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# An Overview of Credit Derivatives

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## Credit risk

- The distribution of financial loss due to a **broken financial agreement**, for example
  - Agency downgrade
  - Chapter 11
  - Failure to pay according to schedule
- Pervades **virtually all** financial transactions
- A credit derivative is a security that facilitates the distribution of credit risk

## Outline

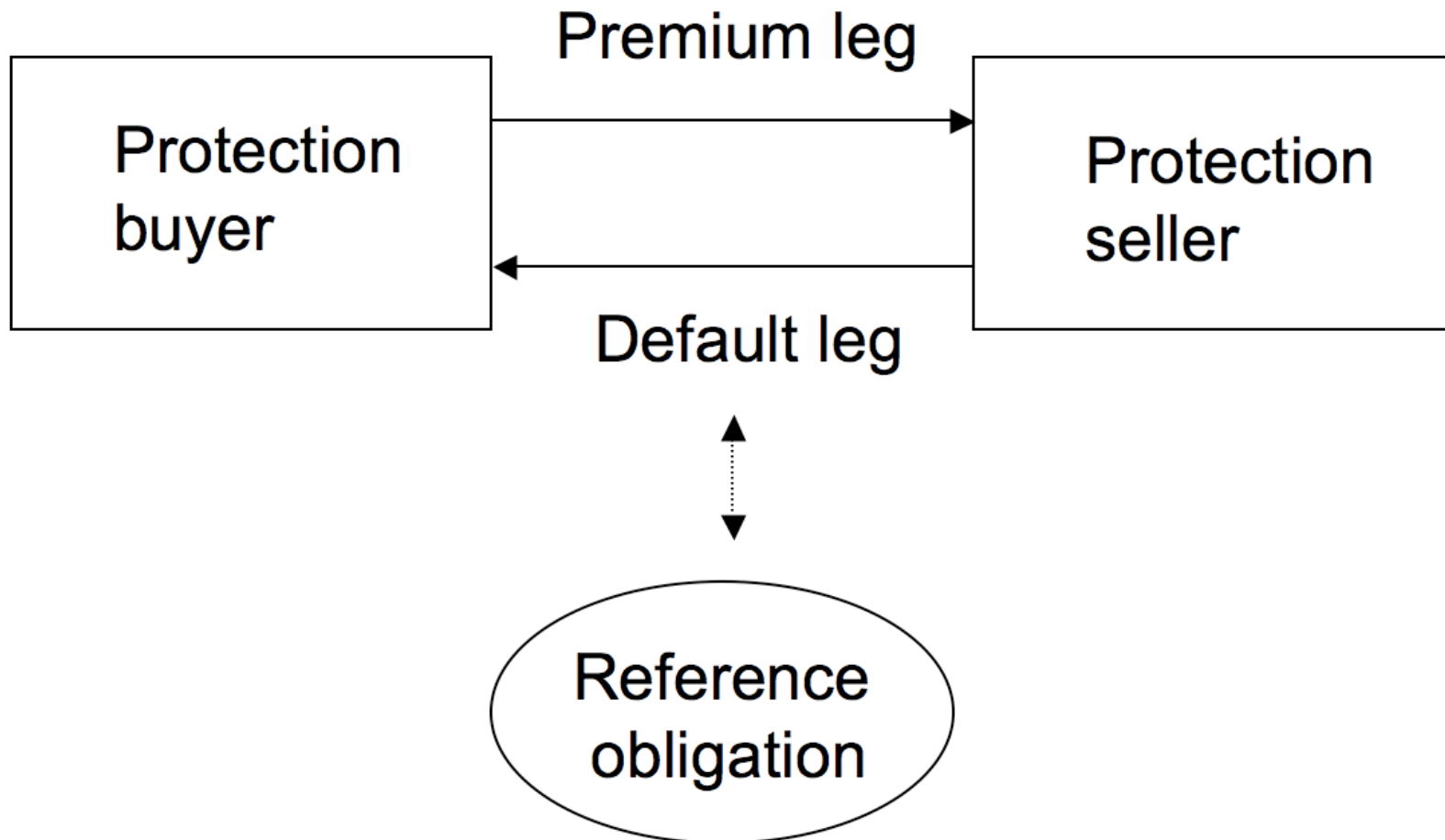
1. Motivating Example: Credit Default Swap (CDS)
2. Point processes
  - (a) Transform
  - (b) Simulation
3. Corporate Bonds and CDS
4. Index Swaps and Tranches

## References

- We will closely follow my 2009 article “An Overview of Credit Derivatives,” which is available on my website [www.stanford.edu/~giesecke](http://www.stanford.edu/~giesecke) along with these slides
- We will provide detailed references

# Credit default swap

## Mechanics



# Credit default swap

## Swap legs

- A CDS is a bilateral over-the-counter transaction
  - **Default leg**

The protection seller compensates the protection buyer for the loss if the reference entity experiences a credit event before the maturity of the contract
  - **Premium leg**

The protection buyer pays a quarterly fee, called the CDS spread and stated as a fraction of the notional per annum, until the credit event or maturity, whichever is first
- The fair spread equates the values of the two legs

# Credit default swap

Transfer of credit risk

- Position of parties
  - The protection buyer is short credit without the risk of short squeezes or repo specials
  - The protection seller is long credit
- Similar to an insurance contract, with the difference that the protection buyer often has no relation to the reference entity
  - New regulation is underway regarding naked positions
- No upfront notional is exchanged—the transaction is unfunded and therefore off-balance sheet

# Credit default swap

## Applications

- Hedging
  - A bank may obtain regulatory capital relief by buying protection on a loan it has made
- Speculation
  - A fixed-income investor can assume credit exposure by selling protection without having to fund a cash bond purchase
  - An investor buying protection bets that the credit risk of the reference name increases
  - Off-balance sheet

