Course Description and Syllabus

This course is intended to provide a thorough background of computational methods for the solution of linear and nonlinear optimization problems. Particular attention will be given to the description and analysis of methods that can be used to solve practical problems. Although the focus is on methods it is necessary to learn the theoretical properties of the problem and of the algorithms designed to solve it.

The general problem addressed may be written as:

$$\min_{x} \; F(x) \;\; \text{subject to} \;\; c(x) \geq 0,$$

where x is an n-vector, $c(x)$ is an m-vector, and $F(x)$ and the elements of $c(x)$ are twice continuously differentiable scalar functions. Smoothness in the functions is critical to the methods described in this course. However, methods to solve this class of optimization problem are critical in solving non-smooth problems and also optimal control problems. The associated problem of solving systems of nonlinear equations is covered in the course.

The flow of the course is to start with problems in one variable and then progressively introduce more complexity is the following manner:

Unconstrained nonlinear problems

Linear equality constrained problems

Linear inequality constrained problems

Nonlinear equality constrained problems

Nonlinear inequality constrained problems

Within each of these categories of problem specific subclasses will also be described. For example, when covering methods for unconstrained optimization we shall also describe special methods for the problem of nonlinear least squares (very common in finding the parameters in modeling) and methods of solving systems of nonlinear equations. Similarly linear and quadratic programming will covered in methods for linear inequality constrained problems.

Aspects covered for all categories of problem include derivation of the optimality conditions, proof of global convergence and asymptotic rate of convergence.

Methods covered may all be viewed as variants or generalizations of Newton's method. We shall need to describe the methods in detail and that leads to careful attention to precisely how the numerical linear algebra operations are performed. This leads among other things to describe how to update matrix factorizations. Specific methods covered include:

Linesearch methods

Trust-region methods

Newton's method

Modified Newton's methods

Quasi-Newton methods

The linear and nonlinear conjugate gradient methods

The Simplex method

Penalty function methods

Barrier function methods

Augmented Lagrangian methods

Sequential linearly constrained methods

Sequential quadratic programming methods