is a *relevant perturbation* and generates a first order phase transition at  $T = 0$  (see Fig. 1). Correspondingly,  $\langle \hat{Q} \rangle$  *jumps* [15] at the degeneracy point. At finite temperatures the width  $\delta n_g$  of this charge step becomes finite,  $\delta n_g \sim T/E_C$ .

On the delocalized side  $v$  scales to large values and  $\alpha \rightarrow 0$  so that the dissipation effectively disappears from the problem, and we recover the well-understood mixed valence fixed point of the Anderson model [5, 20]. At this fixed point the charge susceptibility is finite, and thus the charging step remains smeared out by quantum fluctuations even at  $T = 0$ . The width of the step is roughly determined by the scale  $T^{**}$ , which vanishes exponentially fast as we approach the transition. The corresponding  $T = 0$  phase diagram is sketched in Fig. 1.

We shall now discuss how the thus-far neglected exchange coupling  $j$  changes the above picture. Let us again first examine the degeneracy point,  $\hat{\Delta} \rightarrow 0$ , and let us assume that we are deep in the localized phase. Here charge fluctuations ( $v$ ) are irrelevant, however a small exchange coupling is still generated at the beginning of scaling, and produces a spin Kondo effect at the degeneracy point at some Kondo temperature  $T_K$ . The scale  $T_K$  is

typically small, but could likely be pushed into the measurable range by gradually opening up the quantum dot. Although the exchange coupling somewhat renormalizes  $v$ , it will typically not make it relevant. In other words, in the localized phase, at the degeneracy point one scales at low  $T$  to a special quantum state with large spin fluctuations but suppressed charge fluctuations [16, 21]. As a consequence, at the degeneracy point the conductance of the SET becomes of the order of  $G_Q$  at temperatures  $T \ll T_K$ , even though charge fluctuations of the SET are suppressed and  $v$  scales to zero. Despite the strong spin fluctuations,  $\Delta$  remains a relevant perturbation, and therefore, at  $T = 0$  there is still a *jump* in  $\langle \hat{n} \rangle(n_g)$ . Interestingly, on the  $\langle \hat{n} \rangle \approx 1$  side of this jump, the Kondo effect always develops, the spin is screened, and a Fermi liquid is formed for  $T \ll T_K$  [22]. Therefore the  $T = 0$  conductance is close to the quantum conductance. On the other side of the transition, however,  $\langle \hat{n} \rangle \approx 0$ , spin fluctuations are suppressed, and no Kondo effect takes place. Since the conductance is directly related to the scattering phase shifts and thus the occupation number  $\langle \hat{n} \rangle$  through the Friedel sum rule, the  $T = 0$  conductance in this localized phase must also have a *jump* as a function of  $n_g$ . This regime may be difficult to explore experimentally since we expect  $T_K$  to be small unless both  $\alpha$  and the tunnel coupling  $V$  are sufficiently large.

Exchange fluctuations play another important role too: they increase the value of  $v$  and, as a consequence, the true critical value of  $\alpha$  is somewhat renormalized,  $\alpha_c^{(0)} \rightarrow \alpha_c$ . It is very hard to estimate this critical value of  $\alpha_c$ , but for vanishingly small values of  $v$  we have been able to show that  $\alpha_c(v \rightarrow 0) = \alpha_c^{(0)}(v \rightarrow 0) = 1/2$ . For non-vanishing values of  $v$ , counter-intuitively,  $\alpha_c$  seems to be shifted to somewhat smaller values [23].

To obtain a more quantitative picture of the phase transition, we performed numerical renormalization group calculations. We used the Anderson Hamiltonian, Eqs. (1) and (2), and represented the field  $\varphi$  by fermionic density fluctuations [7]. The computed  $T = 0$  charging steps are shown in Fig. 2. The steps get sharper and sharper as we approach  $\alpha_c$ , and finally a sudden step appears for  $\alpha > \alpha_c$ .

The  $T = 0$  temperature AC conductivity is shown in Fig. 3 in the charge localized phase. The AC conductivity clearly shows the Kondo resonance, whose width remains finite as one approaches the charge step from the  $\langle n \rangle \approx 1$  side, while a *dip* of width  $\sim |\Delta|$  develops on the  $\langle n \rangle \approx 0$  side, consistent with the  $G(\omega = 0)$  conductance having a jump. Similar non-monotonic behavior should appear as a function of temperature.