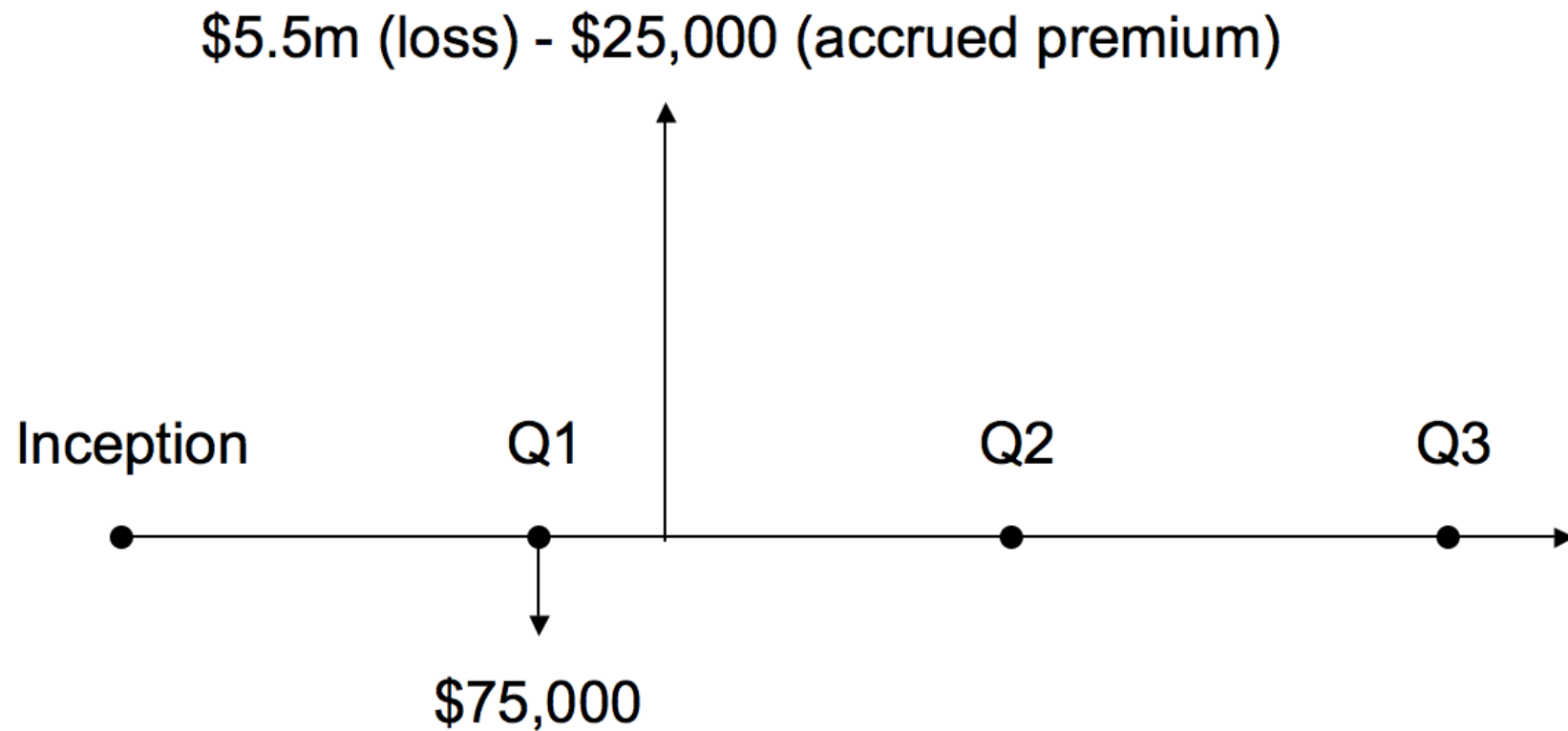
# Credit default swap

## Example

- A protection buyer purchases 5yr protection on a corporate name with notional \$10m at an annual spread of 300bp
- After 3 months, the protection buyer makes the first payment, roughly equal to  $\$10m \times 0.03 \times 0.25 = \$75,000$
- The reference name defaults 1 month after the first payment and the reference obligation has a recovery rate of 45%
  - The protection seller compensates the protection buyer for the loss by paying  $\$10m \times (100\% - 45\%) = \$5.5m$  at default
  - The protection buyer pays the accrued premium, roughly equal to  $\$10m \times 0.03 \times 1/12 = \$25,000$ , at default
- The contract expires

# Credit default swap

Cash flows to the protection buyer



# Credit default swap

## Reverse engineering

- Spreads for various maturities between 0.5 and 10 years are quoted for a growing universe of reference names
- From market spreads we can infer a name's risk-neutral probability of default over future horizons
  - Mark-to-market of credit swap positions
  - Construction of forward spread curve
  - Design of credit trading strategies
- Need a model for default timing and the loss at default

# Point process

## Basic setting

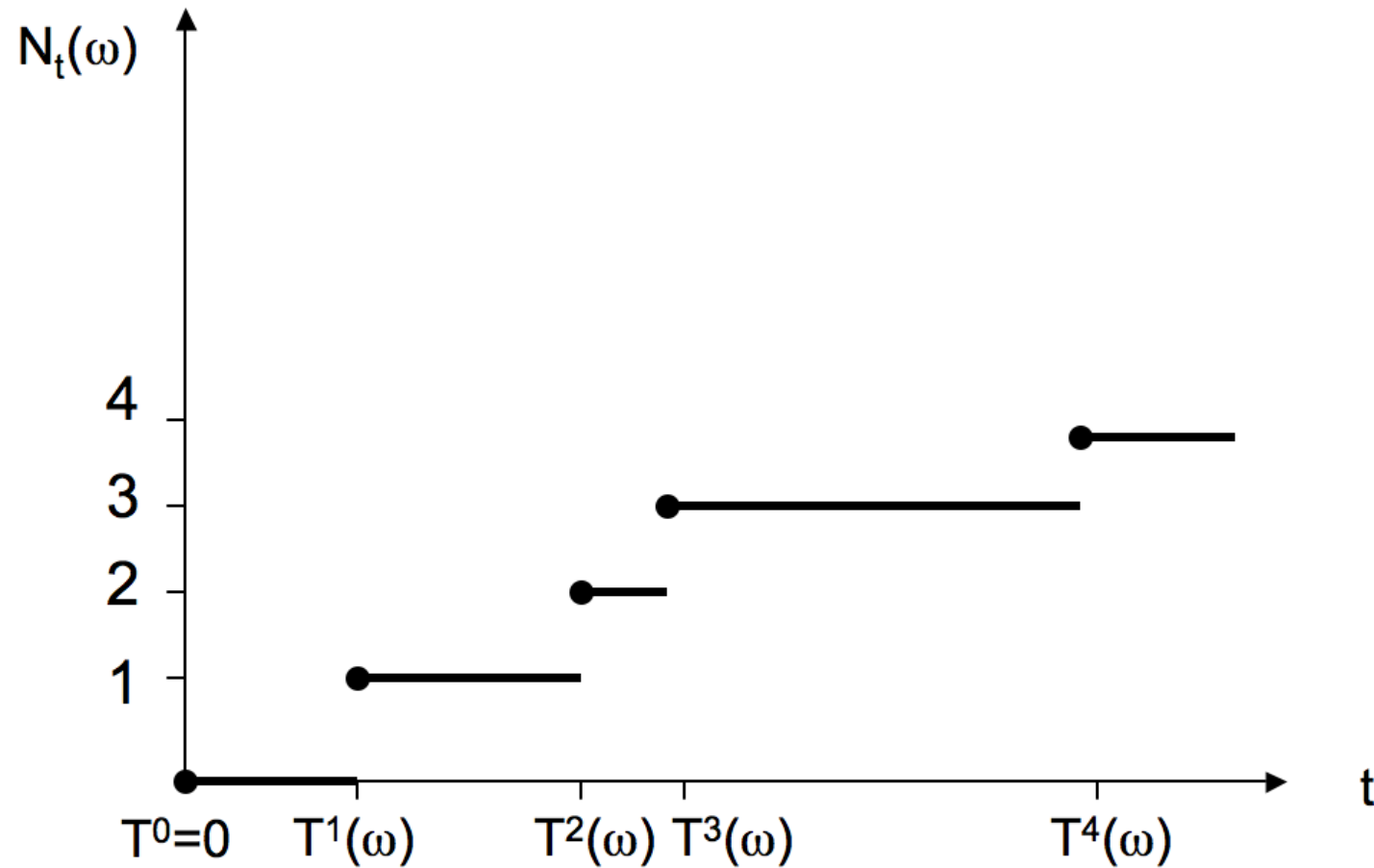
- Probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  with filtration  $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$
- Consider a sequence of default stopping times  $(T^k)_{k \geq 0}$ 
  - $0 = T^0 < T^1 < T^2 < \dots$
  - $\lim_{k \rightarrow \infty} T^k = \infty$
- The stopping times generate the **default process**  $N$  by

$$N_t = \sum_{k \geq 1} 1_{\{T^k \leq t\}}$$

- $N$  is non-explosive:  $N_t < \infty$  almost surely so an infinite number of defaults cannot accumulate at a finite time
- $N$  has the same information as  $(T^k)$

# Point process

The sample paths of  $N$  are right continuous with left limits



# Point process

## Compensator and intensity

- The default process  $N$  is a submartingale:

$$N_t = \mathbb{E}(N_t | \mathcal{F}_t) \leq \mathbb{E}(N_s | \mathcal{F}_t) \quad t \leq s$$

- The Doob-Meyer theorem guarantees that  $N = A + M$  where  $A$  is a nondecreasing, right continuous predictable process called the **compensator** and  $M$  is a (local) martingale
- The **intensity** process  $\lambda \geq 0$  generates  $A$ :

$$A_t = \int_0^t \lambda_s ds$$

- The intensity represents the mean arrival rate: for small  $\Delta$

$$\lambda_t \Delta \approx \mathbb{E}(N_{t+\Delta} - N_t | \mathcal{F}_t)$$

# Point process

The characteristic martingale

- Let  $\psi(u) = 1 - e^{-u}$  and define

$$Z_t(u) = \exp(\psi(u)A_t - uN_t), \quad u \geq 0$$

- By Stieltjes integration by parts,  $Z(u)$  is a (local) martingale since  $M = N - A$  is (local) martingale:

$$Z_t(u) = 1 - \psi(u) \int_0^t Z_{s-}(u) dM_s$$

- If  $\mathbb{E}(\exp(A_T)) < \infty$ , then  $(Z_t(u))_{t \leq T}$  is a martingale

# Point process

Poisson and doubly-stochastic Poisson process

- Suppose  $\lambda$  is deterministic: Then, by the martingale property of  $Z$

$$\mathbb{E}(e^{-u(N_T - N_t)} | \mathcal{F}_t) = e^{-\psi(u)(A_T - A_t)}$$

Thus  $N$  has independent increments,  $N_T - N_t \sim \text{Poi}(A_T - A_t)$ , so  $N$  is a **Poisson process** (one direction of Watanabe's theorem)

