# 7 Action

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### 7.1. Revealed Preference

DEFINITION 7.1.1. Let $X$ be a set of outcomes. Then the *choice rule C,* which maps each *availability set A* (a subset of $X$) into a set of chosen elements of that subset, satisfies the *weak axiom of revealed preference* iff for all $A \subseteq X$ and $A' \subseteq X$, if $x,y \in A$, $x,y \in A'$, $x \in C(A)$, and $y \in C(A')$, then $x \in C(A')$.

The weak axiom stipulates that if $x$ is ever chosen when $y$ is also available, then there can be no availability set in which $y$ is chosen but $x$ is not.

We can define revealed preference in terms of 7.1.1.

DEFINITIION 7.1.2. $P$ is a *revealed preference relation* for a choice rule $C$ on a set of outcomes $X$ iff $P$ is a preference relation on $X$ and for all $x,y \in X$, $xPy$ iff there is some availability set $A \subseteq X$ such that $x,y \in A$, $x \in C(A)$, and $y \notin C(A)$.

## 7.2 Expected Utility Theory

DEFINITION 7.2.1. $P \subseteq X \times X$ is a *von Neumann-Morgenstern preference relation* on $X$ iff for all x,y,z,w $\in X$, and p,q $\in (0,1)$:
(a) *Closure.* $(x,p,y) \in S$.
(b) *Weak ordering.*
- $xPIy$ (*Reflexivity)*
- $xPIy$ or $yPIx$ *(Connectivity)*
- $xPIy$ and $yPIz$ implies $xPIz$ *(Transitivity)*

(c) *Reducibility.* $[(x,p,y),q,y]\ I\ (x,pq,y)$.
(d) *Independence.* If $(x,p,z)\ I\ (y,p,z)$, then $(x,p,w)\ I\ (y,p,w)$.
(e) *Betweenness.* If $xPy$ then $x\ P\ (x,p,y)\ P\ y$.
(f) *Solvability.* If $x\ P\ y\ P\ z$, then there exists $p$ such that $y\ I\ (x,p,z)$.

THEOREM 7.2.2. (J. von Neumann & O. Morgenstern, 1944).
If $P$ is a von Neumann-Morgenstern preference relation on $X$, then there exists a real-valued utility function $u$ defined on $X$, such that
(a) $xPy$ if and only if $u(x) > u(y)$, and $xIy$ if and only if $u(x) = u(y)$;
(b) $u(x,p,y) = pu(x)+(1-p)u(y)$;
(c) $u$ is an interval scale, that is, if $v$ is any other function satisfying 1 and 2, then there exist real

numbers *b*, and *a*>0, such that $v(x) = au(x)+b$.

The proof of 7.2.2 is beyond the level of this course.

EXAMPLE 7.2.3. Paradox: (M. Allais, *Econometrica, 21*:503-546, 1953) [updated version]. Compare the following two situations:

| *Situation 1* | *Situation 2* |
|---|---|
| Choose between: | Choose between: |
| Gamble 1: \$5000 with probability 1 | Gamble 3: \$5000 with probability .11 |
| Gamble 2: \$7500 with probability .10 | \$0 with probability .89 |
| \$5000 with probability .89 | Gamble 4: \$7500 with probability .10 |
| \$0 with probability .01 | \$0 with probability .90 |

Most people prefer gamble 1 to gamble 2, but prefer Gamble 4 to Gamble 3, even though this pattern is inconsistent with the independence axiom. In particular,
gamble 1 *P* gamble 2 can be rewritten as (\$5000,.11,\$5000) *P* [(0,1/11,\$7500),.11,\$5000]; and
gamble 4 *P* gamble 3 can be rewritten as [(0,1/11,\$7500),.11,\$0] ***P*** (\$5000,.11,\$0) (cf axiom 4). Since expected utility theory requires an ordering consistent with the interval function of utility, this pattern of preferences cannot be accommodated. In particular, the preference for gamble 1 over gamble 2 implies that u(gamble 1) > u(gamble 2), and hence that u(\$5000) > .10u(\$7500)+.89u(\$5000)+.01u(0), so .11u(\$5000) > .10u(\$7500)+.01u(\$0). But the preference in situation 2 implies that u(gamble 4) > u(gamble 3); hence .10u(\$7500)+.90u(\$0) > .11u(\$5000)+.89u(\$0), implying .10u(\$7500)+.01u(\$0) > .11u(\$5000), contradicting the inequality derived from the most common preference in situation 1.