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The Usefulness of Core Theory in Economics

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Core theory furnishes a useful framework for studying a wide variety of economic problems. It has an undeserved reputation of being too abstract, owing mainly to the manner in which it is employed in the theory of general equilibrium. In fact, core theory is a highly flexible way of looking at practical economic problems, especially problems in industrial organization. This paper seeks to show how simple numerical examples can illustrate the idea of the core, and, in turn, how the core can illustrate basic principles of economics.

Principles of the Core

The theory of the core begins with the assumption that there are n individuals who can do something either all together, individually or in small groups. For economic applications, a typical example is trade in a market, where all individuals may trade with each other in a single market, or in submarkets, or some may decide not to trade at all. The theory assumes that the individuals can measure the results of their actions. For the example of trade in a market, it is traditional to assume an individual measures the outcome by the utility from the bundle of commodities. Alternatively, an individual can measure the gains from trade in terms of money. For a buyer, this is the maximum amount the buyer would have been willing to pay for the quantities purchased minus the amount actually paid. For a seller, it is the actual receipts minus the amount the seller would have been willing to accept

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for what was sold. Thus the theory of the core has three elements: n individuals; the various groups they can form, called coalitions; and functions that measure the results of the actions taken by the individuals and coalitions.

There are some outcomes that the whole group of individuals cannot improve. These are the outcomes such that it is not possible to make one person better off without making at least one other person worse off. Such outcomes are called Pareto optimal. They involve no deadweight loss. The originator of this theory, F. Y. Edgeworth (1881), went on to obtain remarkable results showing how competition among many traders and coalitions leads toward a Pareto efficient outcome. Edgeworth called the mechanism that produces this result "recontracting." The approach is to consider all possible coalitions of traders, recognizing that any coalition of traders will only participate in the market as a whole if and only if they can do at least as well as they could off by themselves in their own coalition. To put it another way, the best outcomes available to a coalition set lower bounds on what its members would be willing to accept as participants in the whole market. For example, the complete set of 1-person coalitions implies the constraint that each individual will not trade in the market unless the trade makes that person better off. Next consider all 2-person coalitions. In this case, any trades must make the individuals at least as well-off as they would be in choosing any possible 2-person partnership. When this logic is extended from 3-person on up to the n-person coalition, there is a total of $2^n - 1$ possible coalitions, with each coalition placing a constraint on the outcome of trade. The larger the number of traders, the smaller is the range of outcomes without deadweight losses. Under certain conditions, the terms of trade that can satisfy all these constraints constitute a competitive equilibrium.

Core theory examines this process systematically. Outcomes that are unacceptable to some coalition because it can do better for its members are said to be dominated. The set of undominated outcomes constitutes the core. Depending on the number of individuals and the process of recontracting, the core will sometimes consist of a range of outcomes, sometimes a single outcome and sometimes the core will not exist at all.

An Example of Pure Exchange with a Nonempty Core

A simple example can illustrate these concepts. Say there are three individuals and the first two are potential buyers of a house from the third. The potential seller will not sell the house for less than \$100,000. Buyer 1 will not pay more than \$120,000 and buyer 2 not more than \$150,000. (From now on units are understood to be in \$1,000.) Let x denote the return to the seller, y_1 the gain to buyer 1 and y_2 the gain to buyer 2. In case it turns out the owner of the house sells it, x is the price of the house. The owner can ensure $x \geq 100$

because retaining the house is an option worth at least 100. For the potential buyers, $y_1 \geq 0$ and $y_2 \geq 0$, because each buyer can refuse to make a purchase and thereby can ensure a net gain of zero, no matter what anyone else does. These three inequalities are the constraints for the three 1-person coalitions.

A coalition of both buyers can do the same as either one of them separately, and the coalition of this pair will have the same lower bound on the sum of their gains. As a result, the relevant constraint for the coalition of both buyers is that $y_1 + y_2 \geq 0$.

