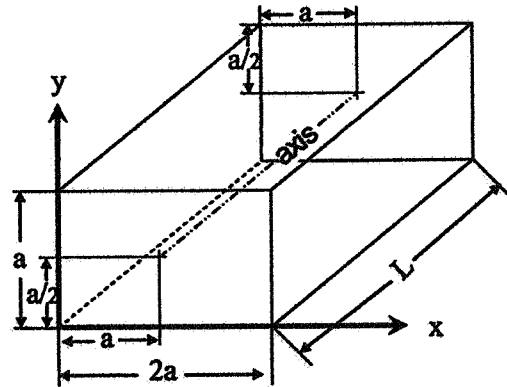


2005

Electricity and Magnetism

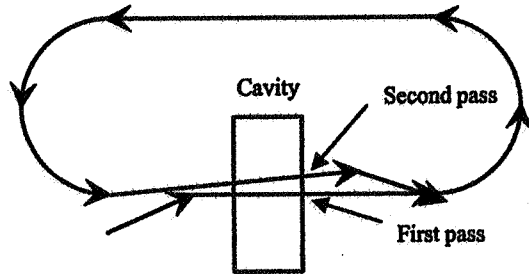
1) A rectangular microwave cavity is excited in the lowest frequency mode that has no magnetic field in the longitudinal (z) direction and a zero of the longitudinal electric field on the axis. The cavity has dimensions $a \times 2a \times L$. The axis is defined by the line $x = a$, $y = a/2$, running through the center of the cavity.



a) If $a = (4.5)^{1/2} \text{ cm} = 3/2^{1/2} \text{ cm}$, what is the frequency?

b) Give an expression for each component of the electric and magnetic fields.

2) A continuous beam of relativistic electrons ($E = \gamma m_0 c^2$) passes through the cavity and is bent back around by a system of magnets so that it passes through the cavity again, a time T later. As suggested in the figure, the first pass is on the cavity axis, while the second pass is off the axis solely due to the influence of the cavity fields on the first pass. The only fields in the cavity are those due to the mode above. The beam current is I and the cavity quality factor is Q ($= \omega U / P_{\text{cavity}}$, where U is the energy stored and P_{cavity} is the power dissipated in the cavity). Assume that as far as the electron beam is concerned, γ is a constant, and the cavity is short enough that its influence on the beam can be treated as an impulse.



The only fields in the cavity are those due to the mode above. The beam current is I and the cavity quality factor is Q ($= \omega U / P_{\text{cavity}}$, where U is the energy stored and P_{cavity} is the power dissipated in the cavity). Assume that as far as the electron beam is concerned, γ is a constant, and the cavity is short enough that its influence on the beam can be treated as an impulse.

Show that if the current is greater than some value, $I_{\text{threshold}}$, the fields can extract energy from the beam and grow exponentially. Below this value the fields will decay to zero.