

## General Physics Problem I

Ground-based optical astronomers are familiar with a phenomenon called Atmospheric Dispersion, wherein stars observed away from the zenith (i.e. the point directly overhead) appear to be distorted as a function of color, due to refraction in the atmosphere. The purpose of this problem is for you to estimate the magnitude of this effect from first principles. The logic and consistency of your physical arguments is more important than the accuracy of your numerical answers.

- (a) Since physical conditions vary significantly with altitude in the atmosphere, you might expect this effect to be sensitive to the details of the atmospheric models. Show, however, that to a good approximation, the difference in the apparent position of a star at a given wavelength,  $\lambda$ , and its true position, depends only on the zenith angle (the angular distance away from the zenith) and the value of the index of refraction of the atmosphere for that wavelength at the altitude of the observatory. Write an explicit expression for the angular spread  $\Delta\theta(\lambda_1, \lambda_2)$  due to atmospheric dispersion for a wavelength range  $\lambda_1 - \lambda_2$ , as a function of the index of refraction evaluated at those two wavelengths and the zenith angle,  $\theta$ .
- (b) Treat the atmosphere as a dilute gas of molecules which can be approximated as classical harmonic oscillators with resonant frequency,  $\omega_0$ . Write (or derive) an expression for the index of refraction as a function of frequency,  $\omega$ , in terms of  $\omega_0$ , the molecule number density,  $N$ , and any other physical constants you think are required.
- (c) Giving simple physical arguments, estimate appropriate values for  $\omega_0$  for air molecules, and for the number density,  $N$ , at standard temperature and pressure. Use these to estimate the index of refraction of air at the extremes of the optical band:  $\lambda_{\min} = 400 \text{ nm}$ ,  $\lambda_{\max} = 900 \text{ nm}$ .
- (d) Typical modern observatories are located at altitudes of  $\sim 3000 \text{ m}$  above sea level. How do you expect the index of refraction values at this altitude to compare to your answer from part (c)? Give a quantitative estimate.
- (e) Use your quantitative answers above to determine the approximate angular size of an unfiltered star image due to atmospheric dispersion alone, when it is observed at a zenith angle of  $60^\circ$ .