- Suppose  $\lambda$  is allowed to be random, but such that conditional on a path of  $\lambda$ ,  $N$  is a Poisson process with rate function  $\lambda$  (two-step randomization: **doubly-stochastic Poisson process**): Then

$$\begin{aligned} \mathbb{E}(e^{-u(N_T - N_t)} | \mathcal{F}_t) &= \mathbb{E}(\mathbb{E}\{e^{-u(N_T - N_t)} | \mathcal{F}_t, (\lambda_s)_{t \leq s \leq T}\} | \mathcal{F}_t) \\ &= \mathbb{E}(e^{-\psi(u)(A_T - A_t)} | \mathcal{F}_t) \end{aligned}$$

# Point process transform

## Measure change

- Since  $Z_0(u) = 1$ , can use  $Z_T(u)$  to define  $\mathbb{P}^u \approx \mathbb{P}$  via

$$\mathbb{P}^u(B) = \mathbb{E}(Z_T(u)1_B), \quad B \in \mathcal{F}_T$$

- Each  $\mathbb{P}^u$  corresponds to a Laplace transform of  $A$ :

$$\mathcal{L}^u(v, t, T) = \mathbb{E}^u(e^{-v(A_T - A_t)} | \mathcal{F}_t), \quad u, v \geq 0$$

- Obtain the  $\mathbb{P}$ -Laplace transform of  $N$  as

$$\begin{aligned} \mathbb{E}(e^{-u(N_T - N_t)} | \mathcal{F}_t) &= \mathbb{E}(e^{-\psi(u)(A_T - A_t)} e^{-u(N_T - N_t)} e^{\psi(u)(A_T - A_t)} | \mathcal{F}_t) \\ &= \mathbb{E}(e^{-\psi(u)(A_T - A_t)} Z_T(u) / Z_t(u) | \mathcal{F}_t) \\ &= \mathbb{E}^u(e^{-\psi(u)(A_T - A_t)} | \mathcal{F}_t) \\ &= \mathcal{L}^u(\psi(u), t, T) \end{aligned}$$

# Point process transform

## Discussion

- The transform of  $N$  is given by the  $\mathbb{P}^u$ -transform of  $A$  evaluated at the characteristic exponent  $\psi(u)$  of the Poisson process
  - The measure change absorbs any correlation between  $\lambda$  and  $N$ , and is therefore called the **correlation-neutral measure**
  - It is redundant in the doubly-stochastic case
- Can be extended to a vector of correlated point processes and to include a stochastic discount factor and random future cash flows, see Giesecke & Zhu (2010)
- Transform of  $A$  is analogous to the price at time  $t$  of a security that pays 1 at  $T$  when the risk-free interest rate is  $v\lambda$ :

$$\mathcal{L}^u(v, t, T) = \mathbb{E}^u(e^{-\int_t^T v\lambda_s ds} | \mathcal{F}_t)$$

# Point process transform

## Laplace transform

- Adopt models for  $\lambda$  from default-free security valuation
  - Affine models, Duffie, Pan & Singleton (2000)
  - Quadratic models, Leippold & Wu (2002)
  - Linear-quadratic models, Cheng & Scaillet (2007)
- Girsanov's theorem implies that
  - The dynamics of any process that does not have jumps in common with  $N$  are not adjusted
  - The intensity of  $N$  under  $\mathbb{P}^u$  is  $e^{-u} \lambda$

## Affine point process

- Take  $\lambda = \Lambda(X)$  for  $\Lambda$  affine
- Under  $\mathbb{P}$ , the risk factor  $X$  solves the SDE

$$dX_t = \mu(X_t) dt + \sigma(X_t) dW_t + \delta dL_t \quad (1)$$

where  $\mu, \sigma^2$  are affine functions

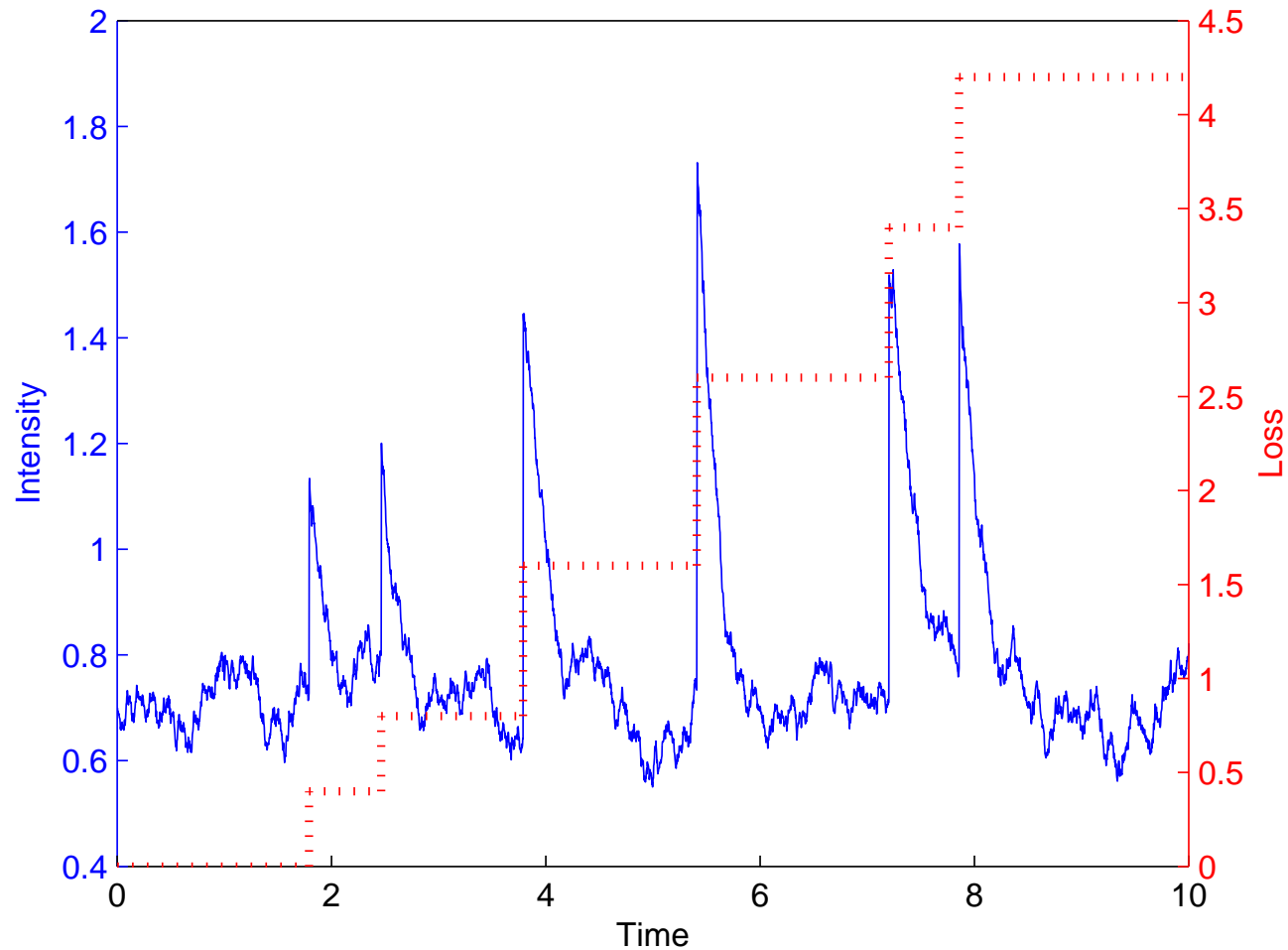
- $L_t = \sum_{k=1}^{N_t} L^k$  is a (loss) point process whose jump times  $T^k$  are those of  $N$ ; random jump sizes  $L^k$  are iid with transform  $\theta$ 
  - Self-exciting model
- From Duffie et al. (2000), know that for  $R$  affine

$$\mathbb{E}(e^{-\int_t^T R(X_s) ds + zX_T} | \mathcal{F}_t) = e^{a(t) + b(t)X_t}$$

where  $b(t) = b(z, t, T)$  and  $a(t) = a(z, t, T)$  satisfy ODEs

# Affine point process

Sample path of  $\lambda$  and  $L$



## Affine point process

- Under  $\mathbb{P}^u$ , the intensity is  $\lambda e^{-u} = \Lambda e^{-u}$
- Applying the previous result,

$$\mathcal{L}^u(v, t, T) = \mathbb{E}^u(e^{-v \int_t^T \Lambda(X_s) ds} | \mathcal{F}_t) = e^{\alpha(t) + \beta(t) X_t}$$

where  $\beta(t) = \beta(u, v, t, T)$  and  $\alpha(t) = \alpha(u, v, t, T)$  satisfy

$$\partial_t \beta(t) = v \Lambda_1 - K_1 \beta(t) - \frac{1}{2} H_1 \beta(t)^2 - e^{-u} \Lambda_1 (\theta(\delta \beta(t)) - 1)$$

$$\partial_t \alpha(t) = v \Lambda_0 - K_0 \beta(t) - \frac{1}{2} H_0 \beta(t)^2 - e^{-u} \Lambda_0 (\theta(\delta \beta(t)) - 1)$$

