Deduction

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Features of formal logic

Symbols
- variables, constants, connectives/operators, [predicates, quantifiers]

Syntax (rules of formation for formulas)

Rules of inference

Axioms

Theorems

Additional features

Semantics

- truth tables, mappings/models, possible worlds)

Proof theory

- $\Gamma \models_{\mathcal{FS}} A, \quad \Gamma \vdash_{\mathcal{FS}} A$

- completeness, consistency, independence, soundness

Syllogistic/term logic (Aristotle)

Sentence types in the square of opposition:

- A-type: Universal and affirmative ("Every philosopher is mortal")
- I-type: Particular and affirmative ("Some philosopher is mortal")
- E-type: Universal and negative ("No philosopher is mortal")
- O-type: Particular and negative ("Not every philosopher is mortal")

Propositional logic (Boole)

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Predicate calculus/first order logic (Frege)

$$(\forall x.(P(x) \wedge Q(x)) \leftrightarrow ((\forall x.P(x)) \wedge (\forall x.Q(x)))$$
$$(\exists x.(P(x) \wedge Q(x)) \rightarrow ((\exists x.P(x)) \wedge (\exists x.Q(x)))$$
$$(\exists x.(P(x) \vee Q(x)) \leftrightarrow ((\exists x.P(x)) \vee (\exists x.Q(x)))$$
$$((\forall x.P(x)) \vee (\forall x.Q(x))) \rightarrow (\forall x.(P(x) \vee Q(x))$$
$$(\exists x.\forall y.R(x,y)) \rightarrow (\forall y.\exists x.R(x,y))$$
$$(\neg(\exists x.P(x))) \leftrightarrow (\forall x.(\neg P(x))$$
$$(\neg(\forall x.P(x))) \leftrightarrow (\exists x.(\neg P(x))$$
$$(\neg(\exists x\rho t.P(x))) \leftrightarrow (\forall x\rho t.(\neg P(x))$$
$$(\neg(\forall x\rho t.P(x))) \leftrightarrow (\exists x\rho t.(\neg P(x))$$
$$(\forall x.(x = t \rightarrow F(x))) \leftrightarrow F(t)$$
$$(\exists x.(x = t \wedge F(x))) \leftrightarrow F(t)$$

Modal logics (C.I. Lewis, Barcan Marcus, Kripke)

(P)	all tautologies of propositional calculus
(K^i)	$[i](\varphi_1 \to \varphi_2) \to ([i]\varphi_1 \to [i]\varphi_2)$
(4^{ij})	$[i]\varphi \to [j][i]\varphi$
(5^{ij})	$\neg[i]\varphi \to [j]\neg[i]\varphi$
(T^u)	$[u]\varphi \to \varphi$
(Incl)	$[u]\varphi \to [i]\varphi$
(Dual)	$\langle i\rangle\varphi \leftrightarrow \neg[i]\neg\varphi$
(MP)	IF $\vdash \varphi_1$ AND $\vdash \varphi_1 \to \varphi_2$ THEN $\vdash \varphi_2$
(N^i)	IF $\vdash \varphi$ THEN $\vdash [i]\varphi$

Other logics

Second/higher order

Intuitionist/constructivist

Multivalued

Probabilistic

Nonmonotonic

Proof methods

Construction

Reductio ad absurdum (proof by negation or contradiction)

Mathematical induction

Automated deduction and resolution

Formal analysis and deductive arguments

- Formality is usually partial even when conclusions are valid [see Nagel and Newman]
- Deduction is often embedded in wider, subjective arguments
- Deduction is used to derive conclusions based on doubted premises [Posner example] and to generate hypothesis [e.g.Higgs boson]
- Everyday deduction is often defeasible/ nonmonotonic [e.g. friend forgot our meeting?]

Some philosophies of mathematics

Realism

Logicism

Formalism

Intuitionism/constructivism

Psychologism

Psychology and syllogism

People are bad at solving most syllogistic problems

Wason selection task

Belief bias

Belief bias example

a

		Belief-Laden		Belief-Neutral
		Valid	Invalid	
Reasoning Condition	True	No poisons are sold at the grocers. Some mushrooms are sold at the grocers. Some mushrooms are not poisonous.	No reptiles can grow hair. Some elephants can grow hair. No elephants are reptiles.	Some monorchids are ground rhumbs. All ground rhumbs are rare. Some monorchids are not rare.
	False	Some green amphibians are toads. All green amphibians are frogs. Some frogs are toads.	No unhealthy foods have cholesterol. Some unhealthy foods are fried foods. No fried foods have cholesterol.	
Baseline		No poisons are sold at the grocers. Some mushrooms are sold at the grocers. Some frogs are toads.		Some monorchids are ground rhumbs. All ground rhumbs are rare. Some gimbel functions are not provable.

b