Pricing Parimutuel Digital Call Auction

The Game we played on the NSF super-ball is called Parimutuel Digital Call Auction (PDCA), where Goldman Sachs and Deutsche Bank jointly launched a series of options on economic data and were jointly marketing the products, which are based on the solution technology developed by US financial technology firm Longitude.

Parimutuel principles are widely used as an alternative to fixed odds gambling in which a bookmaker acts as a dealer by quoting fixed rates of return on specified wagers. A parimutuel game is conducted as a call auction in which odds are allowed to fluctuate during the betting period until the betting period is closed or the auction "called." The prices or odds of wagers are set based upon the relative amounts wagered on each risky outcome.

The following is a mathematical description of the problem. First, there are m states that are mutually exclusive and exactly one of them will be true at the maturity. A contract on a state is a paper agreement so that on maturity it is worth a pre-determined notional $\$w$ if it is on the winning state and worth \$0 if is not on the winning state. There are n orders betting on one or a subset of states and each order has a price limit and a quantity limit. For example, the jth order is given as $(\mathbf{a}_j \in R^m_+,\ \pi_j \in R_+,\ q_j \in R_+)$, where $\mathbf{a}_j$ is the combination betting vector where each component is either 1 or 0

$$\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ ... \\ a_{mj} \end{pmatrix},$$

where 1 is winning state bet and 0 is non-winning state bet; π_j is the price limit for one such a contract, and q_j is the maximum number of contracts the bidder like to have.

Let x_j, $j = 1, ..., n$, be the number of contracts awarded to the jth bidder, and the Auction Settling Price (ASP) of each state be p_i, $i = 1, ..., m$. Then, the jth bidder will pay the amount between $q_j x_j$ and $\mathbf{p}^T\mathbf{a}_j \cdot x_j$. In this project, you would find "good" and/or "fair" pricing models to decide p_i.

- Do literature search and understand the state of art models and technologies in solving the problem. I have attached few. You may also go to http://www.gs.com/econderivs/ or http://www.exchangerepublic.com/ or http://www.stanford.edu/ yyye/cpcam-ec.pdf to learn more of the game.

- There are several possible models to price the auction. Compare them and find what differences are among them. Write down the complementarity condition of each model, and show how the optimal dual solution $\mathbf{p}^*$ satisfies the "fairness" conditions.
- Implement different models using Excel or other solvers, and conduct simulation experiments using our NSF data to support your findings.

Your grade of the project is based on your project report due March 17.