- The transform of  $N$  satisfies

$$\begin{aligned} \mathbb{E}(e^{-u(N_T - N_t)} | \mathcal{F}_t) &= \mathcal{L}^u(\psi(u), t, T) \\ &= \exp(\alpha(u, \psi(u), t, T) + \beta(u, \psi(u), t, T) X_t) \end{aligned}$$

# Point process simulation

## Time change

- Meyer's (1971) time change theorem asserts that the variables

$$S^k = A_{T^k} = \int_0^{T^k} \lambda_s ds, \quad k = 1, 2, \dots, n$$

are the arrival times of a standard Poisson process in the time-scaled filtration defined by  $\mathcal{F}_{A_t}$

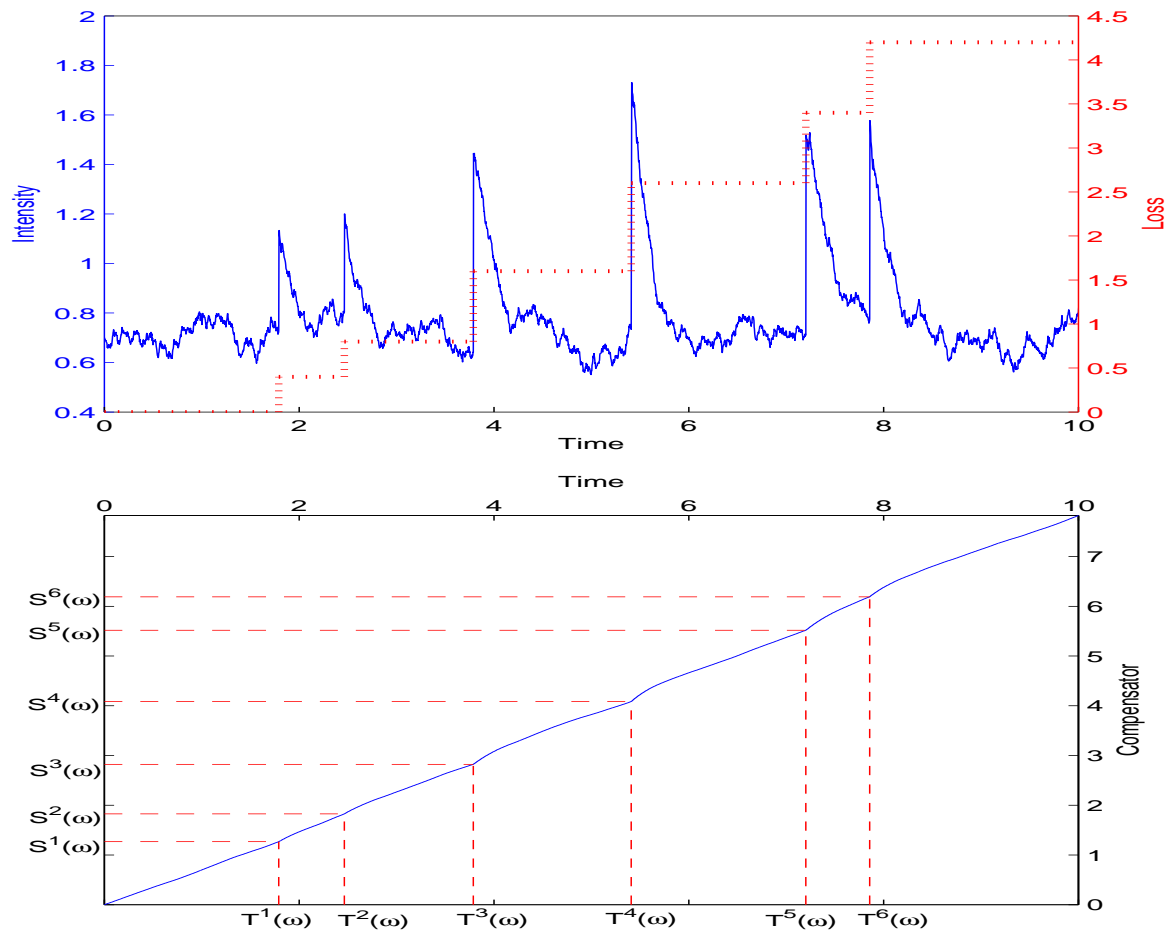
- Conversely, the  $k$ th default time is the hitting time of  $A$  to the random level  $S^k$

$$T^k = A_{S^k}^{-1} = \inf \left\{ t : \int_0^t \lambda_s ds \geq S^k \right\}$$

- Implementation: discretization of  $\lambda$

# Point process simulation

Paths of  $\lambda_t$ ,  $A_t = \int_0^t \lambda_s ds$  and arrivals for the affine model (1)



# Point process simulation

## Discussion

- Due to discretization of  $\lambda$ , simulation estimators are biased
  - Magnitude of bias is hard to quantify
  - Difficult to obtain valid confidence intervals
  - Difficult to determine optimal allocation of computational budget
- **Exact methods** eliminate the need to discretize  $\lambda$ 
  - Thinning scheme: Lewis & Shedler (1979), Glasserman & Merener (2003) for bounded  $\lambda$  and Giesecke, Kim & Zhu (2010) for general case
  - Projection method: Giesecke, Kakavand & Mousavi (2010)
  - Mimicking Markov chain method: Giesecke, Kakavand, Mousavi & Takada (2010)

# Valuation

Alternative formulations of the pricing problem

- Model the point process  $N$  under the **actual measure** that represents the empirical likelihood of events, and then specify the change of measure to a risk-neutral measure
  - Pricing and empirical time series applications that require risk premia specifications (as in Berndt, Douglas, Duffie, Ferguson & Schranz (2005), Eckner (2007), and Azizpour, Giesecke & Kim (2011))
- Model the point process  $N$  under a **risk-neutral measure**
  - Pricing applications

## Single-name valuation

- Let  $\mathbb{P}$  be a pricing measure relative to a constant risk-free rate  $r$
- The issuer's default time is the first jump time  $T^1$  of  $N$ ; the corresponding default process is  $N^1 = \min(N, 1)$
- The financial loss at default is modeled by an  $\mathcal{F}_{T^1}$ -measurable random variable  $\ell^1$ , which is independent of  $T^1$  and has  $\ell = \mathbb{E}(\ell^1)$
- At any time  $t < \min(T, T^1)$ , the firm's risk-neutral conditional survival probability satisfies

$$\begin{aligned}\mathbb{P}(T^1 > T \mid \mathcal{F}_t) &= \mathbb{P}(N_T = 0 \mid \mathcal{F}_t) = \lim_{u \uparrow \infty} \mathcal{L}^u(\psi(u), t, T) \\ &= \mathbb{E}^\infty(e^{-(A_T - A_t)})\end{aligned}$$

where  $\mathbb{P}^\infty(T^1 > T) = 1$

## Corporate zero bond

A zero coupon bond with unit face value and maturity  $T$  pays

- The **face value** 1 at  $T < T^1$

This has value  $F(t, T)\mathbb{P}(T^1 > T \mid \mathcal{F}_t)$ , where  $F(t, T)$  is the price of a unit face value,  $T$ -maturity zero coupon government bond

- The **recovery**  $(1 - \ell^1)$  of face value at  $T^1 \leq T$

This has value

$$\mathbb{E}(F(t, T^1)(1 - \ell^1)N_T^1 \mid \mathcal{F}_t) = (1 - \ell)R_t(T)$$

where  $R_t(T)$  is the pre-default value of a unit recovery payment at  $T^1 \leq T$ . By Stieltjes integration by parts

$$\begin{aligned} R_t(T) &= \mathbb{E}\left(\int_t^T F(t, s)dN_s^1 \mid \mathcal{F}_t\right) \\ &= F(t, T)\mathbb{P}(T^1 \leq T \mid \mathcal{F}_t) + r \int_t^T F(t, s)\mathbb{P}(T^1 \leq s \mid \mathcal{F}_t)ds \end{aligned}$$

