Induction

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Three forms of inference: a schema

[D] Deduction

All P are Q

x is a P

x is a Q

[I] Induction

x is a P

x is a Q

All P are Q

[A] Abduction

All P are Q

x is a Q

x is a P

[I] and [A] are *ampliative* – they go beyond/add to what we know from before

Forms of induction

Enumerative induction

All known instances of P are Q

Therefore, All P are Q

Analogical inference

x is a P and a Q

y is a P

Therefore, y is a Q

Hume on induction

Problem:

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Hume’s explanation:

Believing that the future will resemble the past is a “habit of mind” that may lead to false conclusions.

An extension: statistical generalization

99 out of 100 observed football fields are green

Therefore, 99% of football fields are green

Does this suffer from Hume's problem of induction?

Statistical syllogism – are these valid?

Most trees have leaves

X is a tree

Therefore, X (probably) has leaves

Most P are Q

Most Q are R

Therefore Most P are R

Probability theory

DEFINITION. Let *S* = *{s1,s2,...,sn}* be a finite set of possible outcomes in a context *C*. *S* is a (finite) *sample space* for *C* iff exactly one outcome among the elements of *S* is or will be true in *C*.

EXAMPLE. Let *C* be the particular flipping of a coin. Then *S* = *{Heads, Tails}* is a sample space for *C*. Another sample space for *C* is *S'* = *{Heads is observed, Tails is observed, Cannot observe whether the coin is heads or tails}.* Yet another is *S''* = *{Heads is observed and someone coughs, Heads is observed and no one coughs, Tails is observed whether someone coughs or not}*.

Probability theory (cont.)

DEFINITION. Let S be a sample space, and $\emptyset \neq E \subseteq 2^S$ (E is a nonempty subset of the power set of S, i.e., it is a set of subsets of S). Then E is an *event space* (or *algebra of events*) on S iff for every $A,B \in E$:

(a) $S \setminus A = A^C \in E$ (the S-complement of A is in E) and

(b) $A \cup B \in E$ (the union of A and B is in E).

We call the elements of E consisting of single elements of S *atomic events*.

COROLLARY. If E is an event space on a sample space S, then $S \in E$.

EXAMPLE. If $S = \{Heads, Tails\}$, then $E = \{\emptyset, \{Heads\}, \{Tails\}, \{Heads, Tails\}\}$ is an event space on S. The atomic events are *{Heads}* and *{Tails}*.

Probability theory (cont.)

DEFINITION. Let *S* be a sample space and *E* an event space on *S.* Then a function *P: E* -> [0,1] is a (finitely additive) *probability measure* on *E* iff for every $A,B \in E$:

(a) *P(S)* = 1

and

(b) If $A \cap B = \emptyset$ (the intersection of *A* and *B* is empty, in which case we say that *A* and *B* are *disjoint*

events)*,* then $P(A \cup B) = P(A) + P(B)$ (*additivity)*.

The triple <*S,E,P*> is called a (finitely additive) *probability space*.

Probability theory (cont.)

COROLLARY. If $<S,E,P>$ is a finitely additive probability space, then for all $A,B \in E$:

(a) $P(A^C) = 1 – P(A)$ (*binary complementarity)*

(b) $P(\emptyset) = 0$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

DEFINITION. The *conditional probability* $P(A|B)$ of an event A given event B is defined as follows: $P(A|B) = P(A \cap B) / P(B)$

THEOREM. *Bayes's rule.* For events A and B, $P(A|B) = [P(B|A)P(A)] / P(B)$

Foundations of probability

Logicism

Frequentism

Subjectivism

Confirmation theory: a logicist approach to probability (Carnap)

Nicod's principle: Generalizations are confirmed (supported) by their positive instances and falsified by their negative instances

Equivalence principle: Whatever confirms a generalization confirms as well its logical equivalents

Paradoxes of confirmation

Paradox of the ravens

Paradox of “grue”

Paradoxes based on boundaries

Goodman’s “new riddle of induction”

What distinguishes good generalizations/ inductive inferences from bad ones

Examples of abstract background knowledge

"Blue" is a projectible predicate

"Grue" is not a projectible predicate

Determination rules: Citizenship determines the color of your passport