QM I

QUALIFIER EXAM 2004–5 QUANTUM MECHANICS QUESTION

Consider the Hilbert Space describing the spin degrees of freedom of three distinguishable spin 1/2 particles. Denote Pauli matrices acting on particle a as σ_i^a where $a = 1, 2, 3$ and $i = x, y, z$. Consider the hermitian operators

$$\begin{aligned} A &= \sigma_x^1 \sigma_y^2 \sigma_y^3 \\ B &= \sigma_y^1 \sigma_x^2 \sigma_y^3 \\ C &= \sigma_y^1 \sigma_y^2 \sigma_x^3 \\ D &= \sigma_x^1 \sigma_x^2 \sigma_x^3 \end{aligned}$$

- These operators mutually commute so they can be simultaneously diagonalized. What are the eigenvalues of these operators? Show they obey the operator equation

$$ABCD = -I \tag{1}$$

In the theory of quantum measurement one often contrasts quantum mechanics with a classical "hidden variable" theory. Roughly speaking, in such a theory measurement assigns an ordinary numerical values to any possible observable. If the observable is O then denote its assigned value $v(O)$. Basic consistency requires that $v(O)$ must be equal to one of the eigenvalues of the operator O, the results of quantum mechanical measurement. Any operator equation obeyed by mutually commuting observables will be obeyed by their eigenvalues and hence for consistency must also be obeyed by the values assigned by v. For instance the operator equation (1) implies the value equation $v(A)v(B)v(C)v(D) = -1$. You will now show that no such value function v can exist.

Consider the ten operators $A, B, C, D, \sigma_x^1, \sigma_y^1, \sigma_x^2, \sigma_y^2, \sigma_x^3, \sigma_y^3$ and their values $v(A), v(B), v(C), v(D), v(\sigma_x^1), v(\sigma_y^1), v(\sigma_x^2), v(\sigma_y^2), v(\sigma_x^3), v(\sigma_y^3)$.

- Show that the following operator equations hold.

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$$\begin{aligned} I &= A\sigma_x^1\sigma_y^2\sigma_y^3 \\ I &= B\sigma_y^1\sigma_x^2\sigma_y^3 \\ I &= C\sigma_y^1\sigma_y^2\sigma_x^3 \\ I &= D\sigma_x^1\sigma_x^2\sigma_x^3 \\ -I &= ABCD \end{aligned}$$

Each of the above equations involves a mutually commuting set of operators so they each give rise to an equation on values. Now use these value equations to show that no value function v can exist.

This gives a strikingly simple argument that local realism is inconsistent with quantum mechanics.