MATHEMATICS

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The Department of Mathematics offers programs leading to the degrees of Bachelor of Science, Master of Science, and Doctor of Philosophy in Mathematics, and participates in the program leading to the B.S. in Mathematical and Computational Science. The department also participates in the M.S. and Ph.D. degree programs in Scientific Computing and Computational Mathematics and the M.S. degree program in Financial Mathematics.

ADVANCED PLACEMENT FOR FRESHMEN

Students of unusual ability in mathematics often take one or more semesters of college-equivalent courses in mathematics while they are still in high school. Under certain circumstances, it is possible for such students to secure both advanced placement and credit toward the bachelor's degree. A decision as to placement and credit is made by the department after consideration of the student's performance on the Advanced Placement Examination in Mathematics (forms AB or BC) of the College Entrance Examination Board, and also after consideration of transfer credit in mathematics from other colleges and universities.

The department does not give its own advanced placement examination. Students can receive either 5 or 10 units of advanced placement credit, depending on their scores on the Advanced Placement Examination. Entering students who have credit for two quarters of single variable calculus (10 units) are encouraged to enroll in Mathematics 51-53 in multivariable mathematics, or the honors version 51H-53H. These three-course sequences, which can be completed during the freshman year, supply the necessary mathematics background for most majors in science and engineering. They also serve as excellent background for major or minor degrees in Mathematics, or in Mathematical and Computational Science. Students who have credit for one quarter of single variable calculus should take Mathematics 42 in the Autumn Quarter and 51 in Winter Quarter. Options available in the Spring Quarter include Mathematics 52, 53, or 103. For proper placement, contact the Department of Mathematics.

UNDERGRADUATE PROGRAMS

BACHELOR OF SCIENCE

The following department requirements are in addition to the University's basic requirements for the bachelor's degree:

MAJORS

Students wishing to major in Mathematics must satisfy the following requirements:

1. Department of Mathematics courses totaling at least 49 units credit; such courses must be taken for a letter grade. For the purposes of this requirement, any course cross-listed with another department, such as Math 105 (Statistics 116) and Math 160A and 160B (Philosophy 160A and 160B), count as Department of Mathematics courses.
2. Additional courses taken from Department of Mathematics courses numbered 100 and above or from approved courses in other disciplines with significant mathematical content, totaling at least 15 units credit.
3. A Department of Mathematics adviser must be selected, and the courses selected under items '1' and '2' above must be approved by that adviser, acting under guidelines laid down by the department's Committee for Undergraduate Affairs. The Department of Mathematics adviser can be any member of the department's faculty.
4. To receive the department's recommendation for graduation, a student must have been enrolled as a major in the Department of Mathematics for a minimum of two full quarters, including the quarter immediately before graduation. In any case, students are strongly encouraged to declare as early as possible, preferably by the end of the sophomore year.

Students are normally expected to complete either the sequence 19, 20, 21 or the sequence 41, 42 (but not both). Students with an Advanced Placement score of at least 4 in BC math or 5 in AB math may receive 10 units credit and fulfill requirement '1' by taking at least 39 units of Department of Mathematics courses numbered 51 and above. Students with an Advanced Placement score of at least 3 in BC math or at least 4 in AB math may receive 5 units credit and fulfill requirement '1' by taking at least 44 units of Department of Mathematics courses numbered 42 and above.

Subject to approval of the department's Committee for Undergraduate Affairs, it may, in some cases, be permitted to count freshmen and sophomore seminar courses as part of a choice of courses under item '1'. Other variations of the course requirements laid down above (under items '1' and '2') may, in some circumstances, be allowed. (For example, students transferring from other universities may be allowed credit for some courses completed before their arrival at Stanford.) In all cases, approval must be obtained from the department's Committee for Undergraduate Affairs. Application for such approval should be made through the student's Department of Mathematics adviser.

It is to be emphasized that the above regulations are minimum requirements for the major—students contemplating graduate work in mathematics are strongly encouraged to include the courses 116, 120, 121, and 171 in their selection of courses, and in addition, take at least three Department of Mathematics courses over and above the minimum requirements laid out under items '1' and '2' above, including at least one 200-level course. Such students are also encouraged to consider the possibility of taking the honors program, discussed below.

To help develop a sense of the type of course selection (under items '1' and '2' above) which would be recommended for math majors with various backgrounds and interests, see the following examples. These represent only a few of a very large number of possible combinations of courses which could be taken in fulfillment of the Mathematics major requirements:

Example 1—A "General" program (a balanced program of both pure and applied components, without any particular emphasis on any one field of mathematics or applications) as follows:

Either Mathematics 19, 20 and 21, or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 104; 106; 109; 110; 115

Plus any selection of at least eight of the following courses, including three Department of Mathematics courses: Mathematics 105 (Statistics 116), 108, 131, 132, 143, 147, 148, 152, 161, 173; Physics 41, 43, 45; Computer Science 137; Economics 50. (These specific courses from other departments are only meant as an example. There are many suitable courses in several departments which can be taken to fulfill part or all of requirement 2.)

Example 2—A "Theoretical" program (recommended for those contemplating possible later graduate work (see, also, the discussion of the honors program below), providing an introduction to the main areas of mathematics both broader and deeper than the general program outlined above).

Either Mathematics 19, 20, and 21, or 41 and 42 (or satisfactory Advanced Placement credit)

Either the sequence 51, 52, 53, or the sequence 51H, 52H, 53H; 106 or 116; 114; 120; 171

Plus nine or more of the following courses, including at least one from each group: algebra sequence 121, 152, 153, 156; analysis sequence 131, 132, 151, 174A,B (formerly 134A,B), 175; geometry/topology sequence 143, 145, 147, 148, 173; logic and set theory sequence 160A,B, 161.

In addition, those contemplating eventual graduate work in mathematics should seriously consider including at least one graduate-level math course such as Mathematics 205A, 206A, or 210A. Such students should also consider the possibility of entering the honors program.

(Students taking 51, 52, 53 rather than 51H, 52H, 53H should consider taking 113 before attempting 114.)