There are two more coalitions involving pairs of traders. Each is a coalition between the seller and either one of the buyers. A trade between the seller and buyer 1 must ensure them a return equal to the larger of the two values $\{100, 120\}$. The first value comes from the fact that the seller will only participate if the gain is at least 100; thus the gain to the coalition of the two must also be at least 100. But what determines the minimum gain is the willingness of buyer 1 to pay 120. Given the existence of an outside, higher offer for the house, with the possibility of reselling it to buyer 1 afterwards, the coalition between the seller and buyer 1 would reject any offer below the valuation either places on the house because each member of this coalition can bid for the house in competition with an outside offer. Similarly, the coalition between the seller and buyer 2 will demand a return of at least 150 because this is the larger of the two values $\{100, 150\}$ applicable to this coalition. These conditions set two additional constraints on the outcome: $x + y_1 \geq 120$ and $x + y_2 \geq 150$.

Lastly, the valuation for the coalition of all three traders equals the maximum of $\{100, 120, 150\}$. To put it another way, the cumulative return for the coalition of the whole must be at least as much as the most generous buyer is willing to pay. This condition puts an upper bound on the sum of the returns, given by $x + y_1 + y_2 \leq 150$.

Any triplet $\{x, y_1, y_2\}$ that can satisfy all these inequalities is undominated and is said to be in the core of the market. As an example, consider the triplet $\{115, 0, 35\}$, where buyer 2 purchases the house at a price of 115. This triplet is not in the core; it does not satisfy the inequality that the gains to the coalition of the seller and buyer 1 must sum to at least 120. Thus, the seller and buyer 1 can form a coalition leading to the triplet $\{118, 2, 30\}$ in which buyer 1 buys the house from the seller for 118 and resells it to buyer 2 for 120 so that buyer 2 gains 30.

But this set of trades is not in the core either, because it does not fulfill the inequality that the gains to the coalition of the seller and buyer 2 must be at least 150. The imputation $\{120, 0, 30\}$, in which the seller deals directly with buyer 2, *is* in the core. It is undominated and satisfies all the constraints. More generally, all imputations in which $y_1 = 0$, $x + y_2 = 150$, $x \geq 120$ and $y_2 \geq 0$ are undominated and form the core of the market. Thus, any outcome where buyer 2 buys the house for at least 120, but no more than 150, is in the core and nothing else is in the core.

When a nonempty core exists, it means that any trader or group of traders prefers the outcome determined by the whole market to those they could get in any possible submarket involving a subset of traders. These submarkets present feasible alternatives that place limits on the prices that can emerge from the market as a whole. When the market has a nonempty core, it can survive all possible competing alternatives. In core theory, coalitions compete for members by making offers to individuals to induce them to join the coalition. The grand coalition, which includes all the members, can survive only by offering terms that are at least as good as any feasible offer coming from a subcoalition.

Although the example presented here illustrates only the simplest case of pure exchange, it can be extended into a comprehensive analysis of nearly everything there is to say on this topic. Because there is only one seller in the example, competition is present only between the two buyers. More complicated examples would have many sellers and buyers. The core is not empty for the more general case in which there are m sellers and n buyers who each seeks or offers at most one unit of the commodity. It can be shown that all those who sell the commodity must get the same price. This common price is determined by the constraints that emerge as a result of the terms that various coalitions could arrange by dealing among themselves. A still more general model allows the buyers and the sellers to seek or offer more than one unit of the commodity. If the demand schedules of the buyers are downward sloping and the supply schedules of the sellers are upward sloping then there is a nonempty core. Therefore, it remains true that each individual prefers the terms determined in the whole market to those that subsets of traders could agree upon by confining trade among themselves.

However, with multi-unit traders, a wider range of alternatives is consistent with the core constraints than if each trader were replaced by an equivalent set of single-unit traders. This is true because multi-unit traders do not make or tender offers for their commodities one unit at a time unless the forces of competition compel them to. In particular, it need no longer be true that a single price must prevail for the commodity throughout the market in this case. Different sellers could get different receipts per unit and different buyers could pay different prices per unit. In these cases core theory shows how the sizes of the traders could affect the outcome.

