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Problem Set #4
BioE 326B/Rad 226B

1. In vivo correlation times
2. Magic angle spinning
3. MRI of cartilage
4. Saturation
5. MTC imaging

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In vivo rotational correlation times.

Design an experiment to estimate the rotational correlation time, τ_c, of water in *in vivo* human brain gray matter. Any human subjects involved in your experiment should survive the study unscathed. Use the approximate equations for T_1 and T_2. Namely:

$$\frac{1}{T_1} = \gamma^2 \langle B^2 \rangle \frac{2\tau_c}{1+\omega_0^2\tau_c^2} \qquad \frac{1}{T_2} = \gamma^2 \langle B^2 \rangle \left(\tau_c + \frac{\tau_c}{1+\omega_0^2\tau_c^2} \right)$$

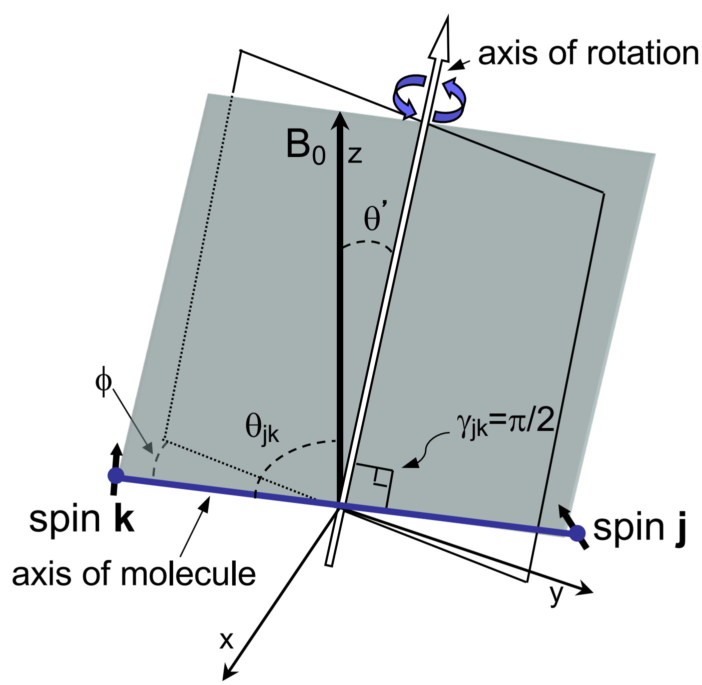
Magic Angle Spinning.

Assume the nuclear spins *j* and *k* are fixed relative to each other within a molecule. In a liquid the molecule will freely tumble in space. However, this is not the case for a solid. Let the molecule rotate about an axis making an angle θ' with respect to the static field B_0, where the frequency of rotation is high compared with the frequencies of interest in an NMR experiment.

a) Consider the simplified case where the radius vector from *j* to *k* is perpendicular to the axis of rotation. As the molecule rotates, the angle θ_{jk} (between B_0 and the internuclear vector) varies with time. Show that the dipolar coupling term in the Hamiltonian is proportional to

$$\langle 3\cos^2\theta_{jk} - 1\rangle_\phi = \frac{1}{2}\left(1 - 3\cos^2\theta'\right)$$

where $\langle\ \rangle_\phi$ denotes averaging over the angle ϕ ranging from 0 to 2π.