*Example 3**—An "Applied Mathematics" program:

Either Mathematics 19, 20, and 21; or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 103; 105 (Statistics116); 106; 108; 109; 110; 115; 131

Plus at least 15 units of additional courses in Applied Mathematics, including, for example, suitable courses from the departments of Physics, Computer Science, Economics, Engineering, and Statistics.

* Students with interests in applied mathematics, but desiring a broader-based program than the type of program suggested in Example 3, including significant computational and/or financial and/or statistical components, are encouraged to also consider the Mathematics and Computational Sciences program.

MINORS

To qualify for the minor in mathematics, a student should successfully complete, for a letter grade, at least six Department of Mathematics courses numbered 51 or higher, totaling a minimum of 24 units. It is recommended that these courses include either the sequence 51, 52, 53 or the sequence 51H, 52H, 53H.

HONORS PROGRAM

The honors program is intended for students who have strong theoretical interests and abilities in mathematics. The goal of the program is to give students a thorough introduction to the main branches of mathematics, especially analysis, algebra, and geometry. Through the honors thesis, students may be introduced to a current or recent research topic, although occasionally more classical projects are encouraged. The program provides an excellent background with which to enter a master's or Ph.D. program in mathematics. Students successfully completing the program are awarded a B.S. in Mathematics with Honors.

It is recommended that the sequence 51H, 52H, 53H be taken in the freshman year. Students who have instead taken the sequence 51, 52, 53 in their freshman year may be permitted to enter the honors program, but such entry must be approved by the Department of Mathematics Committee for Undergraduate Affairs.

To graduate with a B.S. in Mathematics with Honors, the following conditions apply in addition to the usual requirements for math majors:

1. The selection of courses under items '1' and '2' above must include all the math courses 106 or 116, 120, 171 and also must include seven or more additional courses, with at least one from each of the groups: algebra sequence 114, 121, 152, 153, 156; analysis sequence 131, 132, 151, 174A, 174B (formerly 134A, 134B), 175, 176; geometry/topology sequence 143, 145, 147, 148, 173; logic and set theory sequence 160A, 160B, 161.
2. Students in the honors program must write a senior thesis. In order to facilitate this, the student must, by the end of the junior year, choose an undergraduate thesis adviser from the Department of Mathematics faculty, and map out a concentrated reading program under the direction and guidance of the adviser. During the senior year, the student must enroll in Mathematics 197 for a total of 6 units (typically spread over two quarters), and work toward completion of the thesis under the direction and guidance of the thesis adviser. The thesis may contain original material, or be a synthesis of work in current or recent research literature. The 6 units of credit for Mathematics 197 are required in addition to the course requirements laid out under items '1' and '2' above and in addition to all other requirements for math majors.

In addition to the minimum requirements laid out above, it is strongly recommended that students take at least one graduate-level course (that is, at least one course in the 200 plus range). Mathematics 205A, 206A, and 210A are especially recommended in this context.

Students with questions about the honors program should see the Director of Undergraduate Advising.

BACHELOR OF SCIENCE IN MATHEMATICAL AND COMPUTATIONAL SCIENCE

The Department of Mathematics participates with the departments of Computer Science, Management Science and Engineering, and Statistics in a program leading to a B.S. in Mathematical and Computational Science. See the "Department of Mathematical and Computational Science" section of this bulletin.

GRADUATE PROGRAMS

MASTER OF SCIENCE

The University's basic requirements for the master's degree are discussed in the "Graduate Degrees" section of this bulletin. Students entering Stanford in 2001 or later should pay particular attention to the University's course requirements for graduate degrees. The following are specific departmental requirements:

Candidates must complete an approved course program of 45 units of courses beyond the department requirements for the B.S. degree, of which at least 36 units must be Mathematics Department courses. The mathematics courses must include at least 18 units numbered 200 or above. The candidate must have a grade point average (GPA) of 'B' over all course work taken in mathematics, and a GPA of 'B' in the 200-level courses considered separately. Course work for the M.S. degree must be approved during the first quarter of enrollment in the program by the department's Director of Graduate Studies.

For the degree of M.S. in Computer Science, see the "Computer Science" section of this bulletin.

TEACHING CREDENTIALS

For information concerning the requirements for teaching credentials, see the "School of Education" section of this bulletin or address inquiries to Credential Secretary, School of Education.

MASTER OF ARTS IN TEACHING (MATHEMATICS)

In cooperation with the School of Education, the department offers a program leading to a Master of Arts in Teaching (Mathematics). It is intended for candidates who have a teaching credential or relevant teaching experience and wish to strengthen their academic preparation. Detailed requirements are outlined under the "School of Education, Master of Arts in Teaching" section of this bulletin.

DOCTOR OF PHILOSOPHY

The University's basic requirements for the doctorate (residence, dissertation, examinations, and so on) are discussed in the "Graduate Degrees" section of this bulletin. Students entering Stanford in 2001 or later should pay particular attention to the University course unit requirements for advanced degrees. The following are specific departmental requirements.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. In accordance with University requirements, Ph.D. students entering in 2001 or later must complete a total of 135 course units beyond the bachelor's degree. These courses should be Department of Mathematics courses or approved courses from other departments. The course program should display substantial breadth in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 'B' or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the University oral examination and pass a reading examination in one foreign language, chosen from French, German, or Russian.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For the Ph.D. degree in Computer Science, see the "Computer Science" section of this bulletin.

For further information concerning degree programs, fellowships, and assistantships, inquire of the academic associate of the department.

APPLIED MATHEMATICS OPTION

This option differs from the standard Ph.D. program in that qualifying examinations in more applied areas are substituted for the regular qualifying examinations. Also, the courses Mathematics 220 (basic methods in partial differential equations) and Computer Science 237 (numerical methods) are a required part of the curriculum in the first year. Students are required to take 18 units of graduate-level courses in computer science and applied areas such as financial mathematics, fluid mechanics, operations research, or statistics.