A still more general model allows the traders to deal in bundles of continuously divisible commodities. There is a nonempty core in this case if the valuation functions of the traders are continuously increasing concave functions of quantities. The most general analysis assumes a continuum of traders. Think of each trader as indexed by a real number and suppose there are as many traders as there are real numbers in the unit interval. An individual trader is infinitesimally small and has a correspondingly small effect on the outcome of trade. A nonempty core exists for this market under very general conditions on the preferences of the traders. Even with many "crazy" traders, who violate assumptions of rational choice such as transitivity or revealed preference, there is a nonempty core if the "sane" traders are sufficiently more numerous than the "crazy" ones.[1]

Examples of an Empty Core

Sometimes it is not possible to satisfy the conditions for a nonempty core. This happens when the lower bounds on the terms that the coalitions would be willing to accept cannot all be met by the grand coalition. In this event, the core is empty. It means a fully competitive market fails to bring about a Pareto optimal result.

Consider an example which is easily visualized as applying to the cost conditions of the airline industry. Two sellers each operate an airline; they have one airplane apiece. The first airline, A_1, has a small airplane that can carry only two passengers at a total cost of 85. Or, to put it another way, not making the trip will save a cost of 85. (In the preceding example, this condition corresponds to the seller of the house retaining it, as if selling it to himself at the minimal price he would be willing to take for it, which is 100 in that example.) The second airline, A_2, has a bigger airplane that can carry up to three passengers at a total cost of 150. It can avoid this cost entirely by not making the trip. Note that the costs of the airlines are not dependent on whether they fly partly or entirely full, but only on whether they make the trip at all. In this example, let us agree to ignore both fixed and variable costs. This does not affect the validity of the results and it simplifies the arithmetic.

Let there be three potential travelers: B_1, who is willing to pay at most 55 for the trip; B_2, who is willing to pay at most 60 for the trip; and B_3, who is willing to pay up to 70. The total number of coalitions in this example is $2^{2+3} - 1 = 31$. Let us again adopt the terminology that x represents the returns to the sellers while y represents the gains to the buyers. For starters, consider the 1-person coalitions; these have a buyer or a seller acting alone. To make a deal the sellers must receive enough to cover the costs of a trip: that is $x_1 \geq 85$ and $x_2 \geq 150$. The buyers must all perceive themselves as better off by making a purchase; therefore, y_1, y_2 and $y_3 \geq 0$.

Plainly, no coalition of a seller with only one buyer can cover the cost of a trip. Also, a coalition of either two or three buyers cannot gain more than zero. The remaining 2-person coalition, the two airlines without any passengers, cannot get more than the sum of what they could each get by themselves. Therefore, the interesting possibilities involve a seller with at least two buyers.

Consider the alternatives for the small airline A_1 with the three possible pairs of buyers. Again the return for each possible coalition is determined by the maximum of what the various purchasers might pay. For example, the

[1]Models of pure exchange using core theory are in Scarf and Debreu (1963). For a continuum of traders, see Aumann (1964, 1966) and Hildenbrand (1974). Simpler versions are in Telser (1972).

potential gains for the coalition A_1, B_1, B_2 would be determined by the maximum of what the two buyers would pay and the seller would demand, that is the maximum of $\{55 + 60, 85\}$, which equals 115. Similarly, the gains for the coalition A_1, B_1, B_3 would be the maximum of $\{55 + 70, 85\}$ or 125 and the gains for the coalition A_1, B_2, B_3 would be the maximum of $\{60 + 70, 85\}$ or 130. These conditions lead to the following three constraints:

$$x_1 + y_1 + y_2 \geq 115,$$

$$x_1 + y_1 + y_3 \geq 125,$$

$$z_1 + y_2 + y_3 \geq 130.$$

Of course, seller 2 can also make a potential deal with all three buyers. Hence this coalition $\{A_1, B_1, B_2, B_3\}$ can guarantee itself a return of $185 = \max\{150, 55 + 60 + 70\}$. The constraint on the gains of this potential deal is $x_2 + y_1 + y_2 + y_3 \geq 185$.