## Corporate coupon bond

A corporate coupon bond with unit face value, annualized coupon rate  $c$ , coupon dates  $(t_m)$  and maturity  $T$  pays

- The **coupon**  $cC_m$  at each  $t_m < T^1$  where  $C_m$  is the day count fraction for period  $m$  (= portfolio of zero-recovery zero bonds)
- The **face value** 1 at  $T < T^1$  (= zero-recovery zero bond)
- The **recovery**  $(1 - \ell^1)$  of face value at  $T^1 \leq T$
- The **accrued coupon**  $\frac{T^1 - t_{m-1}}{\Delta_m} cC_m$  at  $T^1$  if  $t_{m-1} < T^1 \leq t_m$ , where  $\Delta_m = t_m - t_{m-1}$

Its pre-default value is given by

$$F(t, T)\mathbb{P}(T^1 > T \mid \mathcal{F}_t) + cV_t(T) + (1 - \ell)R_t(T)$$

where  $V_t(T)$  is the *risky DV01*, the pre-default value of a unit stream at coupon times  $(t_m)$  until  $\min(T^1, T)$  plus any accruals

## Credit swap

A credit swap with unit notional, annualized swap spread  $S$ , premium payment dates  $(t_m)$  and maturity  $T$  is a bilateral contract in which

- The **protection seller** pays the default loss  $\ell^1$  at  $T^1 \leq T$   
This has pre-default value  $D_t = \ell R_t(T)$
- The **protection buyer** pays the swap spread  $SC_m$  at each  $t_m < T^1$  plus any accruals  
This has pre-default value  $P_t(S) = SV_t(T)$

The fair spread  $S$  equates the values of the default and premium legs. Since there is no cash flow at inception, the fair swap spread at inception date  $t$  is the solution  $S = S_t(T)$  to the equation  $D_t = P_t(S)$ :

$$S_t(T) = \ell R_t(T) / V_t(T)$$

# Credit swap

## Notes

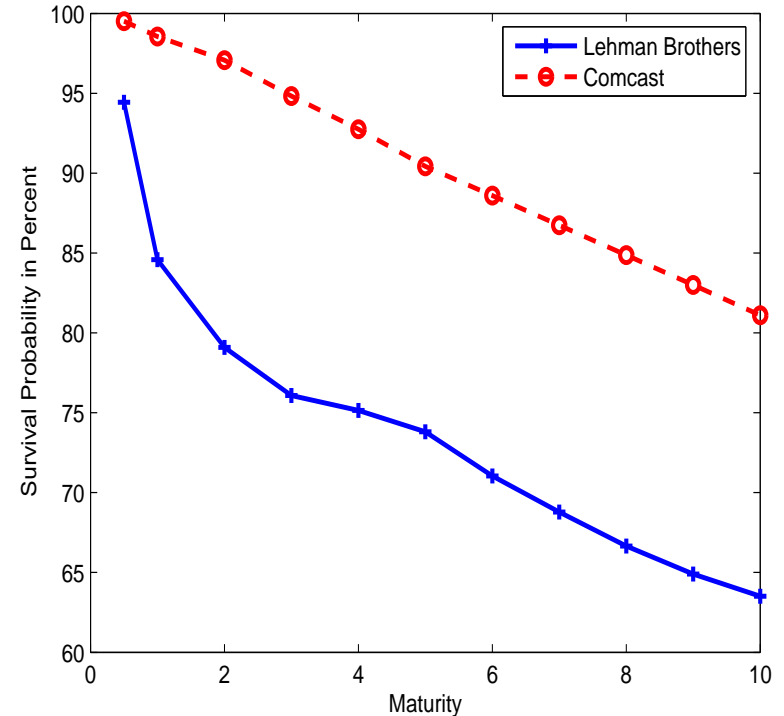
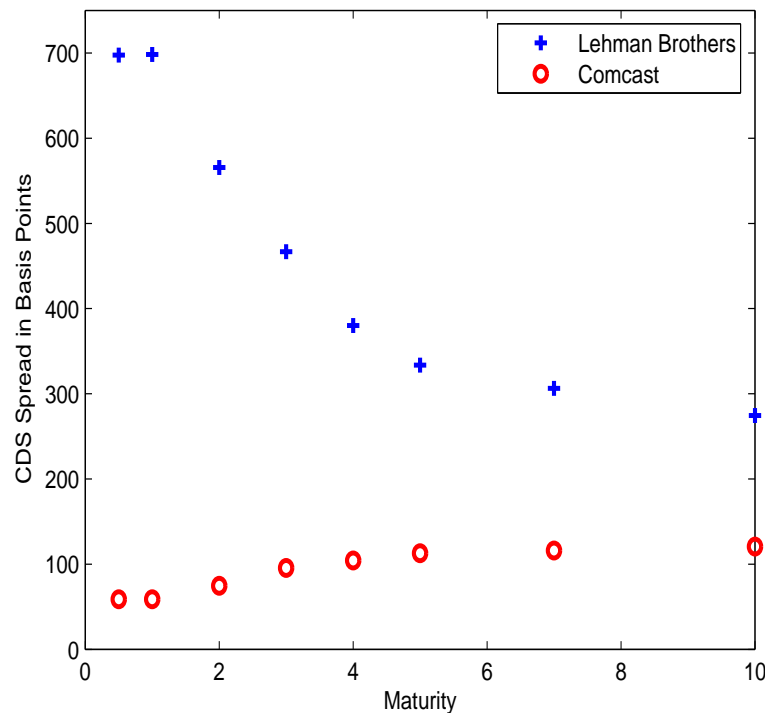
- $S_t(T)$  can be expressed in terms of  $\mathbb{P}(T^1 > s \mid \mathcal{F}_t)$  for various  $s \leq T$ , and the expected loss  $\ell$
- Often traded in terms of points upfront
- We have ignored **counterparty risk**, the risk that the protection seller fails with the reference entity
- Consider an investor who buys protection at  $t = 0$  for the period  $[0, T]$  at a swap spread of  $S_0(T)$ . The **mark-to-market value** of the position at time  $t \geq 0$ , when the market spread is  $S_t(T)$ , is

$$V_t(T)(S_t(T) - S_0(T)) = D_t - P_t(S_0(T))$$

- Forward CDS and swaption: contracts on the mark-to-market

# Credit swap

Market spreads on 9/8/2008 and implied  $\mathbb{P}(T^1 > T)$  for two firms, assuming piece-wise constant intensity  $\lambda$  and  $\ell = 0.6$