Ph.D. MINOR

The student should complete both of the following:*

1. Mathematics 106 or 116, 131, 132
2. Mathematics 113, 114, 120 or 152

These courses may have been completed during undergraduate study, and their equivalents from other universities are acceptable.

In addition, the student should complete 21 units of 200-level courses in mathematics. These must be taken at Stanford and approved by the Department of Mathematics' Ph.D. minor adviser.

* A third coherent sequence designed by the student, subject to the approval of the graduate committee, may be considered as a substitute for items '1' or '2'.

COURSES

(WIM) indicates that the course meets the Writing in the Major requirements.

INTRODUCTORY AND UNDERGRADUATE

The department offers two sequences of introductory courses in single variable calculus.

1. Mathematics 41, 42 present single variable calculus. Differential calculus is covered in the first quarter, integral calculus in the second.
2. Mathematics 19, 20, 21 cover the material in 41, 42 in three quarters instead of two.

There are options for studying multivariable mathematics:

1. Mathematics 51, 52, 53 cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize application. Mathematics 51 covers differential calculus in several variables and introduces matrix theory and linear algebra, 52 covers integral calculus in several variables and vector analysis, 53 studies further topics in linear algebra and applies them to the study of ordinary differential equations. This sequence is strongly recommended for incoming freshmen with 10 units of advanced placement credit.
2. Mathematics 51H, 52H, 53H cover the same material as 51, 52, 53, but with more emphasis on theory and rigor.

The introductory course in modern algebra is Linear Algebra (103 or 113). There are no formal prerequisites for these courses, but appropriate mathematical maturity is expected. Much of the material in 103 is covered in the sequence 51, 52, 53.

19,20,21. Calculus—The content is the same as the sequence 41 and 42 described below, over three quarters rather than two.

19. Calculus—GER:2c
3 units, Aut (Moore)
Win (Staff)

20. Calculus—Continuation of 19. Prerequisite: 19. GER:2c
3 units, Win (Moore)
Spr (Staff)

21. Calculus—Continuation of 20. Prerequisite: 20. GER:2c
4 units, Spr (Moore)

41,42.—Three large lecture classes per week plus two classes in small sections.

41. Single Variable Calculus—Introduction to differential and integral calculus of functions of one variable. Topics: review of elementary functions including exponentials and logarithms, rates of change, and the derivative. Introduction to the definite integral and integration. Prerequisites: algebra, trigonometry. GER:2c
5 units, Aut (Bray)

42. Single Variable Calculus—Continuation of 41. Methods of symbolic and numerical integration, applications of the definite integral, introduction to differential equations. Infinite series. Prerequisite: 41 or equivalent. GER:2c
5 units, Aut (Gibou)
Win (Bray)

51,52,53. Multivariable Mathematics—Recommended for incoming freshmen with 10 units of Advanced Placement credit, and for those interested in science, engineering, or economics. Provides an integrated treatment of multivariable calculus, linear algebra, and ordinary differential equations including numerical algorithms and computer experiments. Applications are stressed.

51. Linear Algebra and Differential Calculus of Several Variables—Geometry and algebra of vectors, systems of linear equations, matrices, vector valued functions and functions of several variables, partial derivatives, gradients, chain rule in several variables, vector fields, optimization. Prerequisite: completion of 21, 42, or a score of at least 4 on the BC Advanced Placement Examination or 5 on the AB Advanced Placement Examination, or consent of the instructor. GER:2c
5 units, Aut (Brumfiel, Carlsson, S. Levandosky, Maclagan,Vakil)
Win (R. Cohen, Gamburd, Zerner)
Spr (Bray)
Sum (Staff)

52. Integral Calculus of Several Variables—Iterated integrals, line and surface integrals, vector analysis with applications to vector potentials and conservative vector fields, physical interpretations. Divergence theorem and the theorems of Green, Gauss, and Stokes. Prerequisite: 51.
5 units, Aut (Stacey)
Win (Goldstein, S. Levandosky)
Spr (Lee)

53. Ordinary Differential Equations with Linear Algebra—Linear ordinary differential equations, applications to oscillations, matrix methods including determinants, eigenvalues and eigenvectors, matrix exponentials, systems of linear differential equations with constant coefficients, stability of non-linear systems and phase plane analysis, numerical methods, Laplace transforms. Similar to 130, integrated with topics from linear algebra (103). Prerequisite: 51.
5 units, Aut (Schlichtkrull)
Win (Dembo)
Spr (J. Levandosky, Zerner)

51H,52H,53H. Honors Calculus—For prospective math majors in the honors program and students from other areas of science or engineering who have a strong mathematics background. Three quarter sequence covers the material of 51, 52, 53, and additional advanced calculus and ordinary and partial differential equations. Provides a unified treatment of multi-variable calculus, linear algebra, and differential equations with a different order of topics and emphasis from standard courses. Students should know one-variable calculus and have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on BC Advanced Placement Exam, or consent of the instructor. Recommended: complete at least the first two quarters. 51H satisfies GER:2c

51H. *5 units, Aut (White)*
52H. *5 units, Win (White)*
53H. *5 units, Spr (S. Levandosky)*

84Q. Stanford Introductory Seminar: Finite Mathematics, Codes, and Cryptography—Preference to sophomores.
3 units, Win (Carlsson)

85Q. Stanford Introductory Seminar: Calculus of Variations—Preference to sophomores.
3 units, Win (Mazzeo)

87Q. Stanford Introductory Seminar: Mathematics of Knots, Braids, Links, and Tangles—Preference to sophomores.
3 units, Spr (Brumfiel)

88Q. Stanford Introductory Seminar: The Mathematics of the Rubik's Cube—Preference to sophomores.
3 units, Win (Bump)

89Q. Stanford Introductory Seminar: The Euler Characteristic in Geometry and Algebra—Preference to sophomores.
3 units, Aut (de Silva)

UNDERGRADUATE AND GRADUATE

Unless explicitly stated, there are no prerequisites for the courses listed below. Where a prerequisite is stated, it may be waived by the instructor.