Lastly, there is the maximum return available to the coalition of all five individuals, the three buyers and the two sellers. This coalition of everybody has two especially relevant alternatives: one where the first airplane flies full and the second does not fly at all; and the other where the second airplane flies full and the first does not fly at all. In the first, the two travelers who value the trip the most fly with the smaller airline; that is, B_2 and B_3 fly with A_1, B_1 does not make the trip and A_2 saves the avoidable cost of its service. For this alternative, the coalition of all five would get $60 + 70 + 150 = 280$. In the second alternative, A_1 does not fly thereby avoiding the cost of 85, and A_2 carries all three passengers generating a value of 185. For the second alternative, the return would be $185 + 85 = 270$. The value for the coalition of five is the larger of the returns under these two alternatives. This is given by $\max\{280, 270\}$. Therefore, the most efficient arrangement does not satisfy the demand of buyer 1; buyer 2 and buyer 3 make the trip on the airplane of seller 1; and seller 2 saves the cost of 150. Consequently, the upper bound on the x's and y's is given by

$$x_1 + x_2 + y_1 + y_2 + y_3 \leq 280.$$

There is no core solution to this market; it is impossible to describe an outcome acceptable to every possible coalition. This is so because the set of inequalities given here has no solution. An outcome in the core must be Pareto optimal so that it gives the most efficient arrangement. Pareto optimality requires that

$$x_1 + y_2 + y_3 = 130,\ x_2 = 150,\ y_1 = 0 \quad \text{and} \quad x_1 + x_2 + y_1 + y_2 + y_3 = 280.$$

The first equation says that airline 1 carries passengers 2 and 3. The second says that airline 2 saves its avoidable cost, and the third says that buyer 1 gains zero either because of not making the trip or because of paying his maximal valuation of the trip, which is 55. The last equation makes the sum of everybody's gains as big as possible.

There are two constraints that set upper bounds on the gains of buyers 2 and 3 necessary to have a nonempty core. The coalition involving passengers 1 and 2 going on airplane 1 can ensure themselves a gain of 115. Since $y_1 = 0$ is necessary for a nonempty core, it follows that the sum of x_1 and y_2 is bounded below by 115. But $x_1 + y_2 + y_3 = 130$ is also necessary for a nonempty core so that the gain to passenger 3 cannot exceed 15 (= 130 − 115). Likewise, the coalition in which passengers 1 and 3 go on airplane 1 can guarantee themselves a return of 125. Again, for a nonempty core, $y_1 = 0$ and $x_1 + y_2 + y_3 = 130$. This puts an upper bound on the gain of passenger 2 given by 5 (= 130 − 125). Therefore, a nonempty core requires that the sum of the gains to passengers 2 and 3 does not exceed 20, or, in symbols, $y_2 + y_3 \leq 20$. However, the second airline can also offer a deal to the three buyers. The lower bound for this deal is 185, so the sum of the gains to the four participants in this coalition satisfies the inequality $x_2 + y_1 + y_2 + y_3 \geq 185$. A nonempty core requires that $y_1 = 0$ and $x_2 = 150$. Therefore, substituting in the preceding inequality, this competition from the second airline would set a lower bound on the gains to buyer 2 and buyer 3 which is

$$y_2 + y_3 \geq 35 = 185 - 150.$$

Consequently, the upper and lower bounds on the sum are contradictory, which proves the core is empty. Telser (1978, ch. 2) contains a detailed analysis of these airplane examples.

Admittedly, the cost conditions in this example are contrived in such a way as to make the core empty. It should be noted, for the record, that extending the model from pure exchange to production need not give an empty core. Also, in the example, the total capacity exceeds the total quantity demanded. Although capacity equal to demand is a sufficient condition for a nonempty core, it is not a necessary condition. The core can be empty for other reasons. The key lesson of this example is that introducing production into the model does add many complications to the theory, which in turn lead to important practical ramifications. To understand why this is so, let us look more closely at how the model treats coalitions.

