Stanford University, Dept of Management Science and Engineering

MS&E 318 (CME 338) Large-Scale Numerical Optimization

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Homework 3, Due Monday May 4

1. Consider the LP problem
$$\begin{aligned} \min \quad & c^Tx \\ \text{s.t.} \quad & Ax = b, && \text{(a)} \\ & x \ge \ell, && \text{(b)} \end{aligned}$$
where A is $m \times n$ $(m < n)$. The Primal Simplex method is an active-set method that moves from vertex to vertex within the feasible region. A vertex is defined by a set of n independent equations drawn from the constraints (a) and (b). In terms of the usual basis partition
$$AP = \begin{pmatrix} B & N \end{pmatrix}, \qquad x = P\begin{pmatrix} x_B \\ x_N \end{pmatrix},$$
write a single matrix equation that defines the current basic and nonbasic variables x_B, x_N as a vertex. You may assume that ℓ is finite and the nonbasic variables are currently on their lower bounds. The matrix equation should involve B and N and a few other items.

2. Suppose $P = I$ and the above basic solution is optimal, with dual variables (y, z) satisfying $B^Ty = c_B$ and $z = c - A^Ty$. Also suppose the last nonbasic variable has bound $\ell_n = 1$ and reduced gradient $z_n = c_n - a_n^Ty = 1.0$. Now suppose ℓ_n is changed from 1 to -1. Is the previous solution still optimal? (Yes/No. Why? What does z_n tell us?)

3. Primal Simplex proceeds by *moving a nonbasic variable away from its current value*, while remaining feasible. Show how it can be restarted at the previous optimal solution (with $x_n = 1$). What will happen during the first iteration?

4. Some systems would set $x_n = -1$ before restarting. What does the first basic solution x_B look like? (What linear system defines x_B? Is the solution sure to be feasible? Why was it a good idea to start with $x_n = 1$?)

5. Suppose the vector x satisfies $\ell \le x \le u$ and d is a search direction. Write some MATLAB commands to solve the 1D optimization problem
$$\max \ \alpha \quad \text{s.t.} \quad \ell \le x + \alpha d \le u.$$

6. When the pth column of B is replaced by $\bar{a}$, we have $\bar{B} = B + (\bar{a} - a)e_p^T = BT$, where $T = I + (v - e_p)e_p^T$ is a special matrix constructed from a sparse vector v. Note that Primal Simplex needs v anyway in order to compute the direction Δx that led to this basis change. The original product-form (PF) update was therefore remarkably simple: Pack the nonzeros of v into a sequential file (with v_p first so we know where it is).

   Later iterations require the solution of $Tx = w$ and $T^Tx = w$ for various rhs's w. By thinking of T as a permuted triangle, show how to solve such systems. (Observe that we *don't* have to work with T^{-1}. Triangles are already ideal!)