103. Matrix Theory and Its Applications—Linear algebra and matrices, emphasizing the computational and algorithmic aspects and the scientific problems in which matrix theory is applied. Solution of linear equations. Linear spaces and matrices. Orthogonal projection and least squares. Determinants, eigenvalues, and eigenvectors. GER:2c
3 units, Aut (Bernstein, Goldstein)
Win (Kiem, Maclagan, Stacey)
Spr (Schlichtkrull, Stacey)
Sum (Staff)

104. Matrix Theory and Its Applications—Continuation of 103. Positive definite matrices, extremum problems, computations with matrices, elements of linear programming, and Markov chains.
not given 2001-02

105. Theory of Probability—(Enroll in Statistics 116.)
3-5 units, Aut (Siegmund)
Spr (Taylor)
Sum (Staff)

106. Functions of a Complex Variable—Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy integral formula, residues, elementary conformal mappings. Prerequisite: 52.
3 units, Aut (Lindenstrauss)
Sum (Staff)

108. Introduction to Combinatorics and Its Applications—Topics: graphs, trees (Cayley's Theorem, application to phylogony), eigenvalues, basic enumeration (permutations, Stirling and Bell numbers), recurrences, generating functions, basic asymptotics. Prerequisites: 51 or 103 or equivalent.
3 units, Aut (Diaconis)

109. Applied Group Theory—Applications of the theory of groups. Topics: elements of groups theory, groups of symmetries, matrix groups, group actions, and applications to combinatorics and computing. Applications: rotational symmetry groups, the study of the Platonic solids, crystallographic groups and their applications in chemistry and physics. (WIM)
3 units, Win (Kiem)

110. Applied Number Theory and Field Theory—Introduction to number theory and its applications to modern cryptography. Topics: congruences, finite fields, primality testing and factorization, public key cryptography, error correcting codes, and elliptic curves, with emphasis throughout on algorithms. (WIM)
3 units, Spr (Brumfiel)

112. Symmetric Functions and Algebraic Combinatorics—Unified treatment of topics in classical enumeration via the study of symmetric polynomials. Classical symmetric functions, Schur functions, Young tableaux, Schensted correspondence, character theory of the symmetric group, introduction to random matrix theory. Prerequisite: 109 or 120, or equivalent.
not given 2001-02

113. Linear Algebra and Matrix Theory—Algebraic properties of matrices and their interpretation in geometric terms. The relationship between the algebraic and geometric points of view and matters fundamental to the study and solution of linear equations. Topics: linear equations, vector spaces, linear dependence, bases and coordinate systems; linear transformations and matrices; similarity; eigenvectors and eigenvalues; diagonalization.
3 units, Aut (Kiem)
Win (Lindenstrauss)

114. Linear Algebra and Matrix Theory—Continuation of 113. Deeper study of 113 topics plus additional topics from invariant subspaces, canonical forms of matrices; minimal polynomials and elementary divisors; vector spaces over arbitrary fields; inner products; Jordan normal forms; Hermitian and unitary matrices; multilinear algebra; applications.
3 units, Win (Katznelson)
Spr (Milgram)

115. Functions of a Real Variable—The development of real analysis in Euclidean space: sequences and series, limits, continuous functions, derivatives, integrals. Basic point set topology. Honors math majors and students who intend to do graduate work in mathematics should take 171. Prerequisite: 51.
3 units, Aut (Katznelson)
Win (Ornstein)
Sum (Staff)

116. Complex Analysis—Analytic functions, Cauchy integral formula, power series and Laurent series, calculus of residues and applications, conformal mapping, analytic continuation, introduction to Riemann surfaces, Fourier series and integrals. Applications of complex analysis to electrostatics, hydrodynamics, and theoretical physics. Prerequisite: 52.
3 units, Win (J. Li)

120. Modern Algebra I—Basic structures in algebra: groups, rings, and fields. Elements of group theory: permutation groups, finite Abelian groups, p-groups, Sylow theorems. Polynomial rings, principal ideal domains, unique factorization domains. (WIM)
3 units, Aut (Rubin)

121. Modern Algebra II—Continuation of 120. Fields of fractions. Solvable and simple groups. Elements of field theory and Galois theory. Prerequisite: 120.
3 units, Win (Rubin)

130. Ordinary Differential Equations—Separable, exact, and linear first order equations; second order linear equations; series solutions, numerical solutions; Laplace transforms; systems of equations. Students with some background in matrix theory should take 53, which integrates linear algebra with differential equations. Pre- or corequisite: 51 or consent of instructor.
not given 2001-02

131. Partial Differential Equations I—First-order equations, classification of second-order equations. Initial-boundary value problems for heat, wave, and related equations. Separation of variables, eigenvalue problems, Fourier series, existence and uniqueness questions. Prerequisite: 53 or 130 or equivalent.
3 units, Aut (Levy)
Win (Lindenstrauss)

132. Partial Differential Equations II—Initial and initial-boundary value problems in infinite domains. Fourier transforms. Boundary value problems for Laplace equation. Bessel functions and Legendre polynomials. Prerequisite: 131 or equivalent.
3 units, Win (S. Levandosky)
Spr (J. Levandosky)

135. Nonlinear Dynamics and Chaos—Topics: one- and two-dimensional flows, bifurcations, phase plane analysis, limit cycles and their bifurcations. Lorenz equations, fractals and strange attractors. Prerequisite: 51 and 53 or equivalent.
3 units, Win (Levy)

143. Differential Geometry—Geometry of curves and surfaces in three-space and higher dimensional manifolds. Parallel transport, curvature, and geodesics. Surfaces with constant curvature. Minimal surfaces.
3 units, Aut (Ni)

145. Algebraic Geometry—Real algebraic curves, Hilbert's Nullstellensatz, complex affine and projective curves, Bezout's theorem, the degree/genus formula, Riemann surfaces, Riemann-Roch theorem. Prerequisites: 106 or 116 and either 120 or 109. Recommended: familiarity with surfaces, e.g., from one of 143, 147, 148, or 173.
3 units, Spr (R. Cohen)