To achieve a large dissipation strength we propose to make a shallow two-dimensional electron gas (2DEG) and fabricate a highly resistive metallic top gate electrode just above the dot, with a resistance larger than  $R_Q$ . According to our estimates, dissipation strength of the

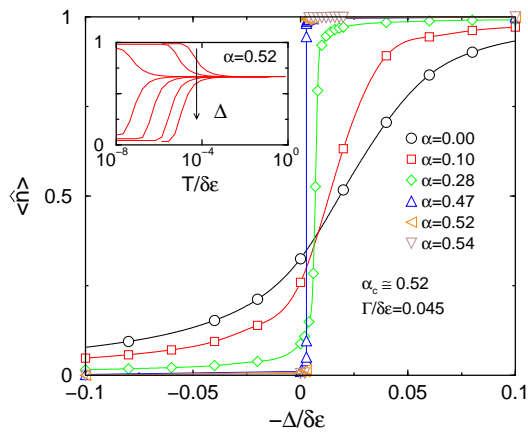


FIG. 2: Charging steps, computed using NRG for a relatively small hybridization. For these parameters  $T_K/\delta\epsilon \sim 10^{-10}$  at  $\alpha \approx \alpha_c$ . Inset: Temperature dependence of the occupation number  $\langle \hat{n} \rangle$  at the critical dissipation,  $\alpha \approx \alpha_c$ , for  $\Delta/\delta\epsilon = -0.0028800, -0.0028300, -0.0028200, -0.0028198, -0.0028190, -0.0028140, -0.0028000$  (bottom to top).

order of  $\alpha \approx 1$  can be reached in this way. The SET can be then tuned through the quantum phase transition by either continuously changing the tunneling  $V$ , or by depleting a second 2DEG positioned below the dot and thereby changing the total capacitance of the dot and hence the value of  $\alpha$ .

In summary, we have shown that sufficiently strong dissipation in the gate electrodes can drive the SET through a quantum phase transition into a state where charge degrees of freedom become localized while spin fluctuations lead to a Kondo effect. In this state both the conductance and the expectation value of the charge on the SET display a jump at temperature  $T = 0$ , while at higher tem-

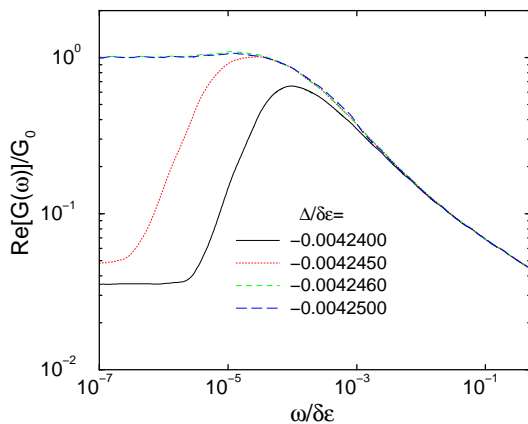


FIG. 3:  $T = 0$  AC conductivity of the SET in the localized phase for  $\alpha = 0.75$  and  $\Gamma/\delta\epsilon = 0.5$  ( $G_0 \sim G_Q$ ).

peratures an anomalous scaling of the Coulomb blockade peaks is predicted. We estimate that this quantum phase transition can be detected by coupling a highly resistive gate electrode to a SET in a shallow 2DEG.

We are grateful to P. Simon, K. Le Hur, Q. Si, O. Sauret, A. Zaikin and Y. Nazarov for valuable discussions. This research has been supported by NSF-MTA-OTKA Grant No. INT-0130446, Hungarian Grants Nos. T046303, NF061726, D048665, and T048782, the European 'Spintronics' RTN HPRN-CT-2002-00302, and at Stanford University by NSF CAREER Award DMR-0349354 and a Packard Fellowship. L.B. is a grantee of the János Bolyai Scholarship.

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and smaller values of  $\alpha$  are sufficient to localize charge fluctuations. Note that Eq. (7) is inappropriate in this Kondo regime, and a strong coupling analysis is needed to obtain the above picture.