2005 QM II

Consider a particle of mass m in a double well potential of the form

$$V(x) = V_0 L^{-4}[x^2 - L^2]^2.$$

a) The only obvious dimensionless quantity that can be made using the parameters of this problem is

$$A = L^2 V_0 m / h^2.$$

Give a physical interpretation to this parameter.

b) In the limit that A >> 1, make an estimate of the ground-state energy, E_0, and the difference in energy, ΔE, between the ground-state and the first excited state. Express all energies in units of V_0. You do not have to get numerical coefficients correctly, but you do need to exhibit correctly the dominant dependence of the answer on A in the limit of large A.

c) Draw a picture of the ground-state and first excited state wave-functions.

d) In the limit that A << 1, make an estimate of the ground-state energy and the difference in energy between the ground-state and the first excited state. Again, express all energies in units of V_0. You do not have to get numerical coefficients correctly, but you do need to exhibit correctly the dominant dependence of the answer on A in the limit of small; A.

e) Draw a picture of the ground-state wavefunction.

Clarification: What we are after in problems b and d is the leading A dependence. If, for instance, at large A, the ground energy were $E_0 = 6.417 A^6 \exp[43.91 A]$ (which it isn't), then an answer worthy of full credit would be $E_0 \sim A^x \exp[\alpha A]$ with the notation that α and x are pure numbers of order 1. If the answer were $E_0 = 6.7 A^4$, full credit would be awarded for $E_0 \sim A^4$.