147. Differential Topology—Smooth manifolds, transversality, Sards' theorem, embeddings, degree of a map, Borsuk-Ulam theorem, Hopf degree theorem, Jordan Curve Theorem. Prerequisite: 115 or 171.
3 units, Spr (Kerckhoff)

148. Algebraic Topology—Fundamental group, covering spaces, Euler characteristic, classification of surfaces, knots. Prerequisite: 109 or 120.
given 2002-03

151. Introduction to Probability Theory—Counting; axioms of probability; conditioning and independence; expectation and variance; discrete and continuous random variables and distributions; joint distributions and dependence; central limit theorem and laws of large numbers. Prerequisite: 52 or consent of instructor.
3 units, Win (Lee)

152. Elementary Theory of Numbers—Euclid's algorithm, fundamental theorems on divisibility; prime numbers, congruence of numbers; theorems of Fermat, Euler, Wilson; congruence of first and higher degrees; Lagrange's theorem, its applications; residues of power; quadratic residues; introduction to the theory of binary quadratic forms.
3 units, Spr (Bump)

153. Combinatorics—Topics in Ramsey's theorem, generating functions, partition functions, and in number theory (sums of integers and van der Waerden's theorem). Recommended: general background in algebra, analysis, and some number theory.
not given 2001-02

156. Group Representations—Designed for undergraduates. Experimental, primarily examining symmetries on objects such as vector spaces ("group representations"), geometric objects ("geometric group actions"), and discrete sets (combinatorics). Topics: group representations and their characters, classification of permutation group representations using partitions and Young tableaus, group actions on sets and the Burnside ring, and spherical space forms. Prerequisites: basic knowledge of linear algebra (51 and 53, or 103 or 113) and group theory (109 or 120).
3 units, Win (Milgram)

160A. First-Order Logic—(Enroll in Philosophy 160A.)
4 units, Win (Visser)

160B. Computability and Logic—(Enroll in Philosophy 160B.)
4 units, Spr (Rathjen)

161. Set Theory—Informal and axiomatic set theory: sets, relations, functions, and set-theoretical operations. The Zermelo-Fraenkel axiom system and the special role of the axiom of choice and its various equivalents. Well-orderings and ordinal numbers; transfinite induction and transfinite recursion. Equinumerosity and cardinal numbers; Cantor's Alephs and cardinal arithmetic. Open problems in set theory.
given 2002-03

162. Philosophy of Mathematics—(Enroll in Philosophy 162.)
4 units, not given 2001-02

171. Fundamental Concepts of Analysis—Recommended for math majors and required of honors math majors. Similar to 115 but altered content and more theoretical orientation. Properties of Riemann integrals, continuous functions and convergence in metric spaces; compact metric spaces, basic point set topology. Prerequisites: either 51, 52, 53; or 51H, 52H, 53H. (WIM)
3 units, Aut (Goldstein)
Spr (Maclagan)

173. Analysis on Manifolds—Differentiable manifolds, tangent space, submanifolds, implicit function theorem, differential forms, vector and tensor fields. Frobenius' theorem, DeRham theory. Prerequisite: 52 or 52H.
3 units, Win (Yang)

174A,B. Honors Analysis—(Formerly 134A,B.) Primarily for students planning graduate work in mathematics or physics and for honors math majors, but of use and interest to other majors at ease with rigorous proofs and qualitative discussion. Coherent, mathematically sophisticated presentation of the basic areas in classical real analysis. Emphasis is on ordinary and partial differential equations. Prerequisite: 53H or 171, or consent of instructor.
174A. *3 units, Win (P. Cohen)*
174B. *3 units, Spr (Mazzeo)*

175. Elementary Functional Analysis—Linear operators on Hilbert space. Spectral theory of compact operators; applications to integral equations. Elements of Banach space theory. Prerequisite: 115 or 171.
3 units, Spr (Lindenstrauss)

176. Spectral Geometry—Relations between the geometry of a region and eigenvalues of the Laplace operator; basic properties of the Laplace and heat operators; "when can you hear the shape of a drum?" Prerequisites: 51, 52, 53 or equivalent.
not given 2001-02

180. Introduction to Financial Mathematics—Financial derivatives: contracts and options. Hedging and risk management. Arbitrage, interest rate, and discounted value. Geometric random walk and Brownian motion as models of risky assets. Initial boundary value problems for the heat and related partial differential equations. Self-financing replicating portfolio. Black-Scholes pricing of European options. Dividends. Implied volatility. Optimal stopping and American options. Prerequisite: 53. Corequisites: 131, 151 or Statistics 116.
3 units, Aut (Dembo)

197. Senior Honors Thesis
1-6 units, Aut, Win, Spr (Staff)

199. Independent Work—Undergraduates pursue a reading program; topics limited to those not in regular department course offerings. Credit can fulfill the elective requirement for math majors. Approval of Undergraduate Affairs Committee is required to use credit for honors majors area requirement. Consult the academic associate for help in finding an adviser.
3 units, Aut, Win, Spr (Staff)

PRIMARILY FOR GRADUATE STUDENTS

201. Practical Training—Registration restricted to students in the M.S. Degree Program in Financial Mathematics. Students obtain employment in a relevant industrial or research activity, chosen to enhance their professional experience, and consistent with the MSFM degree program. Prerequisite: consent of adviser.
1-3 units, Aut, Win, Spr, Sum (Staff)

205A. Real Analysis—Basic measure theory and the theory of Lebesgue integration. Prerequisite: 171 or equivalent.
3 units, Aut (Simon)

205B,C. Real Analysis—Point set topology, basic functional analysis, Fourier series, and Fourier transform. Prerequisites: 171 and 205A or equivalent.
3 units, Win, Spr (P. Cohen)

206A. Complex Analysis—Complex integration. Cauchy's theorem, Residue theorem, argument principle, power series, conformal mapping. Prerequisite: 171 or equivalent.
3 units, Aut (Ni)

206B. Complex Analysis—Riemann mapping theorem, product developments, entire functions, elliptic functions, Dirichlet problem, Picard's theorem. Prerequisites: 171 and 206A or equivalent.
3 units, Win (Ni)

