## Project Title: Overtime Rules in Soccer and their Effect on Winning Percentages <br> Group Members: Elliot Chanen, Lenny Bronner, Daniel Ramos <br> Introduction:

We will examine the overtime rules of soccer to evaluate the randomness and fairness of different rules. Historically speaking there have been four different types of overtime rules: regular overtime, golden goal, silver goal and penalty shootout. We want to figure out which of these four overtime rules are most effective in producing a "fair winner" and if there is any possession advantage to kick-off/starting penalties in any of the systems.

This is relevant to organizations that run soccer tournaments (e.g. FIFA, UEFA etc.) for a number of reasons. First, these organizations may care on the "fairest outcome" - which team deserves to proceed to the next round. On the other hand, however, they may also be concerned in having "bigger teams" qualify as this increases their overall exposure and revenues. In the end, we may be able to draw conclusions about what organisations care about from how they changed these rules in the past. This is then useful for evaluating football associations themselves.

To give teams a preliminary rank, we will use the FIFA World Rankings, which will limit our data to international play but will give a standard ranking that is easy to replicate. We will then see if the "better" team, according to the FIFA rankings, ended up winning the game. Using that data, we will determine which overtime is the most random and which overtime preserves a win for the better team.

## History and Overview of Soccer Overtime Rules:

In association football, or soccer, as it is commonly referred to in the United States, overtime rules have undergone multiple changes and continue to have varying implementations across levels of play. In our focus of professional, international soccer, there have been four types of overtime rules. Overtime occurs when both teams are level after 90 minutes of play, however, not all contests enter overtime. The most common instances of overtime occur in knockout stages of tournaments, which is why we focused on international tournaments as described above.

Typically, group stages of tournaments do not enter overtime ${ }^{1}$. The four types of overtime rules are as follows: classic (or extra time), golden goal, silver goal, and penalty shoot outs (or PKs).

Classic overtime refers to the original overtime rules that include added extra time and penalty shoot outs, before the introduction of golden and silver goals. Under this rule, after 90 minutes of play, another two halves of 15 minutes are added to the time of play and afterward, if necessary, the game enters penalty shoot outs. First possession is determined with a coin toss.

Penalty shoot outs is the final stage of overtime in which each team takes turns attempting a shot from the penalty mark with only the opposing team's goalkeeper defending. However, unlike a normal penalty, once the ball has been kicked it cannot be played off the miss. The default number of penalties is five. However, depending on the order of the shots and the shots made and missed by each team, the match can end before each team has taken their five penalty shots. Under classic, golden goal, and silver goal rules, if the teams remain level after extra time, the winner of the math is determined by a penalty shoot out.

Golden goal is a type of sudden death and under this rule, the first team that scores in extra time is declared the winner. The match is ended immediately upon the goal. It was formally introduced in 1992 not as a strategic mechanism, but for entertainment purposes. However, it was short lived. While the intention of the golden goal rule was to stimulate offense and reduce the number of games decided by penalty shoot outs, it had the opposite effect. Instead of devoting their effort to score, teams were focusing on not conceding goals. Thus, it was abolished in FIFA authorized tournaments in 2004.

Silver goal rules are similar to golden goal rules. However, the match does not end immediately upon the goal. Instead, the team leading at the end of the extra time period is declared the winner. Moreover, the team leading after the first fifteen minute half is declared winner, but if the teams remain level, then the team leading after the next fifteen minute period is declared winner. If no team leads after each fifteen minute period, then the game enters penalty shoot outs.

## Data Collection:

For data collection we focused entirely on international games. This was because regular league games do not use any overtime rules, allowing for draws, and in cup games it was difficult, if not impossible, to identify an "objectively better team". We examined all games of the following tournaments:

O Olympics
O FIFA World Cup
O Gulf Cup
O UEFA Euro
O CONMEBOL Copa America
O CONCACAF Gold Cup
O African Cup of Nations
O AFC Asia Cup

- FIFA Confederations Cup

And identified all games that went into overtime or penalty shootout, which amounted to approximately 300 games. We went through every tournament of each of these cups and looked at each game that had ended in overtime- we noted the winner of each time and later checked what type of overtime had been played.

We then decided to focus on games since 1993, as that is when FIFA (International Federation of Football Associations) started releasing its monthly world ranking. This made sure that we were able to compare what the relative strengths of teams before the games took place. This cut our data down to half, as since 1993 approximately 150 games have gone into overtime. ${ }^{2}$ Finally, we went through all our games again and used the historical FIFA World Ranking ${ }^{3}$ to

2 " Interestingly between 1934 and 1993 around 150 games went into overtime, and between 1993 and 2013150 games went into overtime. That itself may be interesting to analyze.

3"http://www.fifa.com/fifa-world-ranking/ranking-tools/compareteams.html\#dataform=true\&t1=GER\&t2=ARG\&t3=COL\&t4=BEL\&timeRange=fifarankingcreation\&fromDa te=1993-08-01\&toDate=2014-11-26
check if the better or worse ranked team had won.