The return to a coalition of n individuals stems from the activities of its members. The coalition can survive if and only if it can offer its members more than they could get by breaking away and forming their own subcoalition or joining another coalition. Each coalition must also ask whether it can make its present members better off by expanding its membership. There will be a gain if the incremental return from adding a member exceeds the current return per member. Or, to put it another way, at the optimal size of a coalition, the return per member is a maximum. When the coalition of everybody maximizes the return per member so that it is optimal for everybody to join the grand coalition, the core is not empty. Applied to markets, this means there is a core if the traders are better off with the terms they get in the whole market than they would be in any submarket. Therefore, a market with a nonempty core attracts all the trade.

Consider a coalition of individuals who constitute the demand for various commodities. The return to a member of such a coalition equals the valuation of the commodities minus the sum of the quantities bought at prices equal to the marginal cost of producing these commodities. An expansion of the size of the coalition corresponds to an increase in the demand for the commodities from the new members of the coalition. This does not harm the present members provided the marginal costs of the commodities they buy, which equals the prices they pay, do not increase when the demand expands. When adding more and more members to the coalition does not raise marginal costs, the optimal coalition is the coalition of everybody and the core exists. Therefore, a nonempty core requires constant or increasing returns to scale. Another way to see this is by starting with the coalition of everybody and asking whether it can survive. Survival means that it can offer its members a better deal than they could get in any subcoalition. However, with rising marginal cost, a subgroup of demanders has an incentive to break away from the grand coalition. By doing this, they can obtain their commodities at lower prices because marginal costs are lower.

A specific example of an empty core is an industry with identical firms having U-shaped average cost curves such that marginal cost rises with output and equals average cost at the positive output where average cost is a minimum. This case, presented in many textbooks on elementary economics, is familiar to readers of Jacob Viner's famous article (1931) on cost curves.[2] In an optimal coalition it must be true that firms are producing where their unit cost is at a minimum; otherwise, a coalition would form where the firm changes its output, produces at a lower unit cost, and sells to the demanders in this coalition at a lower price than they would have to pay in the grand coalition. But the efficient equilibrium for the market as a whole must be where the price at the quantity demanded equals the marginal cost of the total quantity produced by the firms. The problem arises because the efficient equilibrium for the industry also involves the optimal number of firms that must reckon with the fact that this number is an integer. Changing from n to $n + 1$ firms affects the total cost of production in two ways: it changes both the total variable cost and the total fixed cost of the industry. Optimality requires a comparison between the increase of the fixed cost from having one more firm and the reduction of the variable cost from having each of the firms producing a smaller output while the total output satisfies the total demand at a price equal to marginal cost. The efficient industry equilibrium will not in general be where the unit cost of the firm is at a minimum. The magnitude of the difficulty

[2]Such industries are called Viner industries and are studied in Telser (1978, ch. 3, sec. 4).

depends on size of the output per firm where unit cost is a minimum compared to the total equilibrium quantity. The gap between a nonempty and an empty core is smaller, the more numerous the firms in the optimal industry equilibrium. Therefore, there is a closer and closer approximation to the standard competitive equilibrium, the bigger the Pareto-optimal number of firms.

Resolving an Empty Core

What happens in a market when the core is empty? The answer to this question is referred to as "resolving" an empty core. For the type of cost conditions in the second example, a general method of resolving an empty core requires imposition of suitable upper bounds on the quantities that may be sold by certain sellers. Such bounds always exist.[3]