# Credit swap

## Bloomberg CDSW

3 EquityCDSW  
 1<GO> to save, 2<GO> to save curve source, 3<GO> to send screen grab  
 CPU: 283

**CREDIT DEFAULT SWAP**

Deal  Curves  Ref. Obligation  Forward CDS Matrix

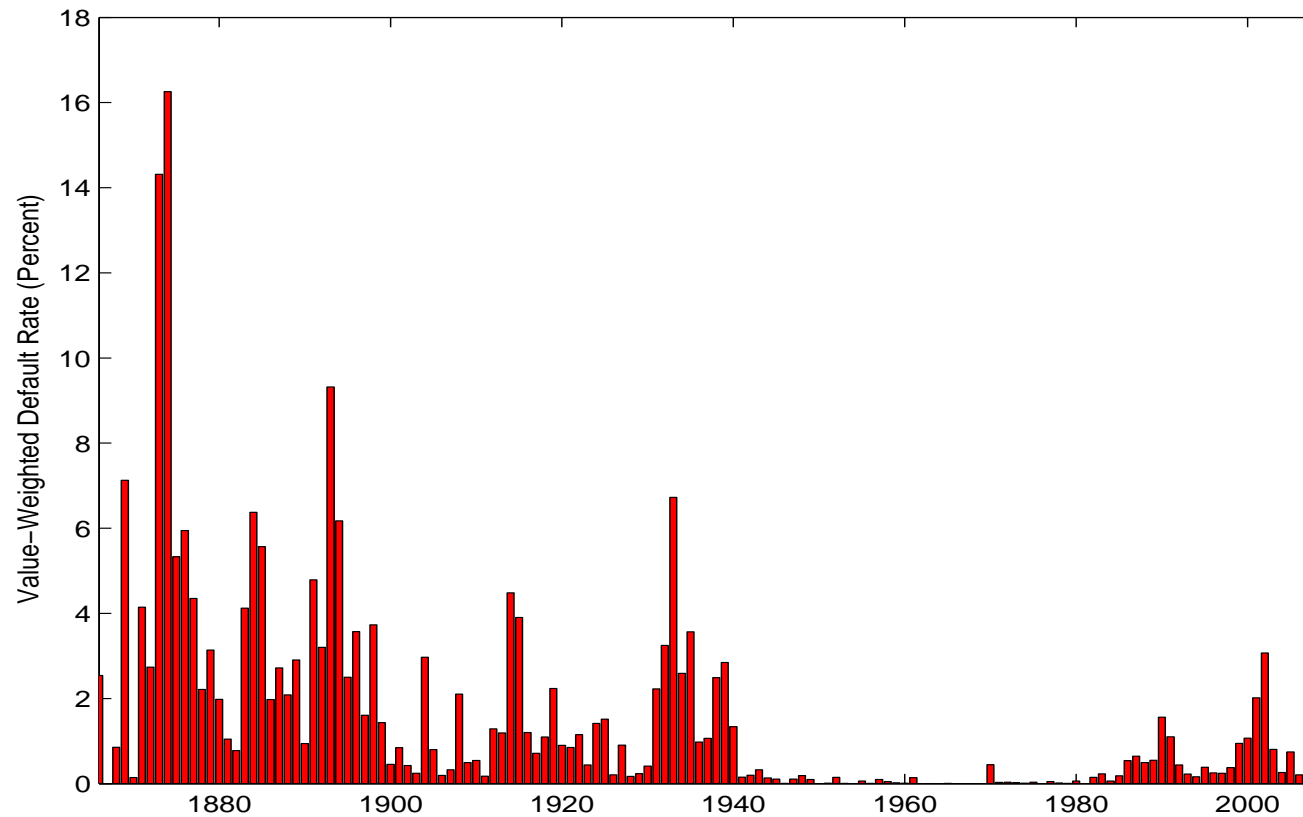
| Deal Information                              | RED Pair:2C033NAAO           | Spreads                   | Date           |
|---|------------------------------|---------------------------|----------------|
| 7)Reference:Comcast Corp                      |                              | Curve Date: 8/19/09       |                |
| Counterparty: <input type="text"/>            | Deal#: <input type="text"/>  | Benchmark: S260 MMid      |                |
| Ticker: / <input type="text"/>                | Series: <input type="text"/> | US ISDA Fixing Swap Crv   |                |
| Business Days: US GB <input type="checkbox"/> | Privilege: U User            | 6) 5yr Fix Diff: -4.09bp  |                |
| Business Day Adj: 1 Following                 | Settlement Code: USD         | Pricing Curve: F Fixing   |                |
| Business Day Adj: 1 Following                 | Currency: USD Amort: N       | Sprds: C Contributor AAsk |                |
| B BUY Notional: 10.00 MM                      | Contract: A SNAC             | 101712 USD Senior IMM I   |                |
| Effective Date: 6/20/09                       | First Accrual Start: 6/22/09 | CDS Spreads               | Default        |
| Maturity Date: 9/20/14                        | Day Count: ACT/360           | Flat: N (bps)             | Prob           |
| Payment Freq: Q Quarterly                     | First Cpn: 9/21/09           | 3/20/10                   | 80.119 0.0079  |
| Pay Accrued: T True                           | Next to Last Cpn: 6/20/14    | 9/20/10                   | 80.119 0.0146  |
| Curve Recovery: T True                        | Date Gen Method: I IMM       | 9/20/11                   | 94.635 0.0329  |
| Recovery Rate: 0.4000                         | Debt Type: 1 Senior          | 9/20/12                   | 109.190 0.0558 |
| Deal Spread: 100.000bps                       | 9)Pts Upf (%): 0.000000      | 9/20/13                   | 115.269 0.0772 |
|   |                              | 9/20/14                   | 121.347 0.1003 |
|   |                              | 9/20/16                   | 120.979 0.1360 |
|   |                              | 9/20/19                   | 121.140 0.1875 |
|   |                              | Frequency: Q Quarterly    |                |
|   |                              | Day Count: ACT/360        |                |
|   |                              | Recovery Rate: 0.4000     |                |

| Calculator               | Mode: 1 Input Sprd      |
|--------------------------|-------------------------|
| Valuation Date: 8/19/09  | Model: B BBG Fair Value |
| Cash Settled On: 8/24/09 | Repl Sprd: 121.347 bps  |
| Price: 99.00771285       | Sprd DV01: 4,605.36     |
| Principal: 99,228.72     | IR DV01: -25.47         |
| Accrued: -16,388.89      | Days: 59 32) Sprd KRR   |
| Cash Amt: 82,839.83      | MTM: 82,836.71          |

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000  
 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2009 Bloomberg Finance L.P.  
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# Corporate defaults cluster

Value-weighted default rate 1865–2008, US nonfinancial (Source: Giesecke, Longstaff, Strebulaev & Schaefer (2011))



## Sources of default clustering

- First, firms are exposed to **common or correlated risk factors**.  
The movements of these factors cause correlated changes in firms' conditional default rates (Duffie, Saita & Wang (2006))
- Second, some of the risk factors may be unobservable **frailties**.  
The uncertainty regarding the values of these factors has an influence on the conditional default rates of the firms that depend on the same frailties (Duffie, Eckner, Horel & Saita (2009))
- Third, a default may be **contagious**, and have a direct impact on the conditional default rates of other firms (Azizpour, Giesecke & Schwenkler (2009))

## Portfolio derivatives

- They facilitate the transfer of the correlated default risk in a portfolio of names
  - There are multiple families of standard reference portfolios, called indices, including the CDX family (North American issuers) and the iTraxx family (European and Asian names)
- They are contingent claims on the portfolio loss point process  $L = \sum_{k=1}^N \ell^k$  where  $\ell^k$  is the loss at the  $k$ th default
- The reference pool often consists of single-name CDS with common notional that we normalize to 1, common maturity date  $T$  and common premium payment dates  $(t_m)$ .
- We pursue a **top-down approach**, modeling the intensity  $\lambda$  of  $N$  without reference to the  $n$  portfolio constituents

# Index swap

In an index swap with swap spread  $S$  and maturity  $T$ ,

- The **protection seller** covers portfolio losses as they occur, i.e. the increments of  $L$ ; this leg has value