## Results:

|  | AET |  | ASDET |  | PSO |  | SILVER GOAL |  | TOTAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Win | Loss | Win | Loss | Win | Loss | Win | Loss | Win | Loss |
| Olympic Games | 3 | 2 | 0 | 2 | 1 | 2 | 0 | 0 | 4 | 6 |
| FIFA World Cup | 6 | 5 | 1 | 1 | 7 | 10 | 0 | 0 | 14 | 16 |
| Gulf Cup of Nations | 0 | 2 | 0 | 0 | 1 | 3 | 0 | 0 | 1 | 5 |
| UEFA Euro Cup | 0 | 1 | 3 | 0 | 6 | 5 | 0 | 1 | 9 | 7 |
| CONMEBOL Copa America | 1 | 0 | 0 | 0 | 9 | 6 | 0 | 0 | 10 | 6 |
| CONCACAF Gold Cup | 3 | 0 | 4 | 3 | 4 | 5 | 0 | 0 | 11 | 7 |
| AFC Asian Cup | 3 | 3 | 1 | 2 | 7 | 2 | 0 | 0 | 11 | 7 |
| African Cup of Nations | 5 | 4 | 0 | 0 | 10 | 7 | 0 | 0 | 15 | 11 |
| FIFA Confederations Cup | 3 | 2 | 0 | 0 | 4 | 1 | 0 | 0 | 7 | 3 |
|  | 24 | 19 | 9 | 8 | 49 | 41 | 0 | 1 | 82 | 68 |
|  | 56\% |  | 53\% |  | 54\% |  | 0\% |  | 55\% |  |

Our data shows that the better team wins games ending in classic overtime $56 \%$ of the time, games ending in golden goal $53 \%$ of the time, and games ending in penalty shoot-out' s $54 \%$ of the time. These results go very well with our hypotheses. Golden goal is the most random because it is based on a short time frame and rewards aggressive random goal scoring. The penalty shootout is also a random overtime. The best overtime for preserving a win for the higher ranked team is classic overtime. This was predictable in that it allows for the longest and fairest format of the overtime options. Note that this is a small sample size and there will be high variance with any form of overtime game.

Win Percentages by Ranking Difference


Consistent with our intuition, the graph above shows that as the difference among rank decreases in size, we arrive closer to a $50 \%$ winning percentage. Also consistent are the two extremes of positive rank difference, meaning a better team, and negative rank difference, meaning
a worse team, and their respective winning percentages. By looking at this data we can see that the overtime rules are quite fair.

## Recurrences:

We also used recurrence analysis to calculate the fairness of overtimes given teams of equal skill level. In particular we created a recurrence relation for penalty shoot-outs and a recurrence relation for golden goal. We didn' t create a recurrence for classic overtime because it is a finite micro soccer game with two fifteen minute halves and didn' $t$ lend itself to recurrence analysis as well.

For penalty shoot-outs, as described earlier, each team has an opportunity to shoot as many as 5 penalty shots and whichever team scores more in those 5 wins the game. We wrote some python code to determine how many states there are in those 10 shots ( 5 for each team). There are $2^{10}$ or 1024 possible states if every combination of 10 shots was possible, but, for example, if a team goes up 3-0 after 6 shots, the game is over. Thus there are 958 possible states with 353 wins for each team and 252 draws. After those 10 shots, if the two teams are tied, each team gets one shot until one team makes it and one team misses.

If w is the probability that team 1 wins and p is the probability that a penalty kick scores, we get the recurrence relation:

$$
w=p^{2} w+(1-p)^{2} w+p(1-p) \rightarrow w=1 / 2
$$

So, assuming each team has the same probability of scoring, a penalty shoot-out should give neither team an advantage.

We also wrote a recurrence relation for golden goal overtimes. Just like the old NFL overtime rules ${ }^{4}$, the first team that scores wins, which would make it seem as though the team to start with the ball has a major advantage. This is easily verified by looking at the recurrence. Let w be the probability that the team with the ball first wins and let p be the probability that the team with the ball scores.

$$
\mathrm{w}=\mathrm{p}+(1-\mathrm{p})^{2} \mathrm{w} \rightarrow \mathrm{w}=1 /(2-\mathrm{p})
$$

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So, assuming that each team has an equal probability to score on their possession, w is greater than or equal to $1 / 2$ because p is between 0 and 1 inclusive. Thus the team that starts with the ball in golden goal has an advantage. The team that starts with the ball in golden goal is determined by a coin toss, so golden goal overtime rules are random in who gets the advantage.

## Conclusion:

In conclusion, most overtime rules for soccer have been relatively fair and relatively random. That means that the overtime rule changes were often to increase entertainment and not really for increasing fairness. Classic overtime is clearly the most fair in letting skill take as large a part as possible in determining the overtime winner. Penalty shoot-outs and golden goal are the most random and are nearly coin flips to determine the winner of a game. Because we want skill to determine the winner of major international tournaments (our data), and believe that soccer officials would want the same, we are strong proponents of the classic overtime rules. We didn' $t$ analyze the added excitement value of penalty shootouts or golden goal, but classic overtime rules are generally the most fair, if we define fair as giving skill a large weight in determining outcome.

## Future Work:

To continue this study, one could calculate television ratings for games that end in ties, classic overtime, penalty shoot-outs, and golden goal. It is difficult to determine which rules each soccer association will want based on fairness, but determining how much money the league stands to gain based on the rules may be better at determining how they will decide their overtime rules.