210A,B,C. Modern Algebra—Groups, rings, and fields, Galois theory, ideal theory. Introduction to algebraic geometry and algebraic number theory. Representations of groups and non-commutative algebras, multilinear algebra. Prerequisite: 120 or equivalent.
210A. *3 units, Aut (Bump)*
210B. *3 units, Win (Bump)*
210C. *3 units, Spr (Milgram)*

216A,B. Introduction to Algebraic Geometry—Basic notions in algebraic geometry. Algebraic curves, algebraic varieties, sheaves, cohomology, Riemann-Roch theorem. Classification of algebraic surfaces, moduli spaces, deformation theory and obstruction theory, the notion of schemes.
216A. *3 units, Win (J. Li)*
216B. *3 units, Spr (J. Li)*

217A. Differential Geometry—Smooth manifolds and submanifolds, tensors and forms, Lie and exterior derivative, deRham cohomology, distributions and the Frobenius theorem, vector bundles, connection theory, parallel transport and curvature, affine connections, geodesics and the exponential map, connections on the principal frame bundle. Prerequisite: 173 or equivalent.
3 units, Aut (Schoen)

217B. Differential Geometry—Riemannian manifolds, Levi-Civita connection, Riemann curvature tensor, Riemannian exponential map and geodesic normal coordinates, Jacobi fields, completeness, spaces of constant curvature, bi-invariant metrics on compact Lie groups, symmetric and locally symmetric spaces, equations for Riemannian submanifolds and Riemannian submersions. Prerequisite: 217A.
3 units, Win (Ni)

217C. Differential Geometry—First and second variation of arc length, index form and variational theory of geodesics, comparison theorems and consequences for manifolds of positive and negative curvature, almost complex manifolds and integrability, Hermitian and Kaehler metrics, connections on complex vector bundles and Chern classes, Hodge theory, vanishing theorems in the Riemannian and Kaehler settings.
3 units, Spr (Yang)

220A,B,C. Partial Differential Equations of Applied Mathematics—Greens functions, integral transforms, variational and distribution theoretic methods for the analysis of differential and integral equations, with illustrative examples. Prerequisite: some familiarity with differential equations and functions of a complex variable.
220A. *3 units, Aut (J. Levandosky)*
220B. *3 units, Win (J. Levandosky)*
220C. *3 units, Spr (Papanicolaou)*

222A. Computational Methods for Fronts, Interfaces, and Waves—High-order methods for multidimensional systems of conservation laws and Hamilton-Jacobi equations (central schemes, discontinuous Galerkin methods, relaxation methods). Level set methods and fast marching methods. Computation of multi-valued solutions. Multi-scale analysis, including wavelet-based methods. Boundary schemes (perfectly matched layers). Examples from (but not limited to) geometrical optics, transport equations, reaction-diffusion equations, imaging, and signal processing.
3 units, Aut (Levy)

224. Ten Great Algorithms of Scientific Computation—Ten basic algorithms that have revolutionized scientific computing: fast fourier transform; conjugate gradient method (and Krylov Subspace Methods); quick sort; metropolis algorithm; fast multipole algorithm; singular value decomposition (and matrix decompositions); simplex method. Their practical and theoretical aspects.
3 units, Win (Diaconis, Golub)

226. Capillary Surfaces—Introduction to the modern theory of capillary free surface interfaces in the contexts of differential geometry and the calculus of variations. Emphasis is on the interactions between the formal mathematics and physical reality. Results from NASA space experiments. Topics: global estimates on configuration of solution surfaces, discontinuous dependence of solutions on boundary data, symmetry breaking, criteria for existence, failure of existence under physical conditions, failure of uniqueness under conditions for which solutions exist, etc.
3 units, Win (Finn)

228A,B. Introduction to Ergodic Theory—Measure preserving transformations and flows, ergodic theorems, mixing properties, spectrum, Kolmogorov automorphisms, entropy theory. Examples. Classical dynamical systems, mostly geodesic and horocycle forms on homogeneous spaces of SL(2,R). Prerequisites: 205A,B.
228A. *3 units, Aut (Ornstein)*
228B. *3 units, Win (Ornstein)*

230A. Theory of Probability—(Enroll in Statistics 310A.)
3-4 units, Aut (Diaconis)

230B. Theory of Probability—(Enroll in Statistics 310B.)
3-4 units, Win (Dembo)

230C. Theory of Probability—(Enroll in Statistics 310C.)
3-4 units, Spr (Lai)

233. Probabilistic Methods in Analysis—Proofs and constructions in analysis obtained from basic results in Probability Theory and a "probabilistic way of thinking." Topics: Rademacher functions, Gaussian processes, entropy.
not given 2001-02

234. Large Deviations—(Same as Statistics 374.) Combinatorial estimates and the method of types. Large deviation probabilities for partial sums and for empirical distributions, Cramer's and Sanov's theorems and their Markov extensions. Applications in statistics, information theory, and statistical mechanics. Prerequisite: 230A or Statistics 310.
3 units, not given 2001-02

235A. Selected Topics in Ergodic Theory—Topics from the Kolmogorov-Sinai theory of entropy; the isomorphism theorem for Bernoulli shifts and Bernoulli flows; K-automorphisms applications to mechanical systems, and automorphisms of compact groups.
3 units, Spr (Ornstein)

236. Introduction to Stochastic Differential Equations—Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Random walk approximation of diffusions. Prerequisite: basic probability and differential equations.
3 units, Win (Papanicolaou)

237. Asymptotic Analysis of Stochastic Equations—Ergodic properties and stability of solutions of stochastic differential equations. Small parameter asymptotics and connections with partial differential equations. Selected applications from physics, biology, and economics.
3 units, Spr (Papanicolaou)

240. Computation and Simulation in Finance—Monte Carlo, finite difference, tree, and transform methods for the numerical solution of partial differential equations in finance. Emphasis is on derivative security pricing.
3 units, Spr (Lee)