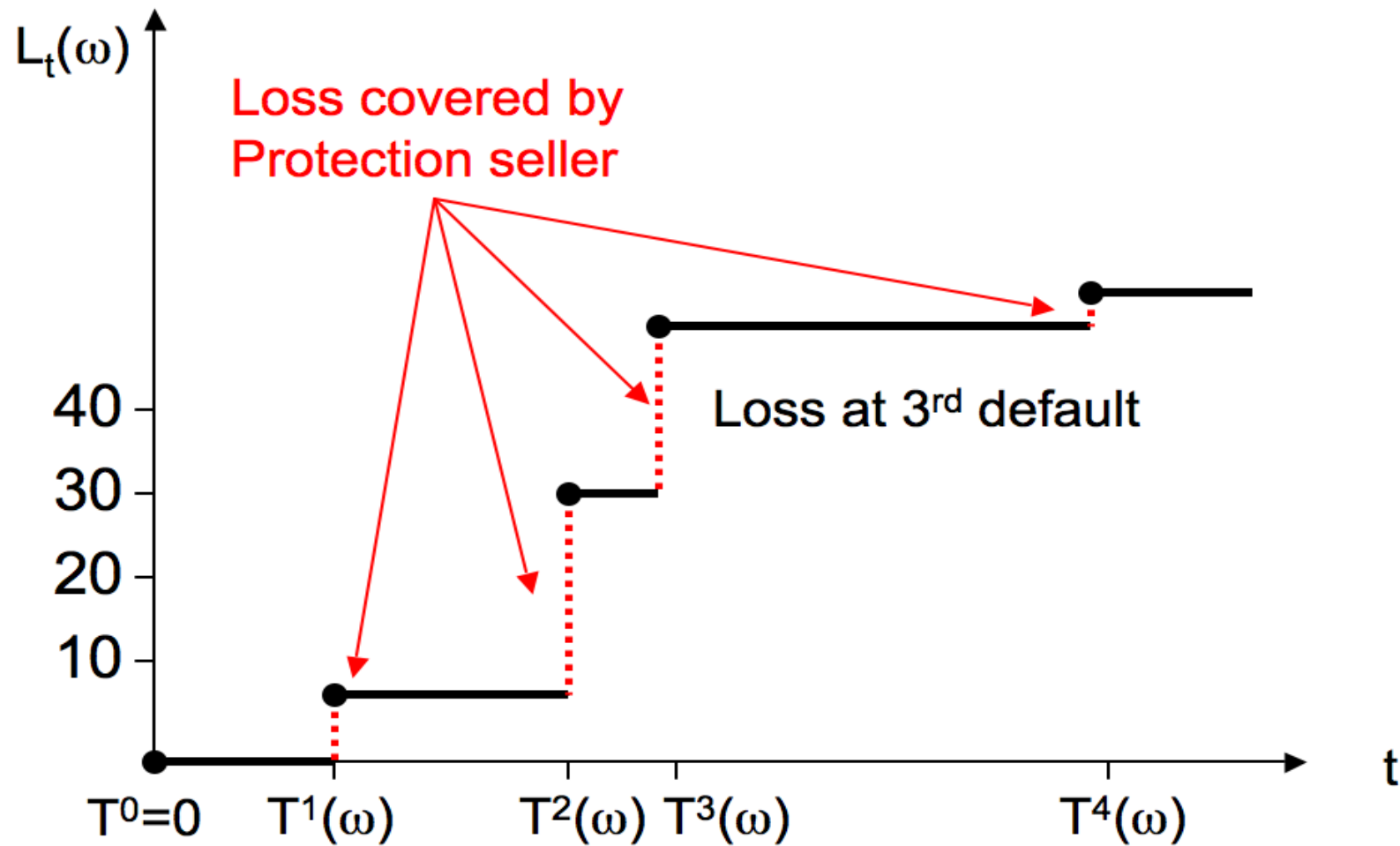
$$\begin{aligned} D_t &= \mathbb{E} \left( \int_t^T F(t, s) dL_s \mid \mathcal{F}_t \right) \\ &= F(t, T) \mathbb{E}(L_T \mid \mathcal{F}_t) - L_t + r \int_t^T F(t, s) \mathbb{E}(L_s \mid \mathcal{F}_t) ds \end{aligned}$$

- The **protection buyer** pays  $SC_m(n - N_{t_m})$  at each date  $t_m$ ; this leg has value

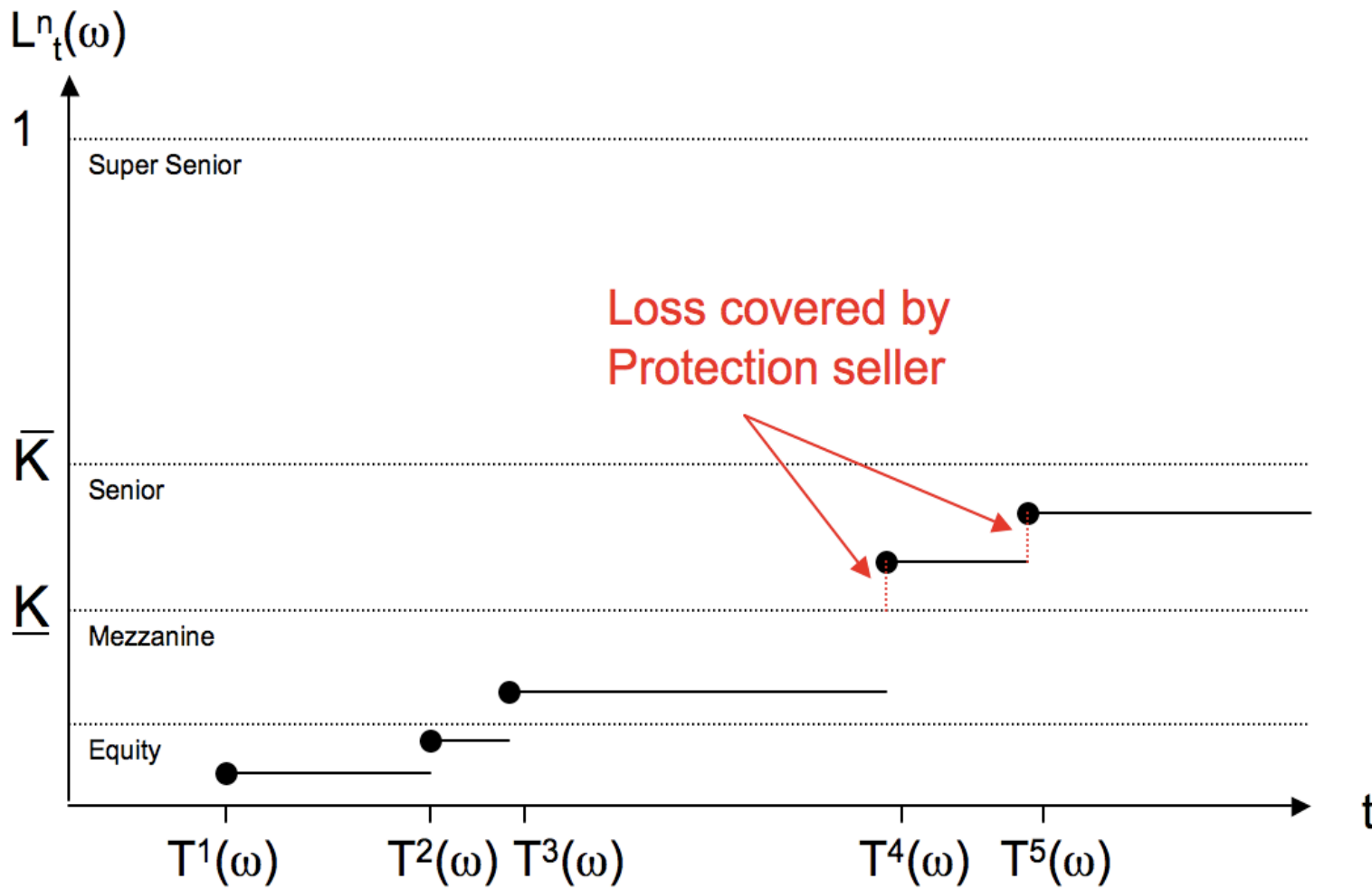
$$P_t(S) = S \sum_{t_m \geq t} F(t, t_m) C_m (n - \mathbb{E}(N_{t_m} \mid \mathcal{F}_t))$$

The fair index swap spread at time  $t$  is the solution  $S = S_t(T)$  to the equation  $D_t = P_t(S)$