It may seem that a proposal for restricting output must be inefficient, since it has the character of a profit-maximizing cartel. However, in the situation where no core exists, such upper bounds can be efficient, if suitably chosen, in the sense that although removing the bounds can help some economic actors, it can do so only at the expense of creating a deadweight loss for the whole group. An example of this occurred in Hyde Park, which has a regular limo service to O'Hare Airport, 25 miles away. Going to O'Hare, this service makes scheduled stops at certain times and locations in the neighborhood where it picks up passengers. Demand is heavy at spring and Christmas breaks. Once, during spring break, we were waiting with several students for the 7 a.m. limo to O'Hare. Just before 7, a Yellow taxi pulled up, flag down, meter off and offered to take up to five customers to O'Hare for $8 each, non-stop, which is below the limo price. The taxi got a full load and left for O'Hare. No one remained for the regular limo service. Although the taxi driver and those who accepted his offer were made better-off, the incentive for providing the regular limo service was impaired. The limo service would not continue, thereby harming the interest of any residents of Hyde Park desiring regular limo service to the airport, unless it could stop this skimming by taxis.

This example closely matches the situation facing shipping conferences. Here a group of shipping firms promises to furnish regular service and enough capacity to handle the cargo of the shippers at certain ports. The conference sets minimum rates. Cargoes accumulate at the port between arrivals of the regular freighters and are transported on the ships of the members of the conference. However, tramps may arrive at the port just before the scheduled stop and offer to take the freight at rates below the conference rates. The shippers who take advantage of these lower rates reduce the incentive for the

[3]For a detailed analysis of suitable upper bounds on output that can give a nonempty core, see Telser (1987, Ch. 5).

conference to provide regular service. The lack of regular service would harm the shippers. Both the shippers and the carriers seek arrangement that can preserve regular service. One way that the conference deals with the problem of tramps is by offering shippers a deferred rebate for loyalty. Shippers who use members of the conference exclusively for a prescribed period, usually a year, get a rebate proportional to their total annual shipping costs at the end of the year. This helps ensure loyalty by the shippers to the conference members and thereby preserves regular service. Studies of shipping conferences by Sjostrom (1989) and Pirrong (1992) support the view that a cooperative arrangement among shipping firms is consistent with efficiency and a competitive return.

To demonstrate the flexibility of core theory as a tool of analysis, the airline example offered earlier can be adapted to illustrate how long-term (forward) contracts or vertical integration can sometimes resolve an empty core problem. Forward contracts work better when each party may have many interests other than the commodity involved in the particular exchange. Vertical integration entails closer relations between the two parties and usually needs agreement on many aspects of their operations so it works better when the two parties have a number of common interests.

Let B_1 own A_1, the smaller airline in the earlier example, and B_2 own A_2, the bigger airline. The third buyer, B_3, buys the service on the open or spot market. Neither B_1 nor B_2 has a big enough demand to cover the cost of using the airline for themselves exclusively. Hence each has an incentive to seek outside business. Notice that by this vertical integration, B_1 and B_2 become involved in the airline business. This would not happen if they made forward contracts for long-term service with the airlines as separate businesses. With three economic actors, there are three singleton coalitions: B_1 must receive the maximum of $\{85, 55\}$; the first for being willing to run its plane, the second for what it is willing to pay for a flight. Similarly, B_2 must receive the maximum of $\{150, 60\}$, and B_3 must not incur a loss by participating.

For the three groups, there are three coalitions of pairs. In a coalition between B_1 and B_2, there are two alternatives: either both fly in the smaller plane and save the cost of using the bigger one (which is 150) or neither flies so that they save $150 + 85$. The potential for gain will be the maximum of $\{55 + 60 + 150,\ 150 + 85\}$, which is 265. In a coalition between B_1 and B_3, both passengers must choose either to fly in the small plane owned by B_1, which has a value of $55 + 70$ or save the cost of the trip which has a value of 85. In this case, the maximum of $\{125, 85\}$ will be 125. Lastly, in a coalition between B_2 and B_3, the pair can either fly in the big plane, owned by B_2, where the trip has a value of $60 + 70$ or they can forego the trip and save 150. The potential for gain is the maximum of $\{130, 150\}$, or 150.

The remaining option is the grand coalition of all three. For this coalition, the best outcome will be a choice between flying in one plane or the other, or not flying at all, with the valuation as the maximum of $\{130 + 150,\ 185 + 85,\ 85 + 150\}$. The latter, $85 + 150$, is what would be gained by not flying at all.