241. Mathematical Finance—(Enroll in Statistics 250.)
3 units, Win (Papanicolaou)

244. Riemann Surfaces—Compact Riemann surfaces: topological classifications, Hurwitz' formula. Riemann-Roch formula, uniformization theorem. Abel's theorem, Jacobian varieties. Some elements of harmonic analysis are developed with applications. Emphasis is on methods which are generally applicable to algebraic curves.
3 units, Aut (Kiem)

245. Topics in Algebraic Geometry—Moduli spaces of algebraic curves and related topics.
not given 2001-02

248A. Algebraic Number Theory—Introduction to algebraic number theory: the arithmetic of local and number fields, and a brief introduction to class field theory. Prerequisite: 210A,B or equivalent.
3 units, Aut (Rubin)

249A. Topics in Representation Theory and Number Theory—Recent and current research in algebraic number theory and arithmetic geometry. Topics chosen from: elliptic curves, Iwasawa theory, special values of L-functions, and class number formulas.
3 units, Win (Rubin)

249B. Topics in Representation Theory and Number Theory—Topics: Selberg trace formula, the spectral theory of quotients of the upper half plane by discrete groups, and related representation theory and number theory.
3 units, Spr (Bump)

253. Regularity of Sets and Mappings—For students interested in any area of analysis. Topics: Lipschitz functions; C^∞ functions; Sobolev functions; various regularity and extension theorems including Rademacher, Kirzbraun, Whitney, Sard, C^1-Sard. Critical sets of real-analytic, complex analytic functions. Affine approximation properties of subsets of R^n, including a discussion of rectifiability and non-rectifiability, structure theorem, and Reifenberg's topological disc theorem.
not given 2001-02

254A,B. Ordinary Differential Equations—Qualitative theory of ordinary differential equations, analytic and geometric methods. Topics from the stability and perturbation theory of dynamical systems; Hamiltonian systems; applications to the theory of oscillations and celestal mechanics.
not given 2001-02

256A,B,C. Partial Differential Equations—Introduction to the theory of linear and non-linear partial differential equations, beginning with linear theory involving use of Fourier transform and Sobolev spaces. Topics: Schauder and L2 estimates for elliptic and parabolic equations; De Giorgi-Nash-Moser theory for elliptic equations; non-linear equations, e.g., the minimal surface equation, geometric flow problems, and non-linear hyperbolic equations.
256A. *3 units, Aut (Simon)*
256B. *3 units, Win (Mazzeo)*
256C. *3 units, Spr (Mazzeo)*

257A,B. Symplectic Geometry and Topology—Linear symplectic geometry and linear Hamiltonian systems. Symplectic manifolds and their Lagrangian submanifolds, local properties. Symplectic geometry and mechanics. Contact geometry and contact manifolds. Relations between symplectic and contact manifolds. Hamiltonian systems with symmetries. Momentum map and its properties.
not given 2001-02

258. Topics in Geometric Analysis
3 units, Win (Mazzeo)

259. Microlocal Analysis—The basic calculus of pseudo-differential operators, focusing on the parametrix construction for elliptic operators, and leading to various applications in geometry (Hodge theorem, index theorem for Dirac operators). Possible topics: pseudo-differential operators on singular and noncompact spaces, the microlocal theory of elliptic boundary value problems, Atiyah-Patodi-Singer index theorem.
not given 2001-02

260A,B. Mathematical Methods of Classical Mechanics—Open to undergraduate students. Differential equations and vector fields. Examples of mechanical systems. Variational principles. Lagrange equations. Hamilton equations. Legendre transform. Lagrange mechanics on manifolds. Symplectic manifolds and Hamiltonian systems. Hamilton-Jacobi equation. System with symmetries and their integrals. Integrable systems. Examples of action-angle variables. Liouville theorem. Perturbation theory. Geometric optic and contact geometry.
not given 2001-02

261A,B. Functional Analysis—Geometry of linear topological spaces. Linear operators and functionals. Spectral theory. Calculus for vector-valued functions. Operational calculus. Banach algebras. Special topics in functional analysis.
261A. *3 units, Aut (Katznelson)*
261B. *3 units, Win (Katznelson)*

263A,B. Lie Groups and Lie Algebras—Definitions, examples, basic properties. Semi-simple Lie algebras, their structure and classification. Cartan decomposition: real Lie algebras. Representation theory: Cartan-Stiefel diagram, weights. Weyl character formula. Orthogonal and symplectic representations. Prerequisite: 210 or equivalent.
given 2002-03

266. Computational Signal Processing and Wavelets—For students interested in theoretical and computational aspects of signal processing. Topics: time-frequency transforms; wavelet bases and wavelet packets; linear and nonlinear multiresolution approximations; estimation and restoration of signals; signal compression.
3 units, Win (Levy)

267A,B. Topics in Functional and Harmonic Analysis—Topics from functional analysis and from the L^p-theory of harmonic analysis—the singular integral theory of Calderon and Zygmund and its extensions, interpolation of operators, multiplier transformations, and smoothness properties of functions. Sets of uniqueness for trigonometric series, spectral syntheses, thin sets, spectral theory of convolution operators, and applications. Prerequisite: knowledge of the elements of Fourier analysis.
not given 2001-02

269. Holomorphic Curves and their Applications in Symplectic Geometry—Holomorphic curves in symplectic manifolds. Gromov's compactness theorem. Invariants of symplectic and contact manifolds, and their Lagrangian and Legendrian submanifolds. Gromov-Witten invariants. Floer homology. Contact homology. Symplectic field theory.
not given 2001-02

270. Geometry and Topology of Complex Manifolds—Complex manifolds, Kahler manifolds, curvature, Hodge theory, Lefschetz theorem, Kahler-Einstein equation, Hermitian-Einstein equations, deformation of complex structures.
3 units, Spr (J. Li)

272A,B. Topics in Partial Differential Equations—Introduction to PDE methods in an intrinsic geometric setting. Topics: Schauder and DeGiorgi-Nash theory in a geometric setting, Sobolev, Poincare, and isoperimetric inequalities. Discussion of nonlinear methods: Leray-Schauder fixed point, and degree and variational methods. Geometric examples introduce basic nonlinear PDEs of geometry (the harmonic map, Yang-Mills, and mean curvature equations) and equations arising from scalar and Ricci curvature. Prerequisite: knowledge of differential geometry through 217A.
not given 2001-02

