CME307/MS&E311 Suggested Course Project I: Sensor Network Localization

(You don't need to answer all the questions posted in the project)

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1 Introduction

Sensor Network Localization (SNL), also closely related to Data Dimension Reduction, Molecular Confirmation, Graph Realization, is one of major topics in Data Science. The SNL problem is: Given possible anchors $\mathbf{a}_k \in R^d$, distance information $d_{ij} \in N_x$, and $\hat{d}_{kj} \in N_a$, find $\mathbf{x}_i \in R^d$ for all i such that

$$\begin{aligned} \|\mathbf{x}_i - \mathbf{x}_j\|^2 &= d_{ij}^2, \ \forall\ (i,j) \in N_x,\ i < j, \\ \|\mathbf{a}_k - \mathbf{x}_j\|^2 &= \hat{d}_{kj}^2, \ \forall\ (k,j) \in N_a, \end{aligned} \tag{1}$$

where $(ij) \in N_x$ ($(kj) \in N_a$) connects points $\mathbf{x}_i$ and $\mathbf{x}_j$ ($\mathbf{a}_k$ and $\mathbf{x}_j$) with an edge whose Euclidean length is d_{ij} ($\hat{d}_{kj}$).

We establishes in class an SOCP relaxation for solving solve (1):: Find a symmetric matrix $\mathbf{x}_i$s such that

$$\begin{array}{ll} \min & \sum_i \mathbf{0}^T \mathbf{x}_i \\ \text{s.t.} & \|\mathbf{x}_i - \mathbf{x}_j\|^2 \le d_{ij}^2, \ \forall\ (i,j) \in N_x,\ i < j, \\ & \|\mathbf{a}_k - \mathbf{x}_j\|^2 \le \hat{d}_{kj}^2, \ \forall\ (k,j) \in N_a. \end{array} \tag{2}$$

We also establishes in class an SDP relaxation for solving solve (1):: Find a symmetric matrix $Z \in S^{d+n}$ such that

$$\begin{array}{ll} \min & \mathbf{0} \bullet Z \\ \text{s.t.} & Z_{1:d,1:d} = I, \\ & (\mathbf{0}; \mathbf{e}_i - \mathbf{e}_j)(\mathbf{0}; \mathbf{e}_i - \mathbf{e}_j)^T \bullet Z = d_{ij}^2, \ \forall\ i,j \in N_x,\ i < j, \\ & (\mathbf{a}_k; -\mathbf{e}_j)(\mathbf{a}_k; -\mathbf{e}_j)^T \bullet Z = \hat{d}_{kj}^2, \ \forall\ k,j \in N_a, \\ & Z \succeq \mathbf{0}. \end{array} \tag{3}$$

Note that $Z_{1:d,1:d} = I \in S^d$ can be realized through $d(d+1)/2$ linear equations.

There is a simple nonlinear least squares approach to solve (1):

$$\min \quad \sum_{(ij)\in N_x} \left(\|\mathbf{x}_i - \mathbf{x}_j\|^2 - d_{ij}^2\right)^2 + \sum_{(kj)\in N_a} \left(\|\mathbf{a}_k - \mathbf{x}_j\|^2 - d_{kj}^2\right)^2 \tag{4}$$

which is an unconstrained nonlinear minimization problem.

Question 1: Run some randomly generated problems (in 1D, 2D and 3D) with few (2, 3 and 4) anchors and tens sensors to compare the three approaches.

2 New Problems and Approaches

Here we describe some possible new types of data and problems in solving in SNL, which is a key topic in Internet of Things (IOT).

2.1 SNL with Noise Data

In practical problems, there are often noises in the distance information. To deal with possible noises, the SDP relaxation approach (3) can be modified to minimize the L_1 norm of the errors:

$$\begin{array}{ll} \min & \sum_{ij\in N_x}(\delta'_{ij} + \delta''_{ij}) + \sum_{kj\in N_a}(\hat{\delta}'_{kj} + \hat{\delta}''_{kj}) \\ \text{s.t.} & Z_{1:d,1:d} = I, \\ & (\mathbf{0}; \mathbf{e}_i - \mathbf{e}_j)(\mathbf{0}; \mathbf{e}_i - \mathbf{e}_j)^T \bullet Z + \delta'_{ij} - \delta''_{ij} = d_{ij}^2, \ \forall\, i,j \in N_x,\ i<j, \\ & (\mathbf{a}_k; -\mathbf{e}_j)(\mathbf{a}_k; -\mathbf{e}_j)^T \bullet Z + \hat{\delta}'_{kj} - \hat{\delta}''_{kj} = \hat{d}_{kj}^2, \ \forall\, k,j \in N_a, \\ & Z \succeq \mathbf{0}. \end{array} \tag{5}$$

The SDP solution from the relaxation

$$\bar{Z} = \begin{pmatrix} I & \bar{X} \\ \bar{X}^T & \bar{Y} \end{pmatrix}$$

often may not be rank d so that $\bar{X} \in R^{d\times n}$ cannot be the best possible localization of the n sensors.

Question 2: Use the SDP solution $\bar{X} = [\bar{\mathbf{x}}_1,\ \bar{\mathbf{x}}_2,\ \ldots.,\ \bar{\mathbf{x}}_n]$ as the initial solution for approach (4) and apply the Steepest Descent Method for a number steps. How is the final solution come out after steepest descent?

2.2 SNL with TDOA Data

Question 3: It may be easier/faster to obtain the distance-difference information using the Time Difference of Arrivals (TDOA) technology, that is, d_{ij} is individually unknown but $t_{ijk} = d_{ij} - d_{jk}$ is known. This technology is popular using sound signals. Formulate a relaxation model for SNL to tackle the problem, and run numerical experiments on one sensor and few anchors. The key using TDOA is to formulate the relation of d_{ij} and d_{ij}^2.

2.3 SNL with Pathlength Data

Question 4: Similarly, some times, d_{ij} is individually unknown but the path length may be estimated using signal relays. That is, for some sensors $i_1, i_2, ..., i_p$, $d_{i_1 i_2} + d_{i_2 i_3} + ... + d_{i_{p-1} i_p}$ is known. Formulate a relaxation model for SNL to tackle the problem, and run numerical experiments on few sensor and few anchors.

2.4 SNL with Angle/Obstacle Data

Question 5: In most real circumstances, besides distance information, some geographical data such as the angle between two sensors from an anchor point, polyhedron where sensors can only be in, etc. How to incorporate these pieces of information into SNL?

3 First-Order Method for Sensor Network Localization

Unfortunately, the current available SDP solvers are still too time consuming in solving large-scale SDP problems. In this part, you are asked to implement one of first-order SDP methods described in class to solve the SDP relaxation problem for SNL.

References

[1] P. Biswas and Y. Ye, "Semidefinite programming for ad hoc wireless sensor network localization," in *Proceedings of the third international symposium on Information processing in sensor networks*, ACM Press, 2004, pp. 46–54.

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[4] L. Doherty, L. E. Ghaoui, and K. S. J. Pister, "Convex position estimation in wireless sensor networks." in *Proceedings of IEEE Infocom*, Anchorage, Alaska, April 2001, pp. 1655 –1663.

[5] Mingjie Gao, Ka-Fai Cedric Yiu, Sven Nordholm, Y. Ye, "On a New SDP-SOCP Method for Acoustic Source Localization Problem," ACM Transactions on Sensor Networks (TOSN), 12(4), 2016.

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