Notice that the maximum valuation of 280 is the same as in the case when the A's and B's were not vertically integrated.

Many solutions are possible that will satisfy all of these inequalities and so the core is nonempty in this case; one of them is the triplet $y_1 = 115$, $y_2 = 150$ and $y_3 = 15$.[4] This solution is based on a particular allocation of the property rights, but with three different buyers and the two airlines, there are actually five other allocations: B_1 owns A_2 and B_2 owns A_1; B_2 owns A_1 and B_3 owns A_2; B_2 owns A_2 and B_3 owns A_1; B_1 owns A_1 and B_3 owns A_2; B_1 owns A_2 and B_3 owns A_1. Allowing one buyer to own both airlines would provide three more allocations but with less competition than the six above. It is a general result that if any allocation of the property gives a nonempty core, then all will do so, although the imputation of the gains differs in each. Because the efficiency of the outcome does not depend on the allocation of the property, one may take this as an illustration of the Coase "theorem."[5]

Vertical integration gives a nonempty core in this example by eliminating certain potential contracts. These would confer gains on those who participate in them but such contracts would prevent the Pareto-optimal outcome. Ownership of an airline by a particular buyer means that the pair involved always operates together in a potential contract with other participants. Therefore, all the earlier constraints involving an x disappear, owing to the vertical integration. Only those constraints involving y's remain and there is a suitable adjustment of the lower bounds on the returns for the coalitions giving rise to these constraints.

The original situation in which the buyers and sellers are separate allows the most leeway for opportunistic behavior to the individuals. Vertical integration by joining certain pairs of buyers and sellers forces them to recognize their common interest and reduces their incentive to take temporary advantage of each other. Core theory shows this formally, when there is vertical integration, by removing some of the constraints from the original situation. However, this does not answer the question of how the members of a vertically integrated pair will divide their gain between them. Since divorce is possible because of disagreement, the situation can revert back to the empty core. Nor is this all.

[4]In particular, these are the relevant inequalities:

$$y_1 \geq 85;\ y_2 \geq 150;\ y_3 \geq 0;\ y_1 + y_2 \geq 265;\ y_1 + y_3 \geq 125;$$

$$y_2 + y_3 \geq 150 \quad \text{and} \quad y_1 + y_2 + y_3 \leq 280.$$

[5]The Coase "theorem" needs much repair when there is an empty core. Aivazian and Callen (1981) give an example with three firms: two factories that pollute the third, a laundry. They show there is an empty core because there is no undominated allocation of the gains among the three. Coase's elaborate analysis in his comment (1981) fails to come to grips with the issues raised by this example. Also, the allocation of the gains that Coase proposes is dominated by every 2-person coalition. Coase's most recent and authoritative exposition of his "theorem" (1988) is silent on the challenge posed by an empty core, although Chapter 6 discusses several criticisms of the theorem.

Even if the constituents in a vertical integration can reach amicable settlements, vertical integration cannot always resolve an empty core problem.

A change in the preceding example shows this. Say there is a third seller, A_3, with a capacity of 2 and an avoidable cost of 90. Also, let there be a fourth buyer, B_4, willing to pay at most 34. The core is empty with the addition of these two individuals. Vertical integration would not change the empty to a nonempty core, for instance, if B_4 integrates with A_3. The emptiness of the core in this new example is not affected by shuffling the property among the buyers (as the reader should be able to verify).

The original version of the airline coalition story can also illustrate how the gains to the participants change as the size of the coalition expands. To this end —and to show how the definition of the return to a coalition is malleable and therefore useful for many different purposes—now define the gain to a coalition as the total value of what they can produce, which is measured by what people are willing to pay for it, minus the cost of producing it. Assume vertical integration in the form that B_1 owns A_1 and B_2 owns A_2. Since no single passenger can cover the cost of a trip, no coalitions made up of a single member will form. Or to put it another way, the singleton coalitions can always decide to do nothing and each guarantees a gain of zero.