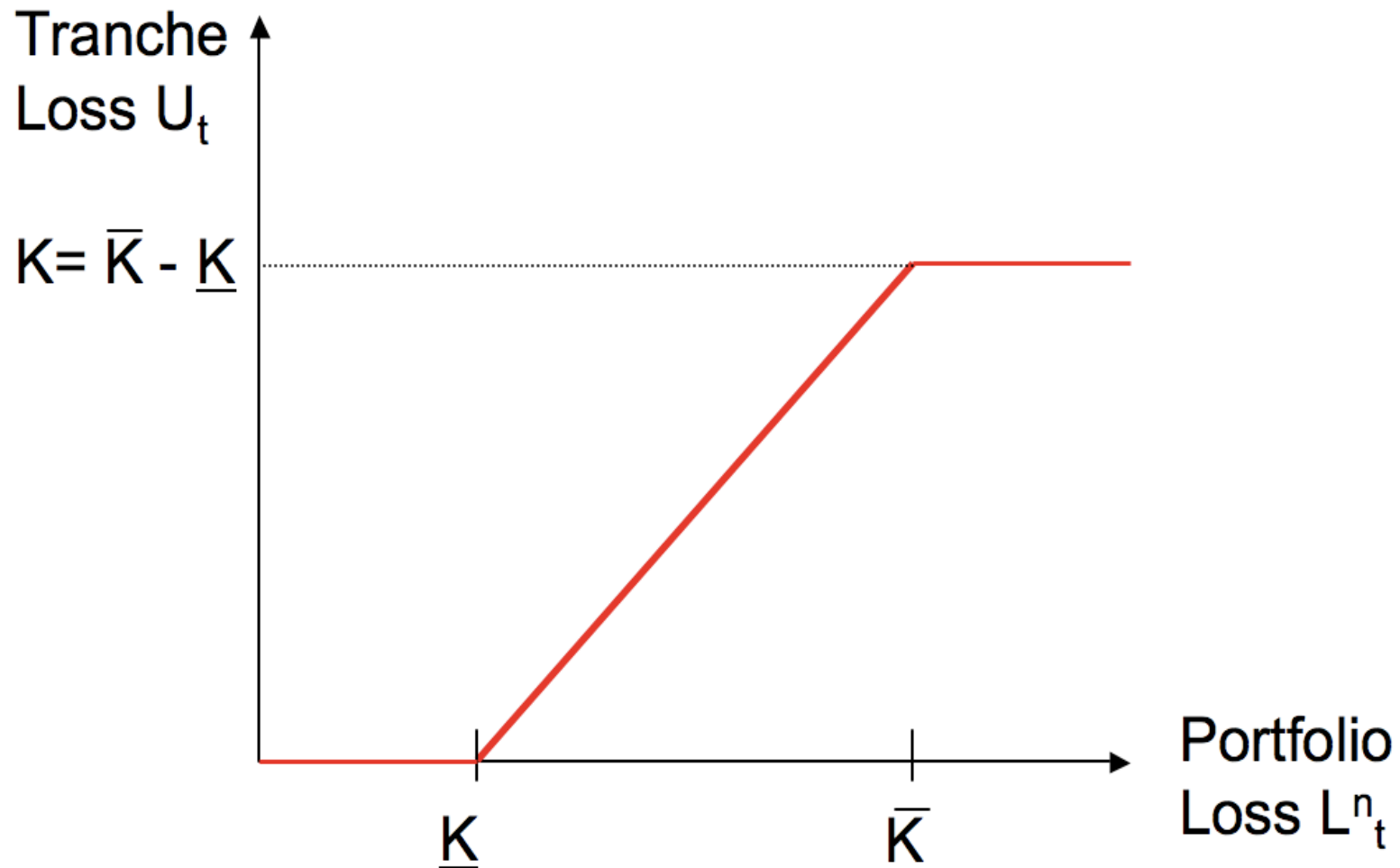
# Index swap default leg



# Tranche swap default leg



# Tranche loss = call spread on portfolio loss



## Tranche swap

In a tranche with lower attachment point  $\underline{K} \in [0, 1]$ , upper attachment point  $\overline{K} \in (\underline{K}, 1]$ , upfront rate  $G$ , and swap spread  $S$ ,

- The **protection seller** covers tranche losses

$U_t = (L_t - \underline{K}n)^+ - (L_t - \overline{K}n)^+$  as they occur; this has value

$$D_t(\underline{K}, \overline{K}) = F(t, T)\mathbb{E}(U_T | \mathcal{F}_t) - U_t + r \int_t^T F(t, s)\mathbb{E}(U_s | \mathcal{F}_t) ds$$

- The **protection buyer** pays  $GKn$  at inception and  $SC_m(Kn - U_{t_m})$  at each date  $t_m$  (assuming  $\overline{K} < 1$ )

$$P_t(\underline{K}, \overline{K}, G, S) = GKn + S \sum_{t_m \geq t} F(t, t_m)C_m(Kn - \mathbb{E}(U_{t_m} | \mathcal{F}_t))$$

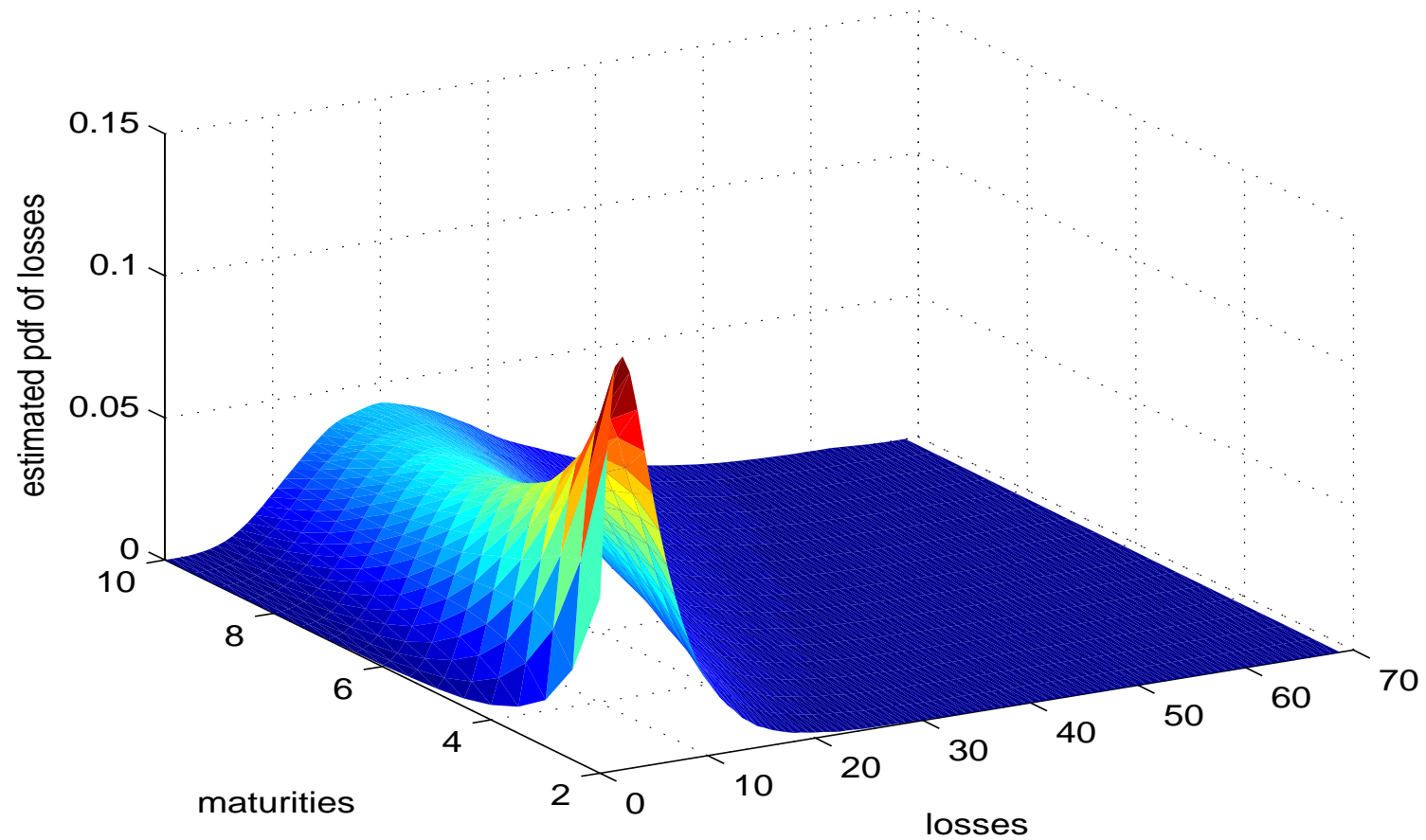
For fixed  $G$ , the fair spread  $S$  is the solution  $S = S_t(\underline{K}, \overline{K}, G, T)$  to  $D_t(\underline{K}, \overline{K}) = P_t(\underline{K}, \overline{K}, G, S)$ . For fixed  $S$ , the fair upfront rate  $G$  is the solution  $G = G_t(\underline{K}, \overline{K}, S, T)$  to  $D_t(\underline{K}, \overline{K}) = P_t(\underline{K}, \overline{K}, G, S)$ .

## Market calibration

Calibrating the affine model (1) to the CDX.HY on 5/11/2007, with  $\lambda = X$ , where  $dX_t = \kappa(c - X_t)dt + \sigma\sqrt{X_t}dW_t + \delta dL_t$ , the  $\ell^k$  iid uniform on  $\{a, b\}$  with  $\mathbb{E}(\ell^k) = 0.6$

| Contract | MarketBid | MarketAsk | Model  |
|----------|-----------|-----------|--------|
| 0-10%    | 70.50%    | 70.75%    | 71.11% |
| 10-15%   | 34.25%    | 34.50%    | 32.85% |
| 15-25%   | 316.00    | 319.00    | 316.80 |
| 25-35%   | 79.00     | 81.00     | 81.47  |
| Index    | 262.85    | 263.10    | 263.46 |
| AAPE     |           |           | 1.47%  |

# Market implied loss surface



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