277. Mathematical Theory of Relativity—Ricci calculus; variational principles and covariance properties; differential geometry of space-time; Cauchy's problem for the differential equations of gravitation and electromagnetism; relativistic hydrodynamics; unified field theories.
not given 2001-02

278. Topics in Mathematical Physics
3 units, Spr (P. Cohen)

281A,B. Introduction to Algebraic and Differential Topology—Fundamental group, covering spaces, embeddings and immersions of manifolds, transversality, homotopy theory, homology and cohomology of complexes, differential forms, Poincare Duality.
281A. *3 units, Aut (Brumfiel)*
281B. *3 units, Win (Schlichtkrull)*

282A. Low Dimensional Topology—The theory of surfaces and 3-manifolds. Curves on surfaces, the classification of diffeomorphisms of surfaces, and Teichmuller space. The mapping class group and the braid group. Knot theory, including knot invariants. Decomposition of 3-manifolds: triangulations, Heegaard splittings, Dehn surgery. Loop theorem, sphere theorem, incompressible surfaces. Geometric structures, particularly hyperbolic structures on surfaces and 3-manifolds.
given 2002-03

282B. Homotopy Theory—Homotopy groups, fibrations, spectral sequences, simplicial methods, Dold-Thom theorem, models for loop spaces, homotopy limits and colimits, stable homotopy theory.
3 units, Win (Carlsson)

282C. Fiber Bundles and Cobordism—Possible topics: principal bundles, vector bundles, classifying spaces. Connections on bundles, curvature. Topology of gauge groups and gauge equivalence classes of connections. Characteristic classes and K-theory, including Bott periodicity, algebraic K-theory, and indices of elliptic operators. Spectral sequences of Atiyah-Hirzebruch, Serre, and Adams. Cobordism theory, Pontryagin-Thom theorem, calculation of unoriented and complex cobordism.
3 units, Spr (R. Cohen)

283. Topics in Algebraic Topology—Etale homotopy theory. Cech homotopy theory, etale coverings, Grothendieck's theory of the fundamental group of a scheme, etale homotopy type as a pro-space, comparison with the complex topology. Applications to K-theory, homotopy theory, and algebraic geometry.
3 units, Spr (Carlsson)

284. Topics in Geometric Topology—Configuration spaces of finite sets of points in manifolds and configurations of polygons in Euclidean space. Associated moduli spaces and connections between configuration spaces and iterated loop spaces. Applications in areas ranging from knot theory to motion planning in kinematics and robotics.
3 units, Win (Milgram)

285. Geometric Measure Theory—Hausdorff measures and dimensions, area and co-area formulas for Lipschitz maps, integral currents and flat chains, minimal surfaces and their singular sets.
not given 2001-02

286. Topics in Differential Geometry
not given 2001-02

289. 3-Dimensional Contact Geometry—Contact structures and foliations, geometric meaning of integrability, tight and overtwisted contact structures, classification of overtwisted contact structures, contact convexity, classification and finiteness results for tight contact structures, holomorphic methods in contact geometry, Legendrian and transversal knots.
not given 2001-02

290A,B. Model Theory—Kripke (possible world) semantics of intuitionistic and modal logics. Completeness results and strategies in automated deduction. Algebraic models. Second order systems. May be taken independently. Prerequisites: 160A,B or equivalent.
290A. *3 units, Aut (Mints)*
290B. *3 units, Win (Mints)*

291A,B. Recursion Theory—Theory of recursive functions and recursively enumerable sets. Turing machines and alternative approaches. Diophantine definability. Definability in formal systems. Gödel's incompleteness theorems. Recursively unsolvable problems in mathematics and logic. Introduction to recursive ordinals and hierarchies. Prerequisites: 160A,B and 162, or equivalents.
given 2002-03

292A,B. Set Theory—The basics of axiomatic set theory; the systems of Zermelo-Fraenkel and Bernays-Gödel. Topics: cardinal and ordinal numbers, the cumulative hierarchy and the role of the axiom of choice. Models of set theory, including the constructible sets and models constructed by the method of forcing. Consistency and independence results for the axiom of choice, the continuum hypothesis and other unsettled mathematical and set-theoretical problems. Prerequisites: 160A,B and 161, or equivalents.

292A. *3 units, Aut (Feferman)*

292B. *3 units, Win (Feferman)*

293A,B. Proof Theory—Gentzen's natural deduction and/or sequential calculi for first-order predicate logic. Normalization respectively cut-elimination procedures. Extensions to infinitary calculi; ordinal complexity of proof trees. Subsystems of analysis and their reduction to constructive theories. Prerequisites: 160A,B and 162, or equivalents.
given 2002-03

294. Topics in Logic—Hilbert's program and its modern modifications. Brouwer-Heyting-Kolmogorov interpretation, realizability, extraction of programs from proofs. Normalization of proofs as computation of programs. Stability of program extraction. Intensional and extensional identity of programs. Continuous cut-elimination and its use for the study of finite proofs and programs. Prerequisites: 160A,B or equivalents.
3 units, Spr (Mints)

297. Algebraic Logic—(Enroll in Computer Science 353.)
3 units, Aut (Pratt)

360. Advanced Reading and Research
any quarter (Staff)

361. Seminar Participation—Participation in a faculty-led seminar which has no specific course number.
any quarter (Staff)

380. Seminar in Applied Mathematics
by arrangement

381. Seminar in Analysis
by arrangement

383. Seminar in Function Theory
by arrangement

385. Seminar in Abstract Analysis
by arrangement

386. Seminar in Geometry and Topology
by arrangement

387. Seminar in Algebra and Number Theory
by arrangement

388. Seminar in Probability and Stochastic Processes
by arrangement

389. Seminar in Mathematical Biology
by arrangement

391. Seminar in Logic and the Foundations of Mathematics
by arrangement

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