There are three possible pair coalitions. The coalition between B_1 and B_2 will either choose not to fly at all, for a gain of zero, or to pay 55 + 60 to fly before subtracting 85 in costs for a gain of 30. Similarly, the coalition between B_1 and B_3 will either not fly at all for a gain of zero, or pay 55 + 70 to fly, minus a cost of 85, for a gain of 40. Finally, the coalition between B_2 and B_3 will either not fly for a gain of zero or will fly the large plane, only two-thirds full, paying 130 with costs of 150. Notice that as this example moves from singleton to pair coalitions, the incremental gain is indeterminate because it depends on who joins whom.

The grand coalition of the three will choose the best of several possibilities. First, it may choose not to fly at all for a gain of zero. Second, it may choose to put the two highest-paying passengers in the small plane, for a gain of 130-85. Or it may choose to put all three passengers in the largest plane, for a gain of 185-150. The second alternative offers the biggest gain of 45, so that will be chosen.

Notice that adding B_1 to a coalition of B_2 and B_3 raises the gain from zero to 45, while adding B_2 to a coalition of B_1 and B_3 raises the gain only from 40 to 45 and adding B_3 to a coalition of B_1 and B_2 raises the gain from 30 to 45. It is a general proposition that when the core is not empty, the return to each person cannot exceed that person's incremental contribution to the grand coalition. In this case, the return to B_1 cannot exceed 45, to B_2 the upper bound is 5 and to B_3 it is 15. These upper bounds are a good starting point to see whether there is an imputation of the gains in the core. Thus, try y_2 at its upper bound, which is 5, and y_3 at its upper bound, or 15. The sum of the gains must equal 45 because it is the maximum available to the grand coalition.

Then for $y_2 = 5$ and $y_3 = 15$, y_1 must be 25 (= 45 − 5 − 15). This gain for B_1 is admissible because it does not exceed the upper bound, which is 45. The triplet {25, 5, 15} is in the core because it satisfies all the inequalities required by a nonempty core. It is an extreme imputation of the gains because it gives the largest possible amount consistent with the core to two of the three actors in this market. Another imputation in the core is at the other extreme where B_1 gets 45, the upper bound, and the other two get zero. Thus {45, 0, 0} is also in the core and is the most favorable imputation for B_1. The theory does not determine which of these imputations or certain others in between will be chosen by the participants.

Conclusion

The existence of production introduces a number of complications into economic analysis. Set-up or avoidable costs are common, as illustrated in the airplane example, and coalitions will form to break down simple marginal cost in this case. There may be lower bounds on the scale of operation, as also illustrated in the airplane example. Continuous changes in output are often more expensive than discontinuous changes in discrete amounts, as in industries like auto assembly, electricity generation and railroad shipping. In addition, the least costly way to satisfy demand usually requires standby capacity available before the actual demand is known. This raises the problem of how to generate enough revenue to cover the cost of the stand-by capacity as well as the out-of-pocket costs of the demand that actually materializes. Both buyers and sellers have an interest in the resolution of these problems.

Core theory offers tools for confronting these challenges explicitly. It shows how opportunistic behavior by customers can destroy an efficient equilibrium and how suitable arrangements between customers and suppliers in the form of vertical integration can sometimes restore an efficient equilibrium.

Core theory has many other interesting uses in areas not discussed here (Telser, 1990). It gives new results that advance understanding of business organizations such as corporations. It explains why they have limited liability, fungible shares and how the nature of their investments relate to the preferences of their owners. Core theory also gives reasons other than the desire to pool risk for joint ventures such as mutual funds.

Rather than being treated as a somewhat arcane topic, suitable only for existence proofs in economic theory classes, the insights and examples of core theory should be brought into the undergraduate economics curriculum—and the tool-kit of every professional economist. Study and application of core theory will deepen understanding of how competition works in many situations ranging from organized markets to the matching of interns to hospitals (Roth and Sotomayor, 1990).

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