

MS&E 337 Information Networks
Homework

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1. Suppose we run the Polya scheme starting with an urn with r red and b blue balls. What is the probability that the ball chosen at step k of the scheme is red?
2. Let $\phi(k) = \beta(1, 2k - 1)$. Divide the interval $[0, 1]$ into n pieces where the length of the k th interval is

$$\psi(k) = \phi(k) \prod_{j=k+1}^n (1 - \phi(j)).$$

Construct a graph on n vertices as follows: for any k , choose one point uniformly at random from the interval $[0, \sum_{i=1}^k \psi(i)]$. If the point is in the i 'th interval, connect vertex k to vertex i . Derive the degree distribution and the resulting graph. Prove that with high probability, its diameter is at most of $\mathcal{O}(\log n)$.

3. Show that the diameter of a graph with constant expansion is at most logarithmic in the number of vertices.