MS&E 337 Information Networks
Fall 2007
Homework #2
Due Fri. 12/14

1. Show that the diameter of a graph $G(V, E)$ with constant expansion is at most logarithmic in the number of vertices n.

2. Given a simple, undirected graph $G(V, E)$, the *switch* operation is as follows:

   - Pick two distinct edges (i, j) and (k, l) uniformly at random.
   - Choose a perfect matching M of $\{i, j, k, l\}$.
   - If the resulting graph remains simple, remove (i, j), (k, l) and add M. Otherwise do nothing.

   Note that a *switch* operation preserves the degree sequence of G. Show that the Markov chain defined by *switch* is ergodic with uniform stationary distribution over the space of graphs with the same degree sequence of G.

3. A coordination game on a graph G with parameter q is a game in which, across each edge (v, w), nodes v and w each receive a payoff of q if they both choose behavior A, they each receive a payoff of $1 - q$ if they both choose behavior B, and they each receive a payoff of 0 if they choose opposite behaviors. It is natural to ask what happens if we consider a more general kind of coordination game on each edge. Suppose in particular that, on each edge (v, w), node v receives payoff u_{XY} if it chooses strategy X while w chooses strategy Y, for any choice of $X \in \{A, B\}$ and $Y \in \{A, B\}$. Moreover, to preserve the coordination aspect, we assume that it is still better to play matching strategies: $u_{AA} > u_{BA}$ and $u_{BB} > u_{AB}$.

   Prove that for any infinite graph G with finite node degrees, and for any choice of payoffs $\{u_{AA}, u_{BA}, u_{AB}, u_{BB}\}$ satisfying $u_{AA} > u_{BA}$ and $u_{BB} > u_{AB}$, there exists a real number q such that the following holds: A finite set S is contagious in G with respect to the coordination game defined by $\{u_{AA}, u_{BA}, u_{AB}, u_{BB}\}$ if and only if it is contagious in G with respect to the coordination game with parameter q.

   (Note: this question is from *Cascading Behavior in Networks: Algorithmic and Economic Issues* by Jon Kleinberg, available at `http://www.cs.cornell.edu/home/kleinber/agtbook-ch24.pdf`, also handed out in class. For background and definitions, see his paper.)