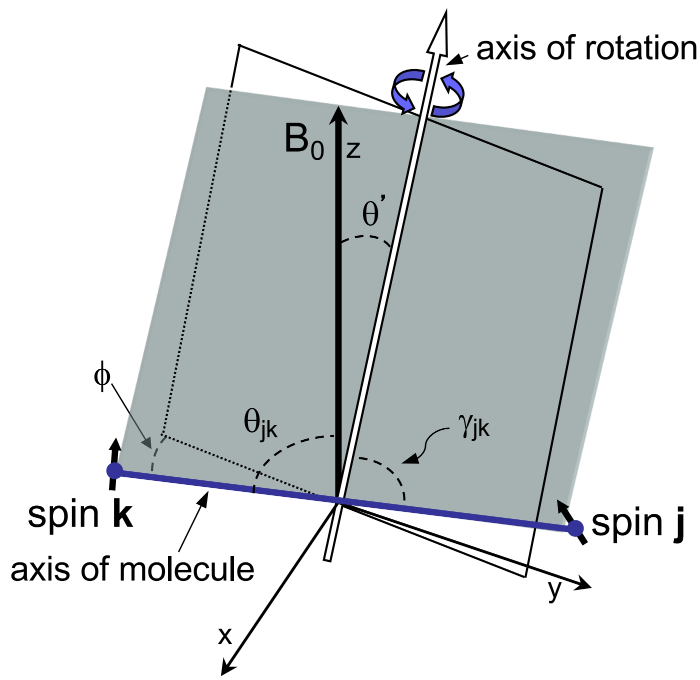


b) What happens to the NMR lineshape if θ' is chosen such that $(1-3\cos^2\theta')=0$?

Magic Angle Spinning. (cont.)

c) (Extra Credit). Consider the more general case in which the radius vector from *j* to *k* makes an angle γ_{ik} with respect to the the axis of rotation. Show that for this case, the dipolar coupling term in the Hamiltonian is proportional to

$$\langle 3\cos^2\theta_{jk} - 1\rangle_\phi = \frac{1}{2}\left(3\cos^2\theta' - 1\right)\left(3\cos^2\gamma_{jk} - 1\right).$$



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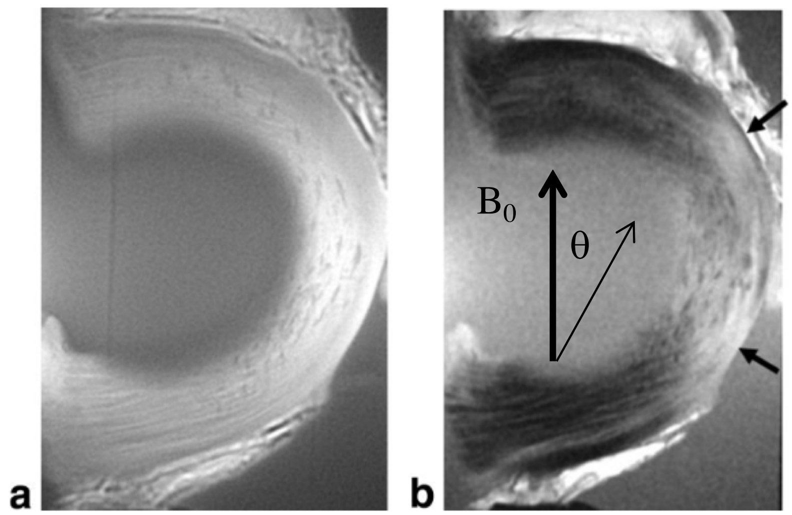
MRI of Cartilage

Cartilage is a highly structured tissue.

a) Explain the contrast seen in the images of normal cartilage below. In particular, why is the signal bright in the regions indicated by the arrows?

Bydder, et al., JMRI, 25:290–300 (2007)

Figure 4. Meniscus. UTE image (**a**) and conventional T1-weighted spin-echo (**b**) with vertical B_0. The UTE image shows high signal (with lower signal from calcification). The conventional spin-echo image shows a marked magic angle effect.



b) Plot T_2 as a function of θ. For what value of θ is T_2 maximized?

c) How does T_1 vary with θ?

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Steady-State NMR and Saturation.

a) Find the steady-state solution to the Bloch equation for the case of a continuous RF excitation, i.e. (in the rotating frame)

$$\vec{B} = [B_1, 0, B_0 - \omega/\gamma].$$

b) How large does B_1 have to be in order for the steady-state magnetization to be driven to approximately zero, a condition known as "saturation" (assume RF excitation is on resonance)?

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MTC imaging.

Design an experiment to help determine if the contrast observed in MT MRI (see Henkelman, et al, "MT in MRI: a review", *NMR in Biomedicine*, 14:57-64, 2001) is due to chemical exchange or a dipole-dipole interaction. This may be an animal/tissue study, and the subject(s) of the experiment need not necessarily survive the study. Explain how your experiment works and hypothesize a result.