Stanford University, Dept of Management Science and Engineering

MS&E 318 (CME 338) Large-Scale Numerical Optimization

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Homework 4, Due Monday May 11

The Bartels-Golub-Reid update uses a product of elementary matrices of the form

$$M = \begin{pmatrix} 1 & \\ \mu & 1 \end{pmatrix} \quad \text{or} \quad \widetilde{M} = \begin{pmatrix} & 1 \\ 1 & \mu \end{pmatrix},$$

where each elementary matrix operates on two rows of U. The choice between M and $\widetilde{M}$ is made to keep $|\mu| \le$ `UpdateTol` $= 10$, 5, or 2 (say) while allowing some freedom to have the "1" operate on the sparsest row of U. (That is, to use the sparsest row of U as pivot row.) The update procedure should be stable as long as the elementary matrices are well-conditioned, or at least behave like well-conditioned matrices.

1. Suppose $\mu > 0$. With the help of Matlab, complete this table in terms of μ:

$$\operatorname{cond}\begin{pmatrix} 1 & \\ \mu & 1 \end{pmatrix} \approx \begin{cases} \ldots, & \mu \ll 1, \\ 2.6180, & \mu = 1, \\ \ldots, & \mu \gg 1. \end{cases}$$

   (Fill in the dots ... with expressions involving μ.)

2. If $M_k = \begin{pmatrix} 1 & \\ \mu_k & 1 \end{pmatrix}$, what is $M_1 M_2$?

3. If $M_1 = \begin{pmatrix} 1 & \\ 90 & 1 \end{pmatrix}$, $M_2 = \begin{pmatrix} 1 & \\ 90 & 1 \end{pmatrix}$, what are $M_1 M_2$ and $\operatorname{cond}(M_1 M_2)$?

4. How does the previous $\operatorname{cond}(M_1 M_2)$ compare with the bound

$$\operatorname{cond}(M_1 M_2) \le \operatorname{cond}(M_1)\operatorname{cond}(M_2)?$$

5. If

$$L_1 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ 10 & & & 1 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & 10 & & 1 \end{pmatrix}, \quad L_3 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 10 & 1 \end{pmatrix},$$

   what are $L_1 L_2 L_3$ and $\operatorname{cond}(L_1 L_2 L_3)$?

6. How does the previous $\operatorname{cond}(L_1 L_2 L_3)$ compare with the bound

$$\operatorname{cond}(L_1 L_2 L_3) \le \operatorname{cond}(L_1)\operatorname{cond}(L_2)\operatorname{cond}